GUJARAT TECHNOLOGICAL UNIVERSITY

BASIC ELECTRONICS (Modified Version January 2014)
SUBJECT CODE: 2110016
B.E. 1st YEAR

Type of course: Basic

Prerequisite: N.A.

Rationale: N.A.

Teaching and Examination Scheme:

Teaching Scheme			Credits	Examination Marks				Total Marks
L	T	P	C	Theory Marks		Practical Marks		
				ESE (E)	PA (M)	ESE Viva (V)	PA (I)	
4	0	2	6	70	30*	30	20**	150

L- Lectures; T- Tutorial/Teacher Guided Student Activity; P- Practical; C- Credit; ESE- End Semester Examination; PA- Progressive Assessment

Content:

Sr. No.	Topic	Teaching Hrs.
1	Circuit Concepts • Electrical Quantities • Lumped Circuit Elements • Kirchhoff's Laws • Meters and Measurements • Analogy between Electrical and other Non-Electrical Physical Systems • A case study	6
2	Circuit Analysis Techniques: • Thevenin and Norton Equivalent Circuits • Node-Voltage and Mesh-Current Analysis • Superposition and Linearity • Wye-Delta Transformation • Computer Aided Circuit Analysis • A Case Study	8
3	Analog Building Blocks and Operational Amplifiers Basic ideas • The Amplifier Block • Ideal Operational Amplifier • Practical Properties of Operational Amplifiers • Applications of Operational Amplifiers • A case study	8
4	Digital Building Blocks • Digital System Building Blocks	8

	• Digital System Components • Computer Systems • Computer Networks • A case study	
5	Signal Processing: • Signals and Spectral Analysis • Modulation, Sampling and Multiplexing • Interference and Noise • A case Study	8
6	Communication Systems • Waves, Transmission Lines, waveguides and Antenna Fundamentals • Analog Communication Systems • Digital Communication Systems • A Case Study	6
7	Basic Control Systems • Feedback Control Systems • Digital Control Systems • A Case Study	6

Reference Books:

1. Introduction to Electrical Engineering, M S Sarma, Oxford University Press

Course Outcome:

After completion of the course, the student will be able to –

- Determine the behavior of simple passive electrical circuits with independent voltage and current sources.
- Design simple analog signal processing functions using operational amplifiers.
- Design simple combinational and sequential functions using gates and flip-flops.
- Explain the functioning of digital system components including DACs, ADCs, memory and display devices,
- Explain the organization of computer systems and computer networks.
- Determine the properties of simple signal processing systems.
- Determine the behavior of analog and digital communication systems.
- Determine the behavior of simple linear feedback control systems

List of Experiments:

The practical/exercises should be properly designed and implemented with an attempt to develop different types of skills so that students are able to acquire the competency. Following is the list of experiments for guidance. A student should perform at least 10 experiments out of the given 14 with at least one experiment from each Unit

S. No.	Unit No.	Practical/Exercise	Apprx. Hrs. Required
1	I	a. Observe the behavior of RLC circuits with ideal and non-ideal voltage sources and current sources. b. Verify Thevenin's and Norton's Theorems	2
2	II	Simulate passive electrical circuits using Multisim simulator and compare the simulated response with that of the actual circuit	2
3	III	Determine the parameters of three commercial Op Amps	2
4	III	Perform simple analog signal processing functions using Op Amps	2
5	IV	Design simple combinational functions as per specifications and verify the correctness of your design	2
6	IV	Design simple sequential functions as per specifications and verify the correctness of your design	2
7	IV	Measure the characteristics of given DACs and ADCs	2
8	V	Simulate simple modulation, sampling, multiplexing, demodulation signal processing functions Multisim	2
9	V	Simulate simple filtering signal processing function Multisim	2
10	V	Measure the performance of a given signal processing system	2
11	VI	Determine the behavior of a given analog communication system through simulation using Multisim	2
12	VI	Determine the behavior of a given digital communication system through simulation using Multisim	2
13	VII	Determine the behavior of a second and third order control systems through simulation using Multisim	2
14	VII	Determine the behavior of a practical control system using ON-OFF and P controllers through simulation using SciLab Determine the behavior of a practical control system using PI and PID controllers through simulation using SciLab	2

Major Equipment's:

(1) CRO (At least 20MHz)

(2) Function Generator (Frequency range up to 20 MHz) – need to have sine, square wave output.

(3) Dual Power Supply (0-12V/15V DC)/3A

(4) Micrometers for measurement of voltage and current with suitable ranges.

(5) Multimeter

(6) Various Electronics Components including different types of Op Amps and digital ICs.

(7) PCs

List of Open Source Software/learning website:

Software: Multisim and SciLab *(*www.scilab.org*)*
Learning Material: http://nptel.iitm.ac.in/. www.spoken-tutotial.org

*PA (M): 10 marks for Active Learning Assignments, 20 marks for other methods of PA;

ACTIVE LEARNING ASSIGNMENTS: Students will prepare power-point slides, which include videos, animations, pictures, graphics for better understanding theory and practical work – The faculty will allocate chapters/ parts of chapters to groups of students so that the entire syllabus of Basic Electronics is covered. The power-point slides should be put up on the web-site of the College/ Institute, along with the names of the students of the group, the name of the faculty, Department and College on the first slide. The best three works should be sent to achievements@gtu.edu.in.

**PA (I): The faculty may also allocate additional marks out of PA for Practical Work, in addition to the regular practicals. This work may be as follows:

(i) Practical work planned by students, with the approval/ guidance of the students.
(j) Preparation of a case study on a present day electronic system of choice.

CHAPTER 1
CIRCUIT CONCEPTS

TOPICS COVERED IN THIS CHAPTER

1. **Introduction**
2. **Various Electrical Quantities**
3. **Lumped Circuit Elements**
4. **Kirchhoff's Laws**
5. **Meters and Measurements**
6. **Analogy between Electrical and Non-Electrical Physical Quantities**
7. **A Case Study – Resistance Strain Gauge**
8. **Questions**

1.1 INTRODUCTION:

- Electric circuits are collections of **circuit elements**. An electric circuit is an inter connection of simple electric devices that have at least one closed path in which current may flow.

- A **circuit element** is any individual circuit component which gets connected to the other components in a network.

- An electric circuit is an idealized mathematical model of some physical circuit. The ideal circuit elements are the resistor, the inductor, the capacitor and the voltage and current sources.

- Electrical engineering is concerned with the analysis and design of electrical circuits, systems and devices. In this chapter we will learn the fundamental concepts of all circuits.

1.2 ELECTRICAL QUANTITIES:

- To understand the operation of electric circuits we must familiar with electrical quantities such as charge, current and voltage.

1.2.1 Charge and Electric Force:

- There are two types of charges:
  1 Positive charge (+)
  2 Negative charge (-)

➢ **Conservation law of charges:**
- Charge can be neither created nor destroyed. It can be only transferred.
- The unit of charge is Coulomb. It is denoted by C.

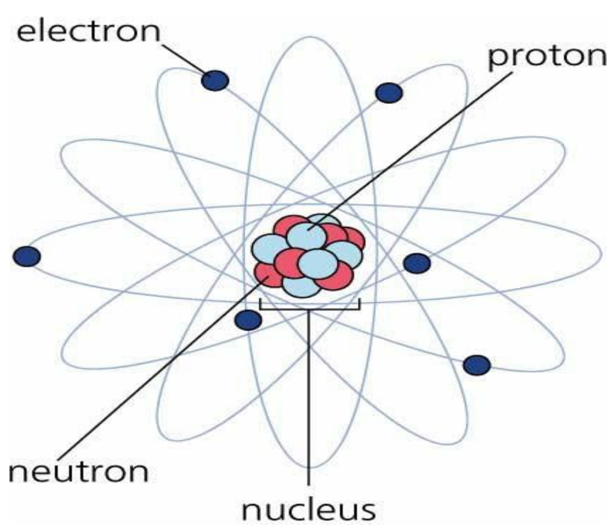


Fig. 1.1: Structure of the atom

- Each atoms consist of three types of particles.
  1 Protons
  - Protons are positive charge (+) and it has charge of + 1.6 X 10^{-19} coulombs (C).

  2 Electrons
  - Electrons are negative charge (-) and it has charge of – 1.6 X 10^{-19} coulombs (C).

  3 Neutron
  - The neutron has zero charge.

➢ **Coulomb's First Law:**
- *"Same kind of charges repel each other and different kind of charges attract each other".*

(a) **(b)**

Fig. 1.2: (a) Repulsion (b) Attraction

➢ **Coulomb's Second Law (Inverse Square Law):**
- *"The force between two charges Q_1 and Q_2 separated by a distance R is proportional to the product between the charges and inversely proportional to the square of distance between them".*
- Coulomb's law has given an expression to calculate the electric force in Newton (N) on one point charge by the other.

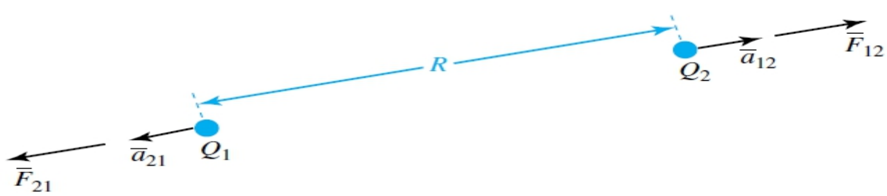


Fig. 1.3: Illustration of coulomb's law.

Force on Q_1 due to Q_2 → $\bar{F}_{21} = [Q_1Q_2/4\pi\varepsilon_0\varepsilon_r R^2]\ \bar{a}_{21} = [K\ Q_1Q_2/R^2]\ \bar{a}_{21}$

Force on Q_2 due to Q_1 → $\bar{F}_{12} = [Q_1Q_2/4\ \pi\varepsilon_0\varepsilon_r R^2]\ \bar{a}_{12} = [K\ Q_1Q_2/R^2]\ \bar{a}_{12}$

Where,

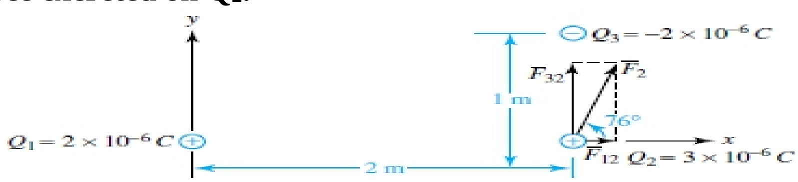
F = Force between two charges in N.

$K = 1 / 4\pi\varepsilon_0\varepsilon_r$

Q_1, Q_2 = Point charges in C.
ε_0 = Permittivity of the free space medium (8.854 X 10^{-12} F/m).
ε_r = Relative permittivity of the medium (for air ε_r = 1).
a_{12} and a_{21} are unit vectors along the line joining Q1 and Q2.
R = Distance between two charges in meter.

- Forces $\bar{F}_{21}$ and $\bar{F}_{12}$ are due to Q_2 and Q_1. They are equal in magnitude and opposite of each other in direction.
- The magnitude of the force is proportional to the product of the charge magnitudes.
- The magnitude of the force is inversely proportional to the square of the distance between the charges.
- The magnitude of the force depends on the medium.
- The direction of the force is along the line joining the charges.

Ex 1: (A) A small region of an impure silicon crystal with dimensions 1.25 × 10^{-6} m ×10^{-3} m ×10^{-3} m has only the ions (with charge +1.6 X 10^{-19} C) present with a volume density of 10^{25}/m^3. The rest of the crystal volume contains equal densities of electrons (with charge −1.6 × 10^{-19} C) and positive ions. Find the net total charge of the crystal.
(B) Consider the charge of part (A) as a point charge Q_1. Determine the force exerted by this on a charge Q_2 = 3μC when the charges are separated by a distance of 2 m in free space as shown in figure.
(C) If the another charge Q_3 = - 2μC is added to the system 1 m above Q_2, as shown in figure, calculate the force excreted on Q_2.



Solution:

- In the region where both ions and free electrons exist, their opposite charge cancel. So the net charge density is zero.
- The volume charge density = volume × charge

$$\rho = (10^{25})(1.6 \times 10^{-19}) = 1.6 \times 10^{6} \text{ C/m}^3$$

- The net total charge is then calculated as:

$$Q = \rho \times v = (1.6 \times 10^{6})(1.25 \times 10^{-6} \times 10^{-3} \times 10^{-3}) = 2 \times 10^{-6} \text{ C}$$

- The force that Q_1 exerts on Q_2 is in the positive direction of x.
- Now as we know that;

$$F = Q_1Q_2 / 4\pi\varepsilon_0\varepsilon_r R^2$$

Where; ε_0 = 8.854 X $^{10-}$12 F/m = 10^{-9} / 36π and εr = 1 (Relative permittivity of air)

$$\bar{F}_{12} = \frac{(3 \times 10^{-6})(2 \times 10^{-6})}{4\pi(10^{-9}/36\pi)2^2}\,\bar{a}_x = \bar{a}_x\, 13.5 \times 10^{-3}\,\text{N}$$

- The force that Q_3 exerts on Q_2 is in the positive direction of y.

$$\bar{F}_{32} = \frac{(3 \times 10^{-6})(-2 \times 10^{-6})}{4\pi(10^{-9}/36\pi)1^2}(-\bar{a}_y) = \bar{a}_y\, 54 \times 10^{-3}\,\text{N}$$

- The resultant force F_2 acting on Q_2 is in the superposition of F_{12} and F_{32} due to Q_1 and Q_3 respectively.

$$\begin{aligned}\bar{F}_2 &= \sqrt{F_{12}^2 + F_{32}^2}\,\angle \tan^{-1}\frac{F_{32}}{\bar{F}_{12}} \\ &= \sqrt{13.5^2 + 54^2} \times 10^{-3}\,\angle \tan^{-1}\frac{54}{13.5} \\ &= 55.7 \times 10^{-3}\,\angle 76^\circ\,\text{N}\end{aligned}$$

1.2.2 Conductors, Insulators and Semi-Conductors:

- Total three types of materials are present,

1. Conductors:
- Materials through which charge can flows easily are called conductors.
- A conductor has a very low resistance to the flow of charge.
- Copper is used mostly for the conductive paths on electric circuit boards and for the fabrication of electric wires.
- Examples: Silver, gold, copper, aluminum and such metals.

2. Insulators:
- Materials that do not allow charge to move easily. Electric current cannot be made to flow through it.
- An insulator has very high resistance to the flow of charge.
- Insulating materials wrapped around the conducting core of the wire.
- Examples: Glass, Plastic, Ceramics, Rubber etc.

3. Semi-Conductors:
- Semiconductors conductivity lies between conductors and insulators.
- Semiconductor has moderate resistance to the flow of charge.
- Examples: Silicon, Germanium, Gallium, Arsenide etc.

1.2.3 Current and Magnetic Force:

➢ **Current:**
- Rate of flow of electrons is known as current.

$$I = Q / t$$

Where,

I = Current, Q = Charge, t = Time

- If charge varies with time, so

$$i = dq / dt$$

Where,

  dq = change in charge
  dt = change in time

- Unit of current is Ampere. It is denoted as A.
- 1 A = 1 C/sec
- 1 Coulomb of charge is the charge carried by 6.25 X 10^{18} electrons.
- 1 mA = 10^{-3} A and 1 μA = 10^{-6} A

➢ **Magnetic Force:**

- ***Ampere's law of force:***
- It is concerned with magnetic forces associated with two loops of wire carrying currents.

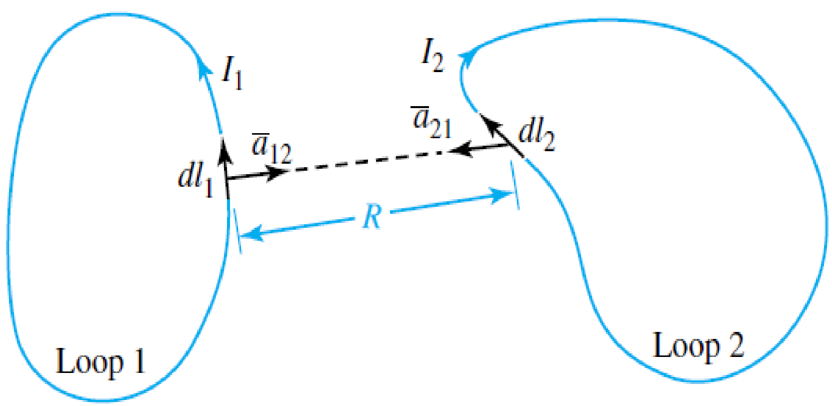


Fig. 1.4 Illustration of ampere's law of force

- It shows two loops of wire in free space carrying currents I_1 and I_2.Considering a differential element dl_1 of loop 1 and a differential element dl_2 of loop 2.
- The differential magnetic force by the differential current element is

$$d\bar{F}_{21} = I_1\, d\bar{l}_1 \times \frac{\mu_0}{4\pi R^2} [\, I_2\, d\bar{l}_2 \times \bar{a}_{21} \,]$$

$$d\bar{F}_{12} = I_2\, d\bar{l}_2 \times \frac{\mu_0}{4\pi R^2} [\, I_2\, d\bar{l}_2 \times \bar{a}_{12} \,]$$

- Where $\bar{a}_{21}$ and $\bar{a}_{12}$ are unit vectors along the line joining the two current elements, R is the distance between the centers of the elements, μ_0 is the permeability of free space.
- The magnitude of the force is proportional to the product of the two currents and the product of the lengths of the two current elements.
- The magnitude of the force is inversely proportional to the square of the distance between the current elements.
- Each current element is acted upon by a magnetic field due to the other current element,

$$d\bar{F}_{21} = I_1\, d\bar{l}_1 \times B_2$$

$$d\bar{F}_{12} = I_2\, d\bar{l}_2 \times B_1$$

- Where B is known as the magnetic flux density with unit of N/A·m, Webers per square meter (Wb/m^2) or tesla (T).
- Current distribution is the source of magnetic field, just as charge distribution is the source of electric field.

$$\bar{B}_2 = \frac{\mu_0}{4\pi R^2}(I_2\, d\bar{l}_2 \times \bar{a}_{21})$$
$$\bar{B}_1 = \frac{\mu_0}{4\pi R^2}(I_1\, d\bar{l}_1 \times \bar{a}_{12})$$

- This equation is known as the **Biot–Savart law.**

- ***Lorentz force equation:***
- Current is due to the flow of charges.
- I = dq/dt and $d\bar{l} = \bar{v}\, dt$, where v is the velocity.

$$d\bar{F} = \frac{dq}{dt}(\bar{v}\, dt) \times \bar{B} = dq\,(\bar{v} \times \bar{B})$$

- Thus the force F experienced by a charge q moving with a velocity v in a magnetic field of flux density B is given by

$$\bar{F} = q\,(\bar{v} \times \bar{B})$$

- Total force acting on a charge q moving with velocity v in a region characterized by electric field intensity E and a magnetic field of flux density B is

$$\bar{F} = \bar{F}_E + \bar{F}_M = q\,(\bar{E} + \bar{v} \times \bar{B})$$

Ex 2: Consider a length of 10^{-6} m of wire whose center is located at the point (1, 0, 0), carrying a current of 2 A in the positive direction of x.
(A) Find the magnetic flux density due to the current element at the point (0, 2, 2).
(B) Let another current element of length 10^{-3} m be located at the point (0, 2, 2), carrying a current of 1 A in the direction of ($-\bar{a}_y + \bar{a}_z$). Evaluate the force on this current element due to the other element located at (1, 0, 0).

Solution:

(a) $I_1 d\bar{l}_1 = 2 \times 10^{-6}\bar{a}_x$. The unit vector $\bar{a}_{12}$ is given by

$$\bar{a}_{12} = \frac{(0-1)\bar{a}_x + (2-0)\bar{a}_y + (2-0)\bar{a}_z}{\sqrt{1^2+2^2+2^2}} = \frac{(-\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z)}{3}$$

Using the Biot–Savart law,

$$[\bar{B}_1]_{(0,2,2)} = \frac{\mu_0}{4\pi}\frac{I_1\, d\bar{l}_1 \times \bar{a}_{12}}{R^2}$$

where μ_0 is the free-space permeability constant given in SI units as $4\pi \times 10^{-7}$ H/m, and R^2 in this case is $\{(0-1)^2 + (2-0)^2 + (2-0)^2\}$, or 9. Hence,

$$[\bar{B}_1]_{(0,2,2)} = \frac{4\pi \times 10^{-7}}{4\pi}\left[\frac{(2\times 10^{-6}\bar{a}_x)\times(-\bar{a}_x+2\bar{a}_y+2\bar{a}_z)}{9\times 3}\right]$$
$$= \frac{10^{-7}}{27}\times 4\times 10^{-6}(\bar{a}_z-\bar{a}_y)\ \mathrm{Wb/m^2}$$
$$= 0.15\times 10^{-13}(\bar{a}_z-\bar{a}_y)\ \mathrm{T}$$

(b) $I_2\, d\bar{l}_2 = 10^{-3}(-\bar{a}_y+\bar{a}_z)$

$$d\bar{F}_{12} = I_2 d\bar{l}_2 \times \bar{B}_1$$
$$= \left[10^{-3}(-\bar{a}_y+\bar{a}_z)\right]\times\left[0.15\times 10^{-13}(\bar{a}_z-\bar{a}_y)\right] = 0$$

Note that the force is zero since the current element $I_2\, d\bar{l}_2$ and the field $\bar{B}_1$ due to $I_1\, d\bar{l}_1$ at (0, 2, 2) are in the same direction.

1.2.4 Definitions:

- **Unit Charge:**
- A charge of 1 coulomb is also called as unit charge.

- **Electric Field:**
- The region around a charged body where another charged body experiences a mechanical force.

- **Electric Flux:**
- Electric flux is defined as the number of lines of force in any particular electric field.
- It is measured in Coulomb (C).

- **Electric Flux Density (D) or Displacement Density:**
- It is defined as the flux per unit area measured in right angle to the direction of flux.
- D = Q/A.
- It is measured in Coulomb per meter square (C/m^2).

- **Electric Field Strength or Field Intensity (E):**
- It is defined as mechanical force experienced by a unit positive charge when it is placed at any point in the electric field.
- E = F/Q.
- It is measured in Newton per Coulomb (N/C).

1.2.5 Electric Potential and Voltage:

- ***Electric potential:***
- Electric potential is defined as Energy per unit charge. Potential is always reference to some point.
- When electrical forces act on a particle, it will possess potential energy.

Electric Potential = Work done / Charge **OR** V = W / Q

- Unit of electric potential is Joule / Coulomb (J/C) or Volt (V).

➢ ***One Volt:***

- A body is said to have a potential of 1 volt, if 1 joule of work is done to give it a charge of 1 coulomb.

➢ ***Voltage or Potential difference (P.D.):***

- Voltage is the difference in electric potential between two points.
- It is expressed in volts (V) or joules per coulomb (J/C).
- Consider two bodies A and B having potential 5 V and 1 V respectively as shown in fig. 1.5.

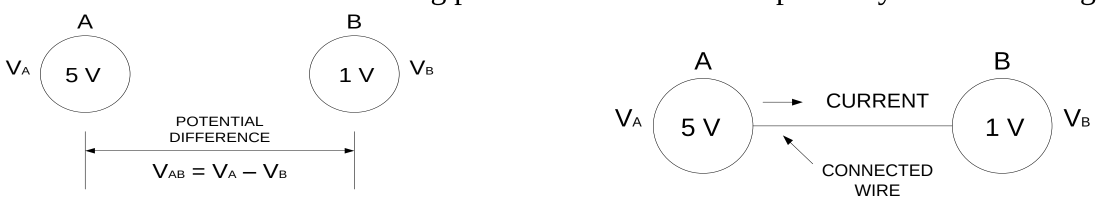


Fig. 1.5: Potential and potential difference

- So body A is at higher potential then B and the potential difference between A and B is given by V_{AB}.

$$V_{AB} = V_A - V_B = 5 - 1 = 4 \text{ V}.$$

- Voltages can be either positive or negative numbers, and it follows that $V_{BA} = -V_{AB}$.
- If $V_A > V_B$ → Positive potential difference.
- If $V_A < V_B$ → Negative potential difference.

➢ ***Electromotive Force (emf):***

- The force which causes current to flow in the circuit is called emf.
- It defined as E.
- Its unit is volt (V).

➢ ***Difference between emf and Potential Difference:***

emf	Potential Difference
emf means electromotive force due to which charge can flow in the circuit.	Potential difference means the difference between potential energy between two points in a circuit.
Unit is volt.	Unit is volt.

1.2.6 Energy and Power:

➢ ***Energy:***

- Total work done in an electric circuit is called electric energy.
- The energy over a time interval is found by integrating power,

$$w = \int_0^T p\, dt$$

$$E = P \times t$$

- Energy is expressed in watt-seconds or joules (J).
- It is expressed in electric utility bills in kilo watt hours (kWh).1 kWh = 3.6×10^6 J.

➢ ***Power:***

- Power is defined as the rate of doing work or the rate of change of energy dw/dt.
- Power = Work done / Time.

$$P = I^2R = V^2/R$$

- Unit of power is Watt. or J/Sec or Volt Ampere.
- 1 kWh = 1 kW x 1 hr
  = 10^3 x 60 x 60 sec
  = 36 X 10^5 Watt Sec
  = 36 X 10^5 J.

Ex 3: A typical 12-V automobile battery, storing about 5 mega joules (MJ) of energy, is connected to a4-A headlight system.
(a) Find the power delivered to the headlight system.
(b) Calculate the energy consumed in 1 hour of operation.
(c) Express the auto-battery capacity in ampere-hours (Ah)
(d) compute how long the headlight system can be operated before the battery is completely discharged.

Solution:

(a) Power delivered: P = V I = 12×4 = 48 W

(b) Assuming V and I remain constant, the energy consumed in 1 hour will equal

$$E = P \times t = 48(60 \times 60) = 172.8 \times 10^3 J = 172.8\ kJ$$

(c) 1 Ah = (1C/s) (3600 s) = 3600 C, So here;

$$Q = W / V = 5 \times 10^6\ J /12V = 0.417 \times 106\ C$$

Thus the auto-battery capacity is 0.417 × 106 / 3600 ∼=116 Ah

(d) 116Ah/4A = 29 hours the headlight system can be operated.

1.2.7 Source and Loads:

- A voltage rise indicates an electric source, with the charge being raised to a higher potential, whereas a voltage drop indicates a load, with a charge going to a lower potential.

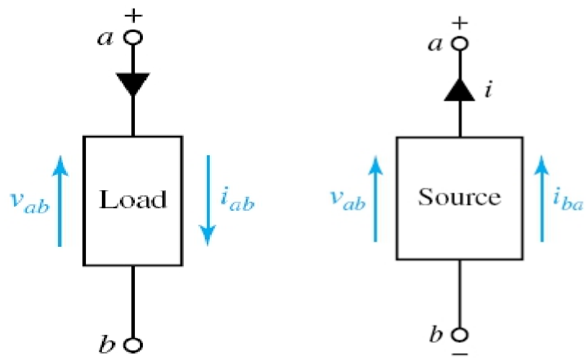


Fig. 1.6: Source and Load

- The voltage across the source is the same as the voltage across the load.
- Source supplies energy and load absorbs energy.

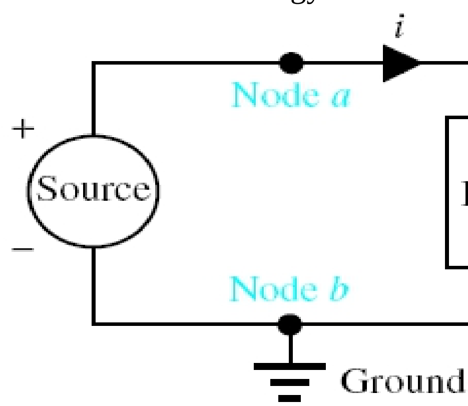


Fig. 1.7: Source–Load combination

- ***Node***: A node is a point at which two or more components or devices are connected together.

- ***Branch****:* part of a circuit containing only one component, source, or device between two nodes is known as a branch.

- When current flows out of the positive terminal of an electric source, it implies that nonelectric energy has been transformed into electric energy.
- Examples: **Generator Source** - mechanical energy → electric energy, **Battery Source** - chemical energy → electric energy, S**olar-Cell Source** - solar energy → electric energy.

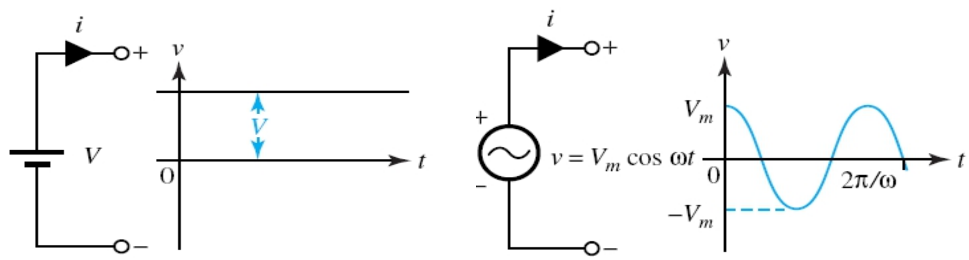
- When current flows in the direction of voltage drop, it implies that electric energy is transformed into nonelectric energy.
- Examples: **Electric Heater -** electric energy → thermal energy, **Motor Load -** electric energy mechanical energy, **Charging Battery** - electric energy → chemical energy.

1.2.8 Sources of Electrical Energy:

- There are two types of sources;

1. **Independent sources (Ideal sources):**

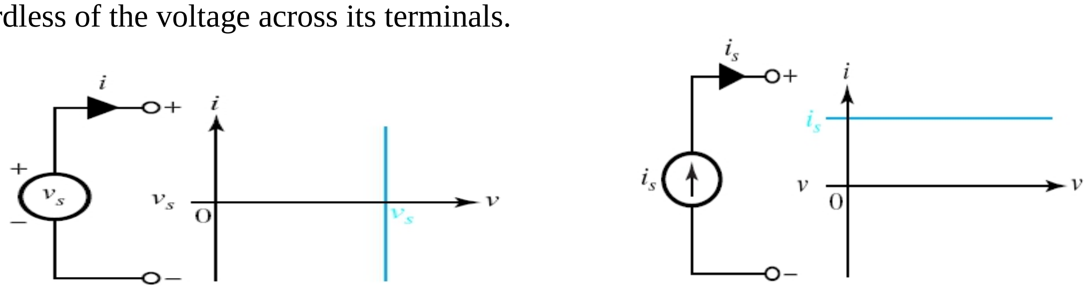
- When the source voltage or current is independent of all other voltages and currents, such sources are known as independent sources.
- An ideal voltage source is one whose terminal voltage V is a specified function of time, regardless of the current i through the source.
- An ideal battery has a constant voltage V with respect to time, is known as a dc source, because i = I is a direct current
- A sinusoidal voltage source V = Vm cosωt is produce an alternating current.



(a) **(b)**

Fig. 1.8: Voltage sources. (a) Ideal dc source (battery). (b) Ideal sinusoidal ac source.

- An ideal current source is defined as one whose current i is a specified function of time, regardless of the voltage across its terminals.



(a) **(b)**

Fig. 1.9: Circuit symbols and i–v curves. (a) Ideal voltage source. (b) Ideal current source

- Ideal sources could theoretically produce infinite energy, as infinite values are physically impossible.

2. **Dependent sources:**

- The sources whose voltage or current does depend on the value of some other voltage or current. Such sources are known as dependent sources or controlled sources.

(a) **(b)**

Fig. 1.10: (a) Dependent voltage source (b) Dependent current source

1.2.9 Waveforms:

- A waveform is a graph magnitude of a quantity with respect to time.
- The quantity plotted on x-axis is time and quantity is plotted on y-axis is voltage.
- Different types of waveforms are shown as below.

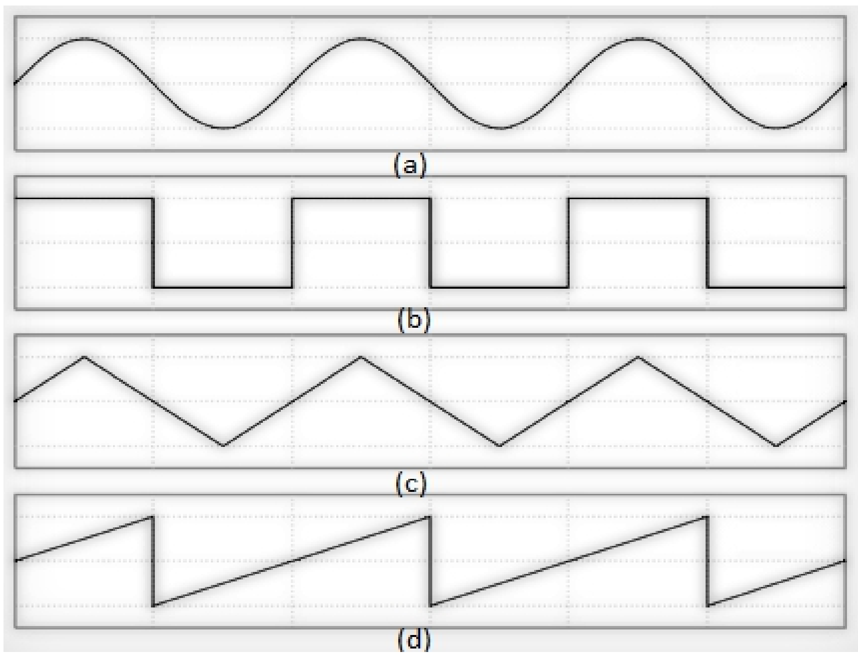


Fig. 1.11: (a) Sinusoidal wave (b) Square wave (C) Triangular wave (D) Sawtooth wave

- Sinusoidal waveform can be expressed as below,

$$f(t) = E_m \sin(\omega t + \varphi)$$

Where,

E_m = Amplitude
ωt = Angular frequency
φ = Phase angle

1. Instantaneous value:

- The value of an alternating quantity (voltage, current, power etc.) at any instant is called its instantaneous value.

2. Cycle:

- One complete set of positive and negative value of an alternating quantity is known as cycle.

3. Amplitude:

- The maximum value (positive or Negative) of an alternating quantity is known as its amplitude.

4. Time Period or Periodic Time (T):

- It is defined as the time taken in second by the any waveform of an ac quantity to complete one cycle.

$$T = 1 / f$$

5. Frequency:

- The number of cycle completed by an alternating quantity per second is known as frequency.

$$f = 1 / T$$

- Its unit is Hertz (Hz) or $(\text{Second})^{-1}$

6. Angular frequency:

- It is defined as below;

$$\omega = 2\pi f = 2\pi / T$$

7. Average value (F_{avg}):

- The average value of a periodic waveform f(t) is the net positive area under the curve of one period, divided by the period.

$$F_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

- The average value of $f(t) = E_m \sin(\omega t + \varphi)$ is as below;

$$F_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

$$F_{avg} = \frac{1}{T}\int_0^{2\pi} \text{Em} \sin(\omega t + \varphi)dt$$

$$F_{avg} = \frac{1}{2\pi} COS\,(1-1) = 0$$

8. RMS value (Effective value):

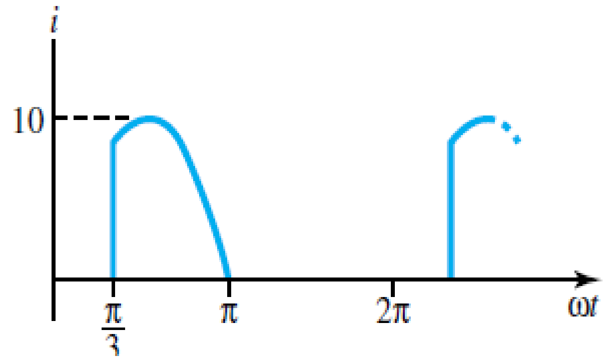
- RMS value of periodic waveform is equal to the square root of the average of $f^2(t)$.

$$F_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt}$$

- The average value of $f(t) = E_m \sin(\omega t + \varphi)$ is as below;

$$F_{rms} = \frac{A}{\sqrt{2}} = 0.707\ A$$

Ex 4: A periodic current waveform in a rectifier is shown in Figure. The wave is sinusoidal for $\pi/3 \leq \omega t \leq \pi$, and is zero for the rest of the cycle. Calculate the rms and average values of the current.



Solution:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi/3} i^2\, d(\omega t) + \int_{\pi/3}^{\pi} i^2\, d(\omega t) + \int_{\pi}^{2\pi} i^2\, d(\omega t)\right]}$$

Notice that ωt rather than t is chosen as the variable for convenience; $\omega = 2\pi f = 2\pi/T$; and integration is performed over three discrete intervals because of the discontinuous current function. Since $i = 0$ for $0 \leq \omega t < \pi/3$ and $\pi \leq \omega t \leq 2\pi$,

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\pi/3}^{\pi} 10^2 \sin^2 \omega t\, d(\omega t)} = 4.49 \text{ A}$$

$$I_{\text{av}} = \frac{1}{2\pi}\int_{\pi/3}^{\pi} 10 \sin \omega t\, d(\omega t) = 2.39 \text{ A}$$

1.3 LUMPED CIRCUIT ELEMENTS:

- Electric circuits or networks are formed by interconnecting various devices, sources, and components Although the effects of each element (such as heating effects, electric-field effects or magnetic-field effects) are distributed throughout space, one often lumps them together as lumped elements.
- Mainly two types of components are,
    1. **Passive components** (like R which is represent heating effect, L which is represent magnetic field, C which is represent electric field)
    2. **Active components** (like transistor, diode, transformer etc.)

1.3.1 Resistance (R):

- An ideal resistor is a circuit element with the property that the current through it is linearly proportional to the potential difference across its terminals,

$$\mathbf{i = v/R = G \times v} \text{ or } \mathbf{v = iR}$$

Which is known as **Ohm's law.**

- R is known as the resistance of the resistor with the SI unit of ohms (Ω), and G is the reciprocal of resistance called conductance, with the SI unit of siemens (S). The circuit symbols of fixed and variable resistors are shown in Fig. 1.12

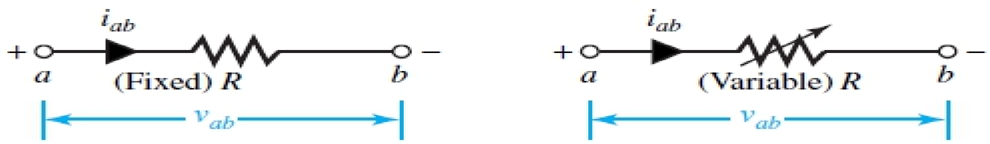


Fig. 1.12: Symbol of fixed & variable resistor

- Resistance is defined as,

$$\mathbf{R = \rho\, l / A = l / (\sigma A)}$$

Where,

ρ = Resistivity (Ω-m)
L = Length of conductor (m)
A = Cross-sectional area of conductor (m^2)

- The resistivity of conductor metals varies linearly over normal **operating temperatures** according to

$$\boldsymbol{\rho_{T2} = \rho_{T1}\, ((T_2 + T) / (T_1 + T))}$$

Where ρ_{T2} and ρ_{T1} are resistivities at temperatures T2 and T1, respectively, and T is a temperature constant that depends on the conductor material. All temperatures are in degrees Celsius.

- Resistance also depends on frequency, spiraling etc.
- The basic construction techniques of resistor are

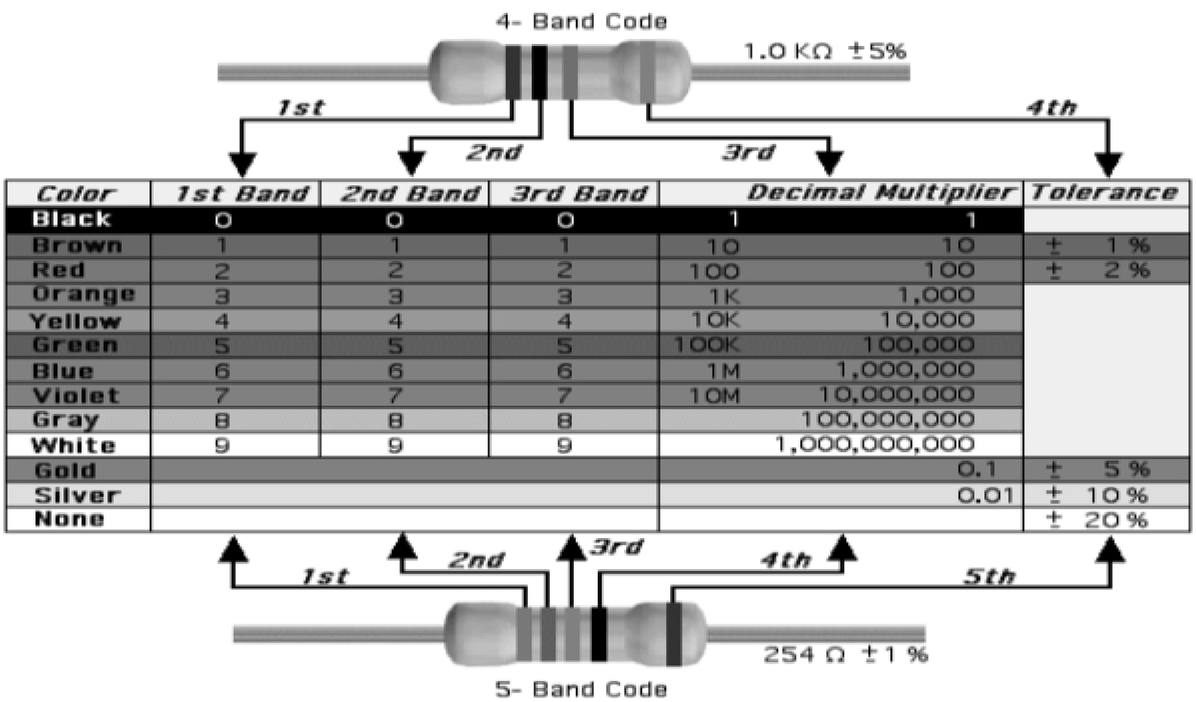
1. **Wire-wound resistors:**
   - Wire-wound resistors are commonly made by winding a metal wire, usually nichrome, around a ceramic, plastic, or fiberglass core. The ends of the wire are soldered or welded to two caps or rings, attached to the ends of the core.

2. **Metal-film resistors:**
   - Metal-film resistors are used when a higher tolerance (more accurate value) is needed. They are much more accurate in value than carbon film resistors. The metal-film resistor is used for bridge circuits, filter circuits and low-noise analog signal circuits.

3. Composition resistors:

- Carbon composition resistors consist of a solid cylindrical resistive element with embedded wire leads or metal end caps to which the lead wires are attached. The body of the resistor is protected with paint or plastic.

- Color-code band for evaluating resistance is as follow;



Color	1st Band	2nd Band	3rd Band	Decimal Multiplier		Tolerance
Black	0	0	0	1	1	
Brown	1	1	1	10	10	± 1 %
Red	2	2	2	100	100	± 2 %
Orange	3	3	3	1K	1,000	
Yellow	4	4	4	10K	10,000	
Green	5	5	5	100K	100,000	
Blue	6	6	6	1M	1,000,000	
Violet	7	7	7	10M	10,000,000	
Gray	8	8	8		100,000,000	
White	9	9	9		1,000,000,000	
Gold					0.1	± 5 %
Silver					0.01	± 10 %
None						± 20 %

Fig. 1.13 (A): Resistor color code chart

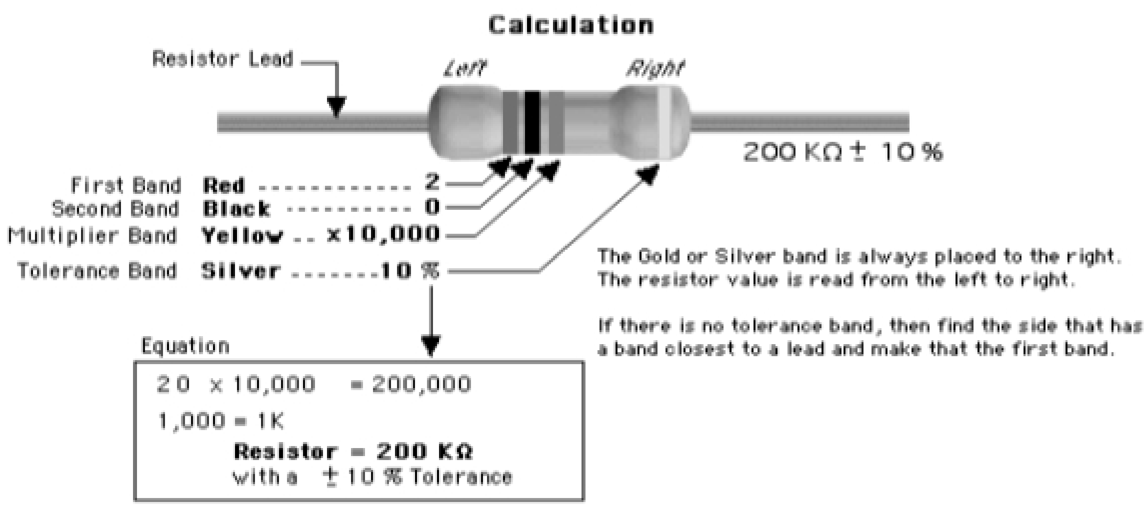


Fig. 1.13 (B): Calculation of Resistor value from color code chart

- Typical power ratings of resistors are 1, 1/2, 1/8, 1/4, 2, 5 up to 1000W.
- Instantaneous power is defined as;

$$\mathbf{P(t) = v(t)i(t) = i^2R = v^2/R = v^2G}$$

- Average power is defined as;

$$\mathbf{Pav = VrmsIrms = i^2rmsR = V^2rms/R = V^2rmsG}$$

- **Series parallel combination of resistors:**

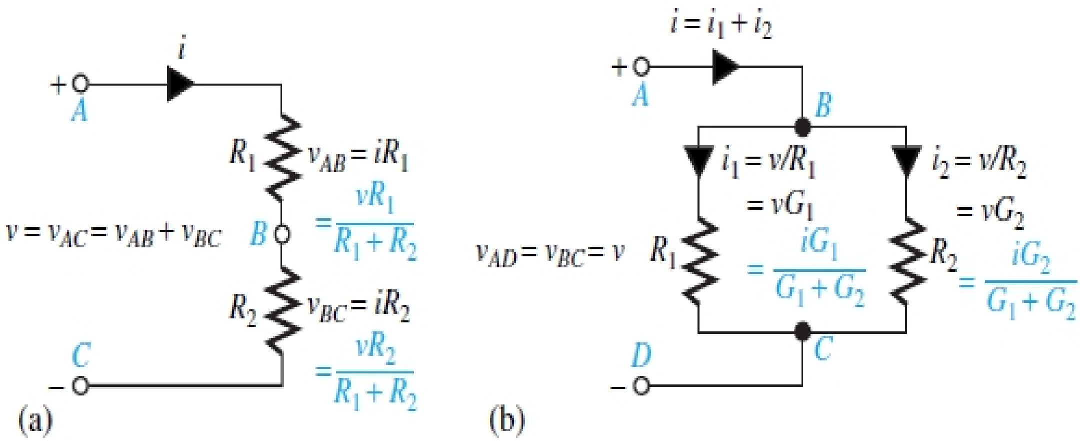


Fig. 1.14: (a) Series combination (b) Parallel combination

- $V = V_{AC} = V_{AB} + V_{BC} = iR_1 + iR_2 = i(R_1 + R_2) = iR_{eq}$
  When R_1 and R_2 are in **series**,

$$\mathbf{Req = R_1 + R_2}$$

- Fig. 1.14(b) shows two resistors in parallel sharing the current i in inverse proportion to their values, while the same voltage v is applied across each of them. At node B,
- $i = i_1 + i_2 = v/R_1 + v/R_2 = v\ (1/R_1 + 1/R_2) = v\ /\ (R_1R_2\ /\ R_1 + R_2) = v\ /\ R_{eq}$
  When R1 and R2 are in **parallel**,

$$\mathbf{R_{eq} = (R_1R_2)\ /\ (R_1 + R_2)}$$

Ex 5: A no. 14 gauge copper wire, commonly used in extension cords, has a circular wire diameter of 64.1 mils, where 1 mil = 0.001 inch.
(a) Determine the resistance of a 100-ft-long wire at 20°C.
(b) If such a 2-wire system is connected to a 110-V (rms) residential source outlet in order to power a household appliance drawing a current of 1 A (rms), find the rms voltage at the load terminals.
(c) Compute the power dissipated due to the extension cord.
(d) Repeat part (a) at 50°C, given that the temperature constant for copper is 241.5°C.

Solution:

(a) $d = 64.1$ mils $= 64.1 \times 10^{-3}$ in $= 64.1 \times 10^{-3} \times 2.54$ cm/1 in $\times$ 1 m/100 cm $= 1.628 \times 10^{-3}$ m. ρ of copper at 20°C is 17×10^{-9} m,

$$l = 100 \text{ ft} = 100 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} = 30.48 \text{ m}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (1.628 \times 10^{-3})^2}{4} = 2.08 \times 10^{-6} \text{ m}^2$$

$$R_{20°\text{C}} = \frac{17 \times 10^{-9} \times 30.48}{2.08 \times 10^{-6}} \cong 0.25\ \Omega$$

(b) Rms voltage at load terminals, V = 110 − (0.25)2 = 109.5 V (rms). Note that two 100-ft-long wires are needed for the power to be supplied.

(c) Power dissipated, $P = (1)^2(0.25)(2) = 0.5$ W.

(d)

$$\rho_{50°\text{C}} = \rho_{20°\text{C}} \left(\frac{50 + 241.5}{20 + 241.5} \right) = \frac{17 \times 10^{-9} \times 291.5}{261.5} = 18.95 \times 10^{-9}\ \Omega \cdot \text{m}$$

Hence,

$$R_{50°\text{C}} = \frac{18.95 \times 10^{-9} \times 30.48}{2.08 \times 10^{-6}} \cong 0.28\ \Omega$$

Ex 6: (a) Consider a series parallel combination of resistors as shown in figure. Find the equivalent resistance as seen from terminal A-B.

(b) determine the current I and power P delivered by a 10 V dc voltage source applied at terminal A-B, with A being at higher potential than B.

(c) Replace the voltage source by an equivalent current source at terminal A-B.

(d) Show the current and voltage distribution clearly in all branches of the original circuit configuration.

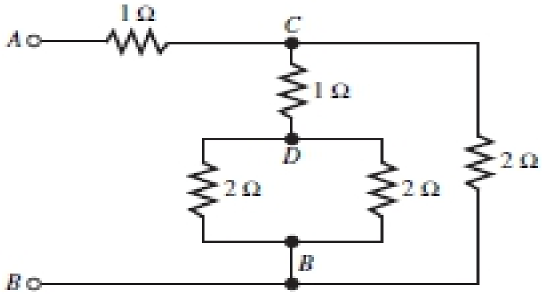
Solution:

(a) The circuit is reduced as illustrated in Figure (b).

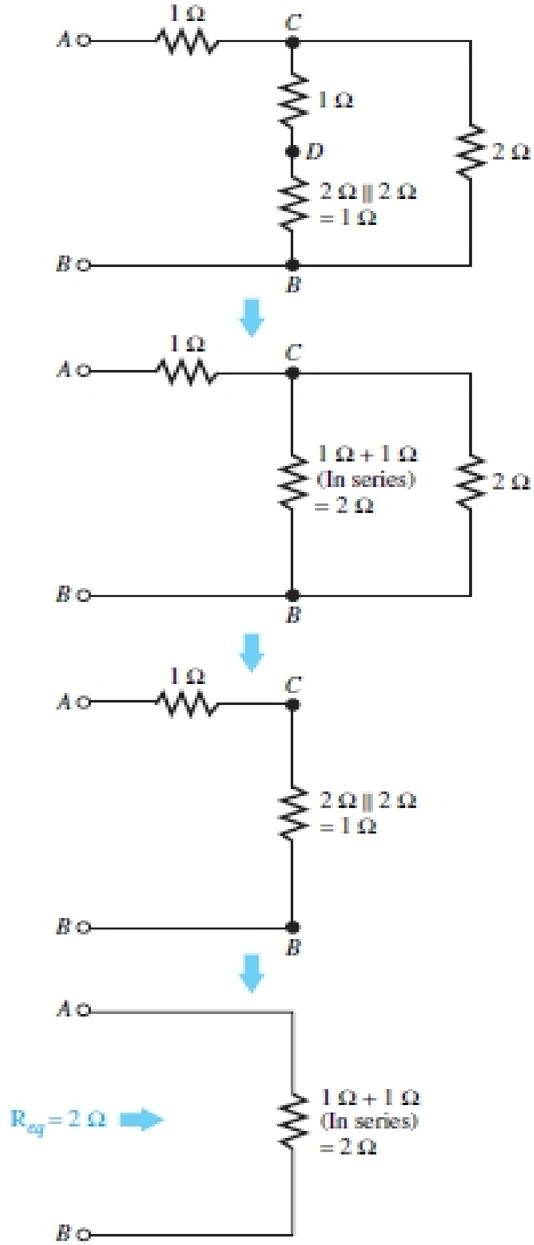
(b) $I = 5$ A; $P = VI = I^2R = V^2/R = 50$ W [see Figure (c)].

(c) See Figure (d).

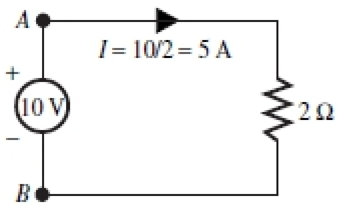
(d) See Figure (e).



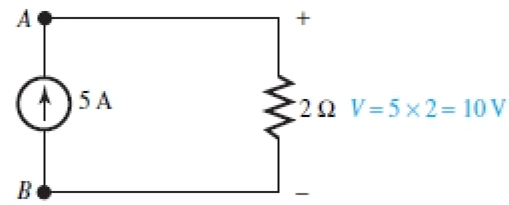
(a)



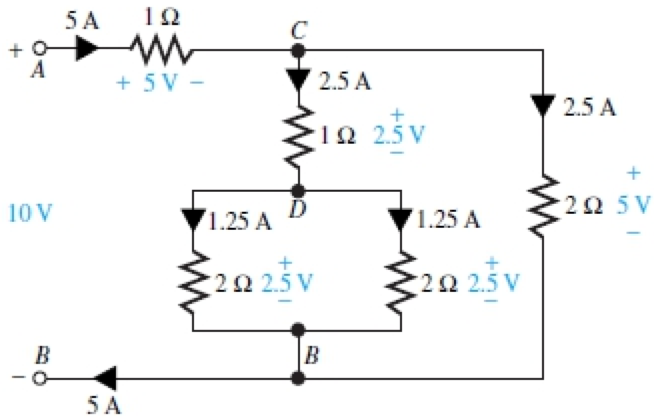
(b)



(c)



(d)



(e)

1.3.2 Maximum Power Transfer:

- **Statement:**

- "*Maximum power will be transferred from a source network to a load if the load resistance R_L is equal to equivalent resistance R_S of the source network*"

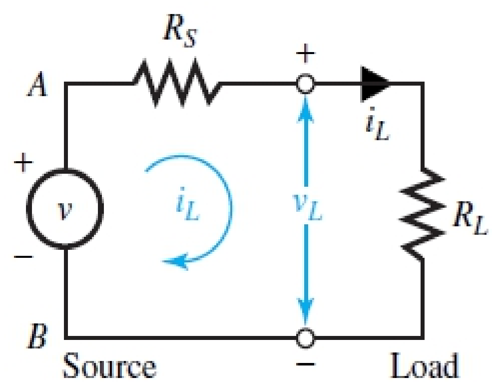


Fig. 1.15: Power transfer between load and source

- Let us consider Figure 1.15, in which a constant voltage source v with a known internal resistance R_S is connected to a variable load resistance R_L.
- When R_L is equal to zero, it is called a short circuit, in which case V_L becomes zero and i_L is equal to V/R_S.

- When R_L approaches infinity, it is called an open circuit, in which case i_L becomes zero and V_L is equal to V. To find the value of the load resistance that will absorb maximum power from the source.
- The power P_L absorbed by the load is given by;

$$P_L = i_L^2 R_L$$

- Where the i_L is given by;

$$i_L = \frac{v}{R_S + R_L}$$

- So ;

$$P_L = \frac{v^2}{(R_S + R_L)^2} R_L$$

- For given fixed values of v and R_S, in order to find the value of R_L that maximizes the power absorbed by the load, one sets the first derivative dP_L/dR_L equal to zero,

$$\frac{dP_L}{dR_L} = \frac{v^2(R_L + R_S)^2 - 2v^2 R_L(R_L + R_S)}{(R_L + R_S)^4} = 0$$

$$(R_L + R_S)^2 - 2R_L(R_L + R_S) = 0$$

$$R_L = R_S$$

- So, in order to transfer maximum power to a load, the load resistance must be matched to the source resistance or, in other words, they should be equal to each other.

- **Source Loading Effect:**

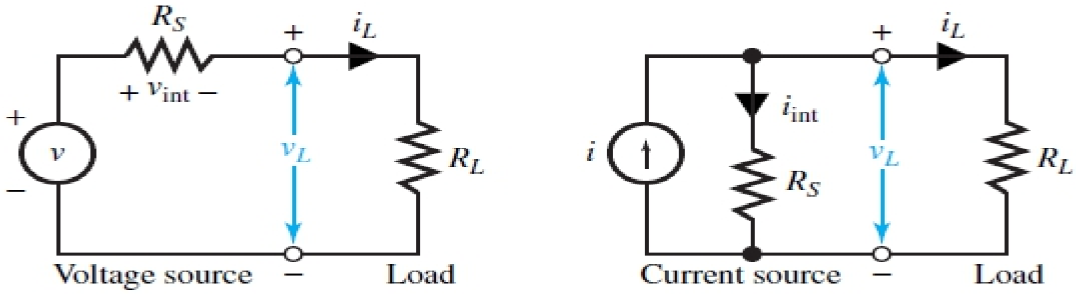


Fig. 1.16: Source loading effect

- From fig. 1.16;

$$v_L = v - v_{int} = v - i_L R_S$$

Where;

V_{int} = internal voltage drop.

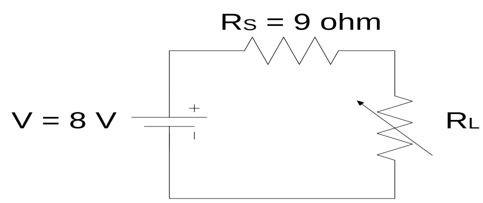
- As seen from above Equation, the voltage actually seen by the load V_L is somewhat lower than the open-circuit voltage of the source.

- When the load resistance R_L is infinitely large, the load current i_L goes to zero, and the load voltage V_L is then equal to the open-circuit voltage of the source V.
- Hence**, it is desirable to have as small an internal resistance as possible in a practical voltage source.**

$$i_L = i - i_{\text{int}} = i - \frac{v_L}{R_S}$$

- Where i_{int} is the internal current drawn away from the load because of the presence of the internal source resistance.
- Thus the load will receive only part of the short-circuit current available from the source.
- When the load resistance R_L is zero, the load voltage V_L goes to zero, and the load current i_L is then equal to the short-circuit current of the source i.
- Hence**, it is desirable to have as large an internal resistance as possible in a practical current source.**

Ex 7: In the given circuit what is the maximum power that can be absorbed by the load R_L if it varied. What is power absorbed by the load if R_L = 7 Ω and R_L = 11 Ω.



- **For maximum power transfer**
  $R_L = R_s$ = 9 Ω

- **Maximum power absorbed by the load**
  $$P_{max} = \frac{V^2}{4R_L} = \frac{8\,X\,8}{4\,X\,9} = 1.78\,Watts$$

➢ **CASE-1: R_L = 7 Ω**

$$I = \frac{V}{R_S + R_L} = \frac{8}{9+7} = 0.5\,Amp$$

- **Power absorbed by the load;**

$$P = I^2 R_L = (0.5)^2 X\,7 = 1.75\,Watts$$

➢ **CASE-1: R_L = 11 Ω**

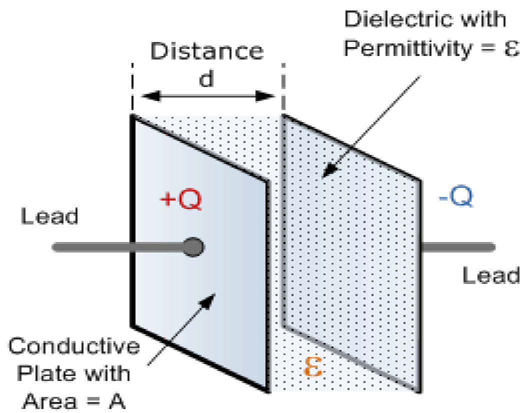
$$I = \frac{V}{R_S + R_L} = \frac{8}{9+11} = 0.4\,Amp$$

- **Power absorbed by the load;**

$$P = I^2 R_L = (0.4)^2 X\,11 = 1.76\,Watts$$

1.3.3 Capacitor:

- Capacitor is an element that store the electrical energy in form of static charges.
- Stored Electric energy is in form of charges.
- Different types of capacitors are Air capacitor, paper capacitor, Ceramic capacitor, Plastic capacitor, Electrolytic capacitor, Mica capacitor, Glass capacitor etc.
- Its unit is Farad (F)



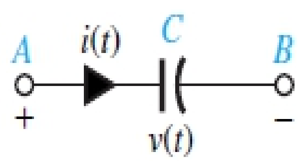


Fig. 1.17: (a) Basic construction (b) Symbol

➢ **Voltage and current equation:**

- The capacitance is defined as follow;

$$C = \frac{Q}{V} \qquad OR \qquad C = \frac{q}{v}$$

- We can write above equation in terms of current

$$i = C\frac{dv}{dt}$$

Where,

V = Voltage across capacitor

i = Current through a capacitor

$$dv = \frac{1}{C}\, i\, dt$$

Taking integrating both side, we get

$$\int_0^t (dv) = \frac{1}{C}\int_0^t i\, dt$$

$$V = \frac{1}{C}\int_0^t i\, dt$$

➢ **Power absorbed by capacitor:**

- The power absorbed by the capacitor is given by;

P = v x i

- Now put value of i from above equation

$$P = v \times C \frac{dv}{dt}$$

- **Energy stored by capacitor:**
- The energy stored by the capacitor is

$$E = \int_0^t p\, dt = \int_0^t v\, X\, C\, \frac{dv}{dt}\, dt$$

$$E = \frac{1}{2} CV^2$$

- **Series parallel combination:**

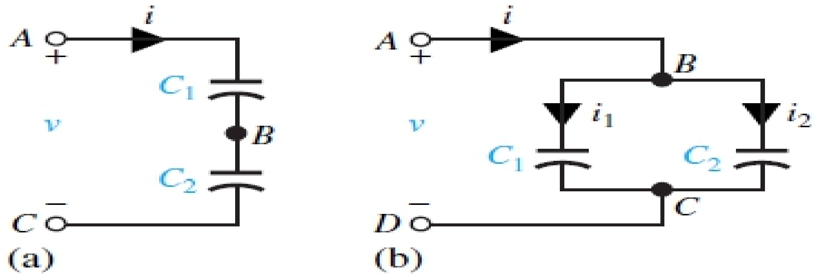


Fig. 1.18: (a) Series connection (b) Parallel connection

- Equation for series connection:

$$\frac{1}{Ceq} = \frac{1}{C1} + \frac{1}{C2}$$

$$Ceq = \frac{C1\, C2}{C1 + C2}$$

- Equation for parallel connection:

$$Ceq = C1 + C2$$

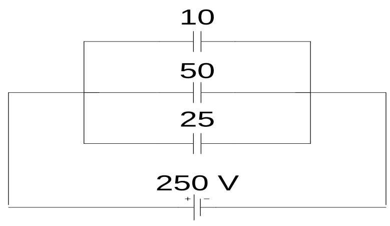
Ex 8: Three capacitors have capacitance of 10, 50 and 25 µF. Calculate;

(a) Charge on each capacitor when they are connected in parallel to a 250 V supply.

(b) Total capacitance

(c) Potential difference across each capacitor when they are connected in series.

Solution:



(a) Parallel connection is as shown in figure;
$Q_1 = C_1\ V = 10 \times 250 = 2500\ \mu C$
$Q_2 = C_2\ V = 50 \times 250 = 12500\ \mu C$
$Q_3 = C_3\ V = 25 \times 250 = 6750\ \mu C$

(b) Total capacitance;
$C_{eq} = C_1 + C_2 + C_3 = 10 + 50 + 25 = 85\ \mu F$

(c) In series connection charge on each capacitor is same so;
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{50} + \frac{1}{25} = \frac{25}{4}\ \mu F$$

$Q = Ceq\ V = (25/4)\ 250 = 1562.5\ \mu C$

Similarly;
$Q = C_1\ V_1$
So; $V_1 = Q / C_1 = 1562.5 / 10 = 156.25$ V

$Q = C_2\ V_2$
So; $V_2 = Q / C_2 = 1562.5 / 50 = 62.5$ V

$Q = C_3\ V_3$
So; $V_3 = Q / C_3 = 1562.5 / 25 = 31.25$ V

Ex 9: A capacitor of capacity of 10 μF is charged from 1000 V d.c. supply and is then discharged through a wire. Find energy stored in capacitor.

<u>Solution:</u>

- Energy stored in the capacitor is, E $= \frac{1}{2}\ CV^2 = \frac{1}{2}\ 10\ X\ 10^{-6}\ X\ 1000^2 = 5\ Joule$

1.3.4 Inductor:

Fig. 1.19: Inductor

- Inductor is an element that store electrical energy in form of magnetic field.
- Inductor is also called a coil or choke.

- Inductor is defined in form of inductance (L). Inductance is a property of inductor to oppose any change in magnitude or direction of current flowing through it.
- Its unit is Henry (H).
- Inductance of a coil is given by,

$$L = \frac{N\varphi}{I}$$

Where,

N = Number of turns

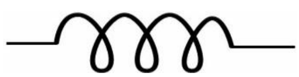
Φ = Flux

I = Current through the coil.

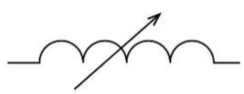
➢ **Types of Inductor:**

- There are two types of inductors;

1 Fixed inductor



2 Variable inductor



➢ **Voltage and Current Equation:**

- The relation between voltage and current is given by,

$$v = L\,\frac{di}{dt}$$

Where,

V = Voltage across inductor

i = Current through inductor

- We can write above equation as,

$$di = \frac{1}{L}\,v\,dt$$

- Now take integration both the side

$$\int_0^t di = \frac{1}{L}\int_0^t v\,dt$$

$$i = \frac{1}{L}\int_0^t v\,dt$$

- **Energy stored by the inductor:**
- The power absorbed by inductor is,

$$P = v\ X\ i$$

$$P = Li\ \frac{di}{dt}$$

- **Energy stored by the inductor:**
- The energy stored by the inductor is,

$$E = \int_0^t P\ dt = \int_0^t Li\ \frac{di}{dt}\ dt$$

$$E = \frac{1}{2}\ Li^2$$

- **Inductor in Series and parallel:**

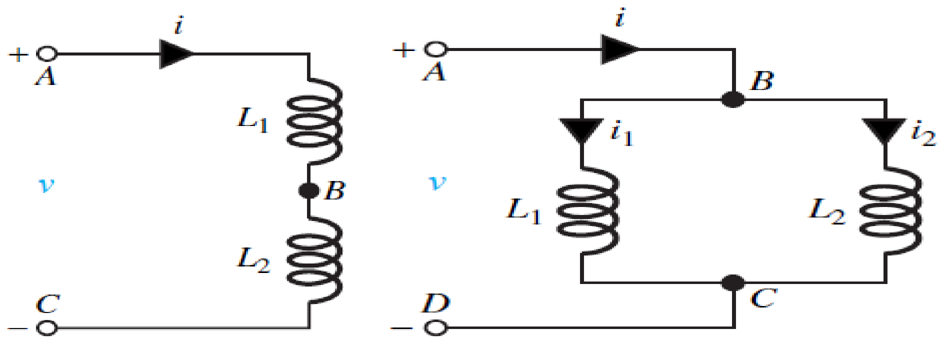


Fig. 1.20: Series and Parallel connection

- Equation for series connection:

$$Leq = L1 + L2$$

- Equation for parallel connection:

$$\frac{1}{Leq} = \frac{1}{L1} + \frac{1}{L2}$$

$$\frac{1}{Leq} = \frac{L1\ L2}{L1 + L2}$$

- **Mutual Inductance:**
- When more than one loop or circuit is present, the flux produced by the current in one loop may link another loop, thereby inducing a current in that loop.
- Such loops are said to be mutually coupled, and there exists a ***mutual inductance*** between such loops.
- The mutual inductance between two circuits is defined as the flux linkage produced in one circuit by a current of 1 ampere in the other circuit.

- Let us now consider a pair of mutually coupled inductors, as shown in Figure 1.21.
- The self-inductances L_{11} and L_{22} of inductors 1 and 2, respectively, are given by

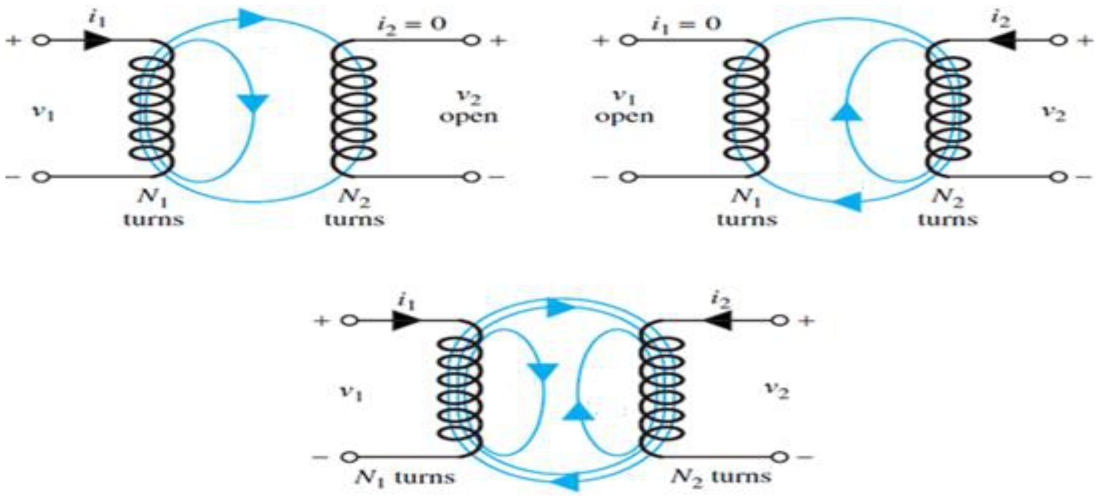


Fig. 1.21: Mutual inductance

- Let ϕ_{12} is flux linkage of coil-2 due to current of coil-1.

$$e_2 = -N_2 \frac{d\phi_{12}}{dt}$$

$$\text{Or } e_2 = -N_2 \frac{d\phi_{12}}{di_1} X \frac{di_1}{dt}$$

$$\text{Or } e_2 = -M \frac{di_1}{dt}$$

Where,

$M = N_2 \frac{d\phi_{12}}{di_1}$ is called mutual inductance of two coil.

- **Coefficient of coupling:**
- It is defined as the ratio of flux linkage to the total flux.
- It is denoted by K.

$$K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

$$\text{As we know} \quad M = N_2 \frac{\phi_{12}}{i_1}$$

$$\text{And} \quad M = N_1 \frac{\phi_{21}}{i_2}$$

$$\text{Multiply above two Eq. so we get,} \quad M^2 = N_2 N_1 \frac{\phi_{12}\,\phi_{21}}{i_1\, i_2}$$

$$\text{Now put} \quad \phi_{12} = K\phi_1 \ and \ \phi_{12} = K\phi_2$$

$$\text{So } M^2 = N_2 N_1 \frac{K^2 \phi_1\, \phi_2}{i_1\, i_2}$$

$$M^2 = K^2 \frac{N_1\phi_1 N_2\,\phi_2}{i_1\, i_2}$$

Now put $L_1 = \frac{N_1\phi_1}{i_1}$ and $L_2 = \frac{N_2\,\phi_2}{i_2}$

$$M^2 = K^2\, L_1\, L_2$$

$$M = K\sqrt{L_1\, L_2}$$

$$K = \frac{M}{\sqrt{L_1\, L_2}}$$

- **Expression for the total emf induced in a coil:**

- The total emf induced in a coil 1 is given by,

$$e_1 = L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

- The total emf induced in a coil 2 is given by,

$$e_2 = L_2 \frac{di_2}{dt} + M_{21} \frac{di_1}{dt}$$

- Total mutually induced emf can be positive or negative but mutual inductance will always be positive.
- Two mutual inductances M_{12} and M_{21} are equal to each other.

$$M_{12} = M_{21} = M$$

- **Dot convention or Dot notation and sign of mutual inductance:**
- M positive if current in the both coils is entering into dot or leaving the dot.
- M negative if current in the one coils is entering into dot and current in other coil is leaving the dot.

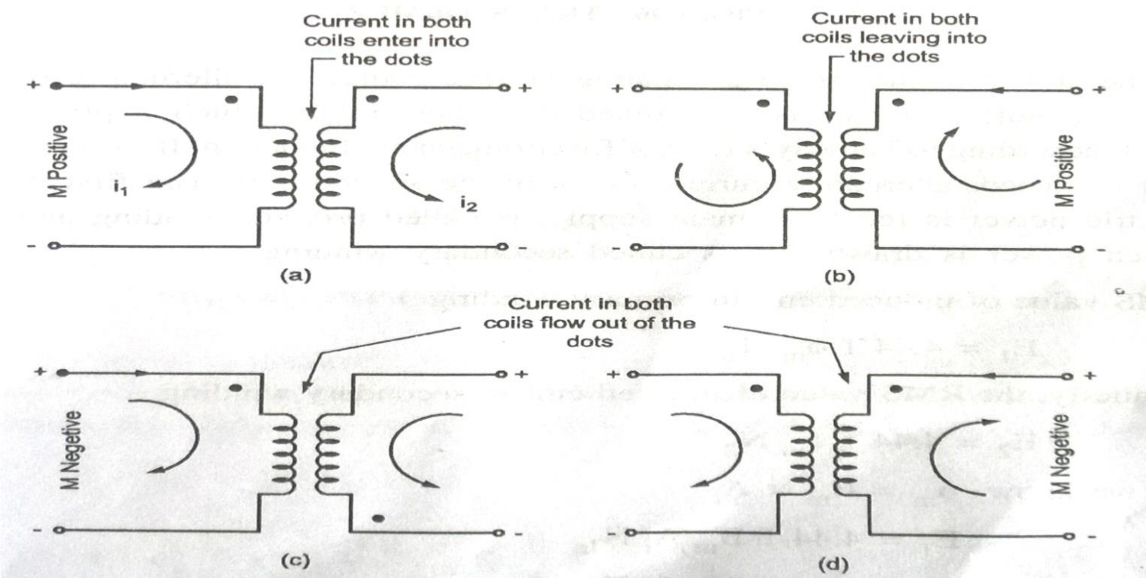


Fig. 1.22: Dot convection of mutual inductance

Ex 10: Three inductors have inductance of 10, 50 and 25 H. Calculate;
(a) Equivalent inductance if all inductors are connected in series.
(b) Equivalent inductance if all inductors are connected in parallel.

Solution:

(a) $L_{eq} = L_1 + L_2 + L_3 = 10 + 50 + 25 = 85$ H

(b) $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{10} + \frac{1}{50} + \frac{1}{25} = \frac{25}{4}$ H

Ex 11: A capacitor of capacity of 20 H and current flowing through it is 1 Amp. Find energy stored in inductor.

Solution:

- Energy stored in the capacitor is, E = $\frac{1}{2}\ LI^2 = \frac{1}{2}\ 20\ X\ 1^2 = 10\ Joule$

1.3.5 Transformer:

- Transformer is a static device because it does not contain any rotating or moving parts.
- It is used to transfer electric power from one ac circuit to other ac circuit. During transfer of power frequency remains same.
- Input and output in the transformer both are ac quantities.
- There are two types of transformer
  1. **Step up**
     - If output voltage is higher than input voltage then it is known as step up transformer.
  2. **Step down**
     - If output voltage is lower than input voltage then it is known as step down transformer.

- The construction of single phase transformer is shown in figure 1.23.

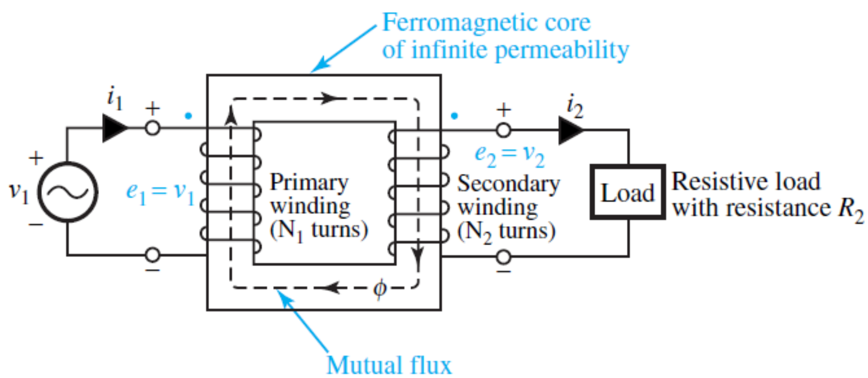


Fig. 1.23 (A): Transformer

- It consist of core and on to the core primary and secondary coil is wounded.
- It also consist of two coils/ windings;
  1. **Primary coil / winding**
  - The coil which is connected to ac supply it is known as primary coil.
  2. **Secondary coil / winding**
  - The coil which is connected to load it is known as secondary coil.
- In transformer primary and secondary windings are isolated from each other as well as from iron coil so there is no physical connections between primary and secondary windings.

- **Principle of operation** of as transformer is as below;

As soon as primary winding is connected to ac supply thus ac current starts flowing through it.

↓

Due to ac current which is produced in primary coil thus flux f is produce in the core.

↓

Most of produced flux f gets linked with the secondary coil through core.

↓

The varying flux f will induce voltage into the secondary coil due to mutual inductance according to the faraday's law of electromagnetic.

➢ **Voltage ratio of the transformer with load:**

- Let RMS produced voltage in the primary coil be E_1 volts and RMS produced voltage in the secondary coil be E_2 volts.

$$E_1 = 4.44\ \phi_m\ f\ N_1 \text{ Volts and } E_2 = 4.44\ \phi_m\ f\ N_2 \text{ Volts}$$

- Now taking ration of above eq.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \quad or \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

➢ **Voltage ratio of the transformer without load:**

- If no load means load is disconnected at secondary side so $I_2 = 0$ so;

$$V_2 = E_2$$

- Also I_1 at the primary side is very small so;

$$V_1 = E_1$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad or \quad \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

➢ **Transformation ratio (a):**

$$a = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{E_1}{E_2}$$

- **Turn ratio of transformer:**

$$\text{Turn ratio} = \frac{N_1}{N_2} = a$$

- **Current ratio of transformer:**

$$power\ input\ = power\ output$$

$$V_1 I_1 \cos \phi_1 = V_2 I_2 \cos \phi_2$$

Here $\cos \phi_1$ and $\cos \phi_2$ are power factors which are same so;

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

- **Ideal transformer:**

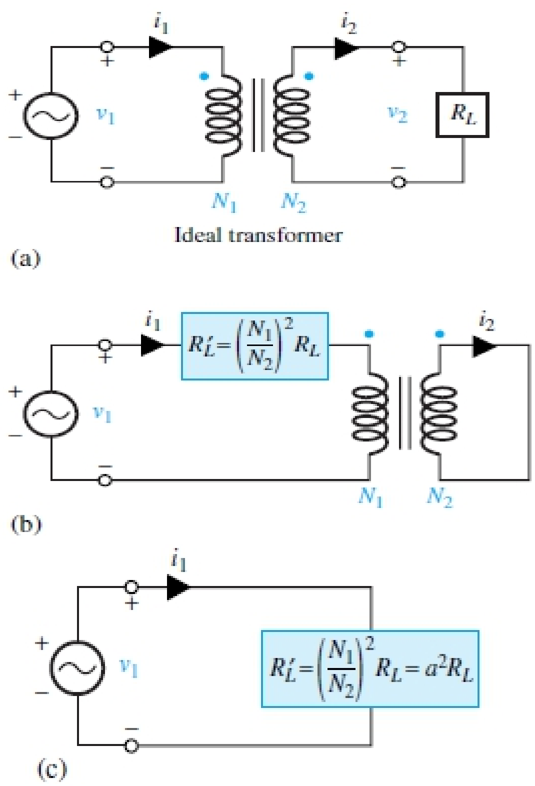


Fig. 1.23 (B): Equivalent circuit viewed from source terminal for ideal transformer

- An ideal transformer is one that has **no losses** (associated with iron or copper) and **no leakage** fluxes (i.e., all the flux in the core links both the primary and the secondary windings).
- In ideal transformer primary and secondary winding resistance are zero.
- For ideal transformer $V_2 = E_2$ and $V_1 = E_1$ so;

$$a = \frac{V_1}{V_2} = \frac{E_1}{E_2}$$

- Efficiency of an ideal transformer is 100% because of no losses taking place.

1.4 KIRCHHOFF'S LAWS:

- Kirchhoff was a German scientist. He formulated two laws related with electrical circuits.
- The two laws are;
  1. Kirchhoff's current law (KCL)
  2. Kirchhoff's voltage law (KVL)
- These are basic laws that must be satisfied among circuit currents and circuit voltages. These laws are fundamental for the systematic analysis of electric circuits.
- These laws are used to determine the current in different branches and voltage at different node of an electric circuit.

➢ **Kirchhoff's current law (KCL):**

- KCL states that, at any node of any circuit and at any instant of time, the sum of all currents entering the node is equal to the sum of all currents leaving the node.
- That is, the algebraic sum of all currents (entering or leaving) at any node is zero.

$$\sum currents\ \ entering\ at\ node\ a = \sum currents\ leaving\ from\ node\ a$$

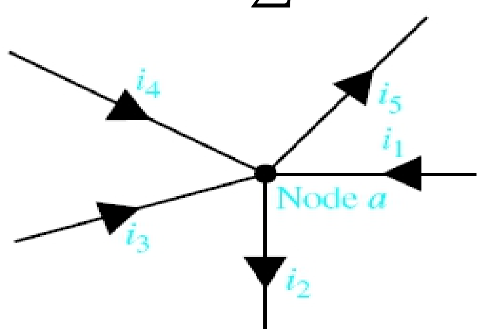


Fig. 1.24: Illustration for KCL

- Figure shows Kirchhoff's current law, in which at node a,
  $i_1 - i_2 + i_3 + i_4 - i_5 = 0$ or $-i_1 + i_2 - i_3 - i_4 + i_5 = 0$
  $i_1 + i_3 + i_4 = i_2 + i_5$

➢ **Kirchhoff's voltage law (KVL):**

- KVL states that the algebraic sum of the voltages (drops or rises) around a closed loop of a circuit must be zero.
- In other words, the sum of the voltage rises is equal to the sum of the voltage drops in a loop. A loop that contains no other loops is known as a mesh.

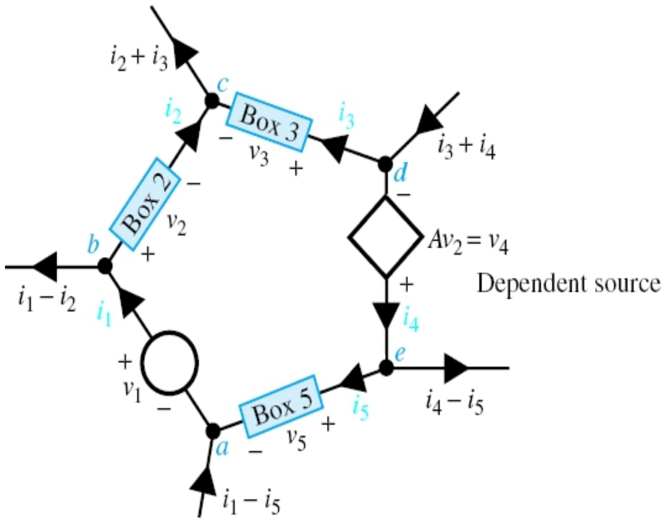
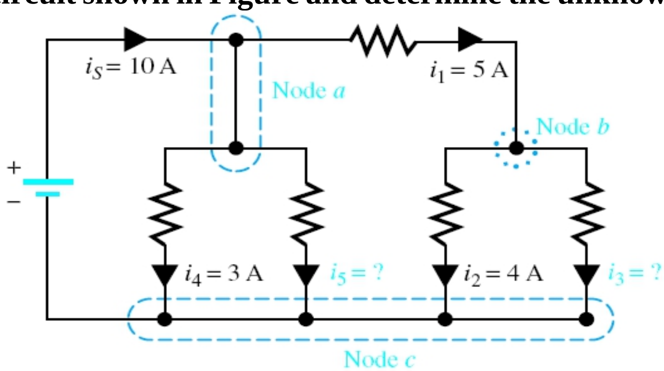


Fig. 1.25: Illustration for KVL

- Figure shows the Kirchhoff's voltage law.
  $-v_1 + v_2 - v_3 - v_4 + v_5 = 0$ or $v_1 - v_2 + v_3 + v_4 - v_5 = 0$
  $v_1 + v_3 + v_4 = v_2 + v_5$

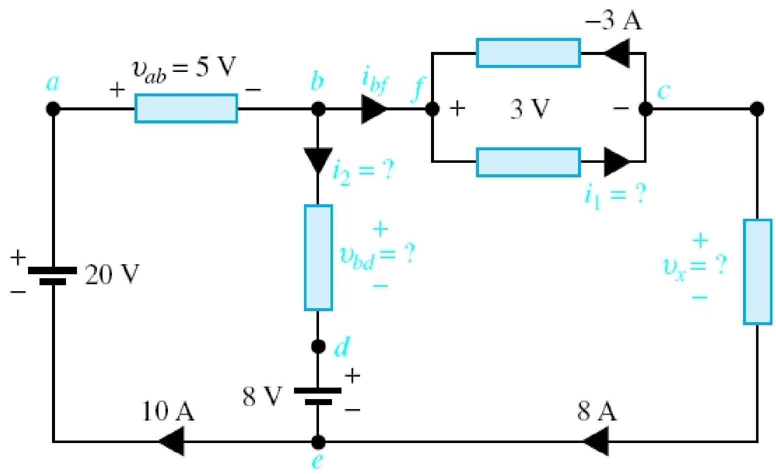
Ex 12: Consider the circuit shown in Figure and determine the unknown currents using KCL.



- Let us assign a + sign for currents entering the node and a − sign for currents leaving the node. Applying KCL at node a, we get
  $+ i_S - i_1 - i_4 - i_5 = 0$
  $10 - 5 - 3 - i_5 = 0$
  $i_5 = 2$ A

- Applying KCL at node b, we get
  $+ i_1 - i_2 - i_3 = 0$
  $5 - 4 - i_3 = 0$
  $i_3 = 1$ A

Ex 13: For the circuit shown in Figure use KCL and KVL to determine i_1, i_2, v_{bd} and v_x. Also, find v_{eb}



- Using KCL at node c, we get
  $i_1 = 8 + (-3) = 5$ A

- Using KCL at node f, we have
  $i_{bf} = i_1 - (-3) = 5 + 3 = 8$ A

- Applying KCL at node b, we get
  $10 = i2 + i_{bf} = i2 + 8$
  $i2 = 2$ A

- Using KVL around the loop abdea in the clockwise direction, we have
  $v_{ab} + v_{bd} + v_{de} + v_{ea} = 0$
  or
  $5 + v_{bd} + 8 - 20 = 0$
  $v_{bd} = 20 - 8 - 5 = 7$ V

- Applying KVL around the loop abfcea in the clockwise direction, we get
  $v_{ab} + v_{bf} + v_{fc} + v_{ce} + v_{ea} = 0$
  $5 + 0 + 3 + v_x + (-20) = 0$
  $v_x = 20 - 3 - 5 = 12$ V
- Note that a direct connection between b and f implies ideal connection, and hence no voltage between these points.

  $v_{eb} = v_{ed} + v_{db}$, we have $v_{eb} = -8 - 7 = -15$ V
  $v_{be} = -v_{eb} = 15$ V a node b is at a higher potential than node e.

1.5 METERS AND MEASUREMENTS:

- **What is Measurement?**

- Measurement is the assignment of numbers to objects or events.
- The subject of electrical measurements is such a large one that entire books have been written on the topic.
- Practical measurements are made with real instruments, which in general disturb the operation of a circuit to some extent when they are connected.
- Measurements may be affected by noise, which are undesirable randomly varying signals.

- **Voltmeter:**

- In order to measure the potential difference between two terminals or nodes of a circuit, a voltmeter is connected across these two points.
- A practical voltmeter can usually be modeled as a parallel combination of an ideal voltmeter (through which no current flows) and a shunt resistance R_V, as shown in Fig. 1.26.
- The internal resistance R_V of an ideal voltmeter is infinite, while its value in practice is of the order of several million ohms.
- There are what are known as DC voltmeters and AC voltmeters. An AC voltmeter usually measures the rms value of the time-varying voltage.

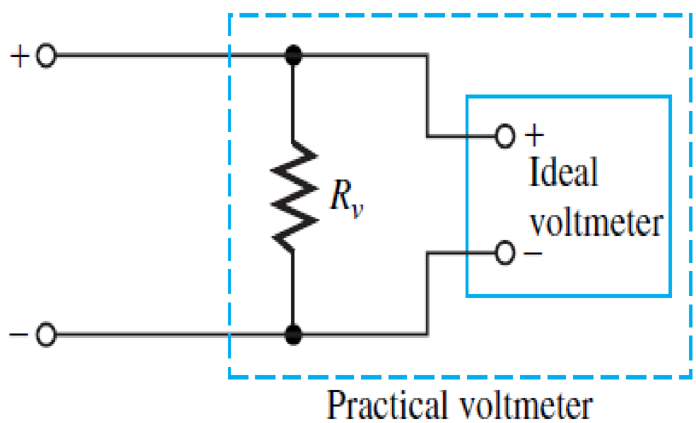


Fig. 1.26: Connection of a voltmeter

- **Ammeter:**

- In order to measure the current through a wire or line of a circuit, an ammeter is connected in series with the line.
- A practical ammeter can usually be modeled as a series combination of an ideal ammeter and an internal resistance R_I.
- The potential difference between the two terminals of an ideal ammeter is zero, which corresponds to zero internal resistance.
- There are what are known as DC ammeters and AC ammeters.
- An AC ammeter usually measures the rms value of the time-varying current.

- Note that for the ammeter to be inserted for measuring current, the circuit has to be broken, whereas for the voltmeter to be connected for measuring voltage, the circuit need not be disassembled.
- Multimeter that measures multiple ranges of voltage and current are available in practice.
- Ohmmeters measure the dc resistance by the use of Ohm's law.
- A Multimeter with scales for volts, ohms, and milli amperes is known as VOM.
- An ohmmeter should not be used to measure the resistance of an electronic component that might be damaged by the sensing current.

➢ **Instrument Transformers:**

- These are generally of two types, **potential transformers (PTs)** and **current transformers (CTs).**
- They are designed in such a way that the former may be regarded as having an ideal potential ratio, whereas the latter has an ideal current ratio.
- The accuracy of measurement is quite important for ITs that are commonly used in AC circuits to supply instruments, protective relays, and control devices.
- PTs are employed to step down the voltage to a suitable level, whereas CTs (connected in series with the line) are used to step down the current for metering purposes.
- Often the primary of a CT is not an integral part of the transformer itself, but is a part of the line whose current is being measured.
- In addition to providing a desirable low current in the metering circuit, the CT isolates the meter from the line, which may be at a high potential.
- Note that the secondary terminals of a CT should never be open-circuited under load.
- The student is encouraged to reason and justify this precaution.
- One of the most useful instruments for measuring currents in the ampere range is the clip-on ammeter combining the CT with one-turn primary and the measurement functions.

➢ **Oscilloscope:**

- To measure time-varying signals (voltages and currents), an instrument known as an oscilloscope is employed.
- It can be used as a practical electronic voltmeter which displays a graph of voltage as a function of time.
- Such a display allows one not only to read off the voltage at any instant of time, but also to observe the general behavior of the voltage as a function of time.
- The horizontal and vertical scales of the display are set by the oscilloscope's controls, such as 5 ms per each horizontal division and 50 V per each vertical division.
- For periodic waveforms, the moving light spot repeatedly graphs the same repetitive shape, and the stationary waveform is seen.
- For non-periodic cases, a common way of handling is to cause the oscilloscope to make only one single graph, representing the voltage over a single short time period.
- This is known as single-sweep operation.
- Since the display lasts for only a very short time, it may be photographed for later inspection.

- Digital meters are generally more accurate and can be equipped with more scales and broader ranges than analog meters.
- On the other hand, analog meters are generally less expensive and give an entire range or scale of reading, which often could be very informative.
- A digital oscilloscope represents the combination of analog and digital technologies.
- By digital sampling techniques, the oscilloscope trace is digitized and stored in the digital memory included with the digital oscilloscope.
- Digital oscilloscopes are generally more costly than analog ones, but their capability in the analysis and processing of signals is vastly superior.

➢ **Wheatstone bridge:**

- Null measurements are made with bridge circuits and related configurations.
- They differ from direct measurements in that the quantity being measured is compared with a known reference quantity.
- The balancing strategy avoids undesirable interaction effects and generally results in more accurate measurement than the direct one.
- By far the most common is the Wheatstone bridge designed for precise measurement of resistance. Fig. 1.27 shows the basic circuit in which the measurement of an unknown resistance R_x is performed by balancing the variable resistances R_a and R_b until no current flows through meter A.

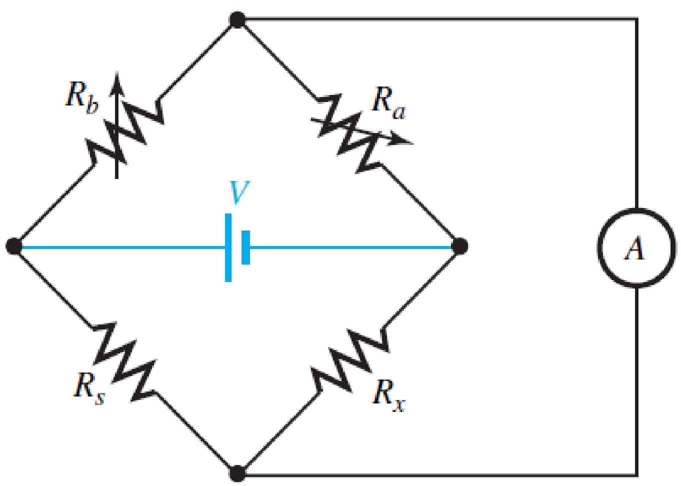


Fig. 1.27: Wheatstone's network

- Under this null condition,

$$R_x = \frac{R_a}{R_b} \cdot R_s$$

Where;

Rs is the known standard resistance.

- There are other bridge-circuit configurations to measure inductance and capacitance.
- Typical instruments utilizing bridge circuits are found in strain gauges measuring stress and in temperature measuring systems with thermocouples and thermistors.

1.6 ANALOGY BETWEEN ELECTRICAL AND NON-ELECTRICAL PHYSICAL SYSTEM:

- The Fig. 1.28 shows analogy between water and electricity.

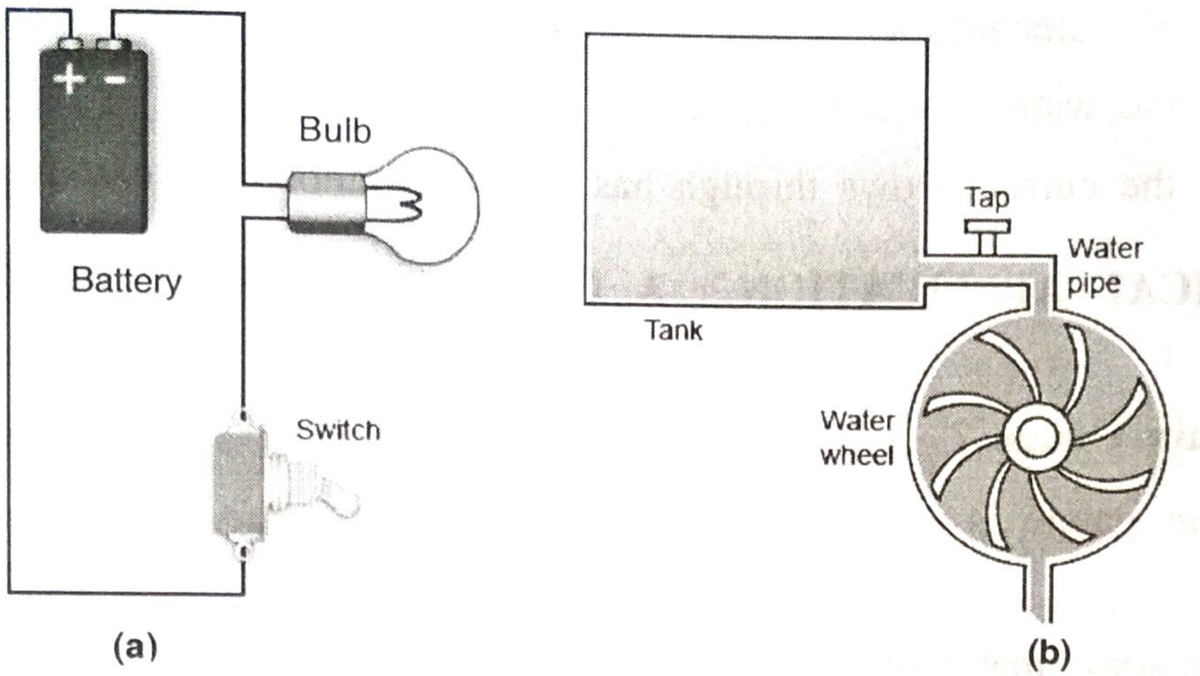


Fig. 1.28: Analogy between electricity and water

1. **Pressure – Voltage:**
   - Fig. 1.28(b) shows full of water tank. This is where the water pressure is stored. The more amount of water in tank, the greater the water pressure.
   - The water tank in fig. 1.28(b) can be compared to the battery in fig. 1.28(a), where a battery in an electric circuit stores the electric pressure (voltage).

   - An empty tank of water with no pressure is similar to an empty battery with no electric pressure.

2. **Flow – Current:**
   - Turning of tap in Fig. 1.28(b) allows water, pushed out of the tank by pressure to flow through the pipe and water wheels. This cause the wheels to rotate.
   - Similarly, in fig. 1.28(a) turning ON switch allows current flow, pushed out of the battery due to electric pressure through the wire and bulb.

   - The flow of water is similar to the flow of current (Amperes).

3. Restriction – Resistance:

- The pipe size and wheel construction cause a restriction to the water flow. The restriction of water flow is similar to resistance in an electric circuit.
- In fig. 1.28(a) the wire and bulb offer a resistance to current flow. The size of wire and bulb affect the amount of current flowing.

- **Thermal – Electrical analogy as below;**

1. Heat ← **Analogues to** → Charge
2. Heat flow ← **Analogues to** → Current
3. Temperature ← **Analogues to** → Voltage
4. Ambient temperature ← **Analogues to** → Reference point (Ground)
5. Heat capacity ← **Analogues to** → Capacitance
6. Thermal resistance ← **Analogues to** → Resistance

1.7 A CASE STUDY – RESISTANCE STRAIN GAUGE:

➢ **What is strain gauge?**

- The strain gauge is used to measure force, load, pressure, strain, stress, thrust etc.

➢ **Principle of strain gauge:**

- There are some materials whose resistance changes when strain is applied to them or when they are stretched and this change in resistance can be measured easily.
- For applying the strain we need force, thus the change in resistance of the material can be calibrated to measure applied stress or strain.
- Thus the devices whose resistance changes due to applied stain, stress or force are called as the strain gauge.
- A typical strain gauge, shown in Fig. 1.29, consists of a metal foil (such as nickel copper alloy) which is formed by a photo etching process in multiple conductors aligned with the direction of the strain to be measured. The conductors are usually bonded to a thin backing made out of a tough flexible plastic. The backing film, in turn, is attached to the test structure by a suitable adhesive.

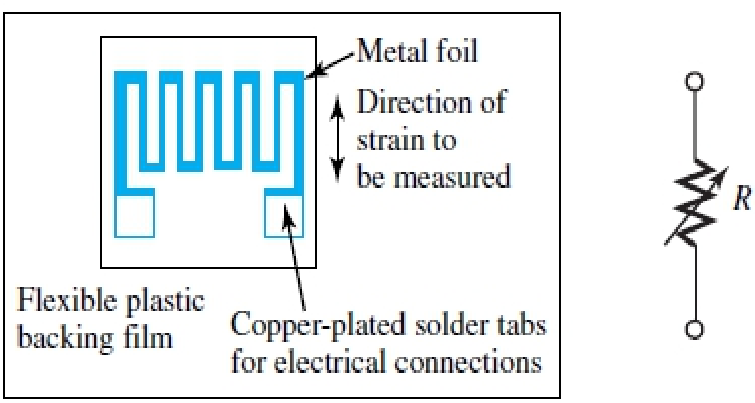


Fig. 1.29: Resistance strain gauge and symbol

- The resistance of a conductor with a circular cross-sectional area A, length l, and conductivity σ is given by

$$\mathbf{R = l / (\sigma A)}$$

- Depending on the compression or elongation as a consequence of an external force, the length changes, and hence the resistance changes. The relationship between those changes is given by the gauge factor G,

$$\mathbf{G = (\Delta R/R) / (\Delta l / l)}$$

  Where, the factor Δl / l, the fractional change in length of an object, **is known as the strain.**
- Alternatively, the change in resistance due to an applied strain ε (= Δl / l) is given by

$$\mathbf{\Delta R = R_0 G \varepsilon}$$

  Where R_0 is the zero-strain resistance, that is, the resistance of the strain gauge under no strain.
- A typical gauge has R_0 = 350 Ώ and G = 2. Then for a strain of 1%, the change in resistance is ΔR = 7 Ώ.
- A Wheatstone bridge is usually employed to measure the small resistance changes associated with precise strain determination.

- Different types of strain gauges are;
  1. Unbounded metal strain gauges.
  2. Bonded metal wire strain gauges.
  3. Bonded metal foil strain gauges.
  4. Vacuum deposited thin metal film strain gauges.
  5. Sputter deposited thin film metal strain gauges.
  6. Bonded semiconductor strain gauges.
  7. Diffused metal strain gauge.

- Desirable properties of strain gauges are;
  1. High resistance
  2. High elastic limit
  3. Linearity
  4. Small size
  5. High sensitivity
  6. More accurate

QUESTIONS

1.1 INTRODUCTION:

1. Give definitions of Circuit and Circuit Elements.

1.2 ELECTRICAL QUANTITIES:

1. Explain charge and electric force in detail.
2. Give coulomb's laws.
3. What is Conductors, Insulators and Semi-Conductors?
4. Explain current and magnetic force in detail.
5. Give definitions of Current and Magnetic force
6. Give definitions of following;
   i. Unit charge
   ii. Electric field
   iii. Electric flux
   iv. Electric flux density
   v. Electric field intensity
7. Explain electric potential and voltage in detail.
8. Give definitions of One volt, Potential difference and emf.
9. Give difference between emf and Potential difference.
10. Explain energy and power in detail.
11. Give definition of Energy and Power.
12. Explain source and load in detail.
13. What is Independent and Dependent source?
14. Give definitions of following;
    i. Waveform
    ii. Instantaneous value
    iii. Cycle
    iv. Amplitude
    v. Time Period
    vi. Frequency
    vii. Angular Frequency
    viii. Average Value
    ix. RMS value
15. Calculate the force on a charge of + 10^2 C placed at a point (0, 0) due to point charge Q_1 and Q_2. The charge Q_1 = + 10^{-6} C is at (-2, 2) meters and Q_2 = - 2 x 10^{-6} C.
    ANS: F_1 = 11.25 N, F_2 = 22.5 N, Resultant force = 25.156 N

16. Two charges of equal magnitude 5 μC but opposite sign are separated by a distance of 10 m. find the net force experienced by a positive charge Q = 2 μC that is placed midway between the two charges.

1.3 LUMPED CIRCUIT ELEMENTS:

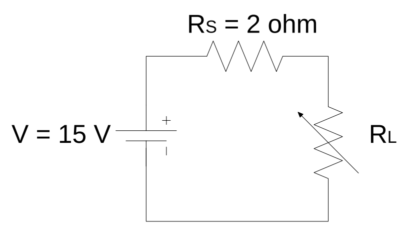
1. Write short note on resistor. (GTU / SUMMER-2014)
2. What is maximum power transfer? Also derive equation for that.
3. Write short note on capacitor. (GTU / SUMMER-2014)

OR

3. Derive voltage, current and energy equations for capacitor.
4. Write short note on inductor.

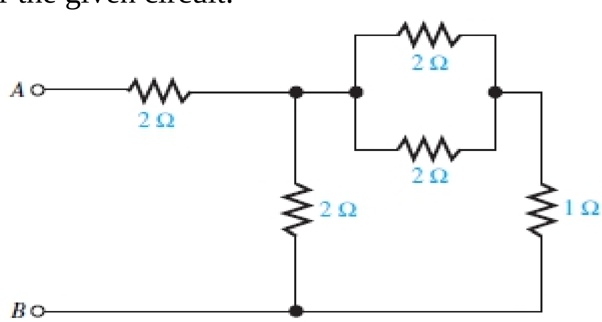
OR

4. Derive voltage, current and energy equations for inductor.
5. What is mutual inductance? Explain concept of mutual inductance also derive its equations.
6. Write short note on transformer.
7. In the given circuit what is the maximum power that can be transferred to load resistor of 3 ohm if it is variable. Also find maximum power absorbed by the load resistor.



ANS: R_L = 2 ohm, P_{max} = 28.125 Watts

8. Determine R_{eq} for the given circuit.



9. Two capacitor C_1 = 4 μF and C_2 = 2 μF are connected in parallel across a 200 V DC supply find (1) Equivalent capacitance (2) Charge across each capacitor (3) If this parallel capacitor combination connected in series with 6 μF then what would be the equivalent capacitance of circuit becomes?

ANS: (1) C_{eq} = 6 μF (2) Q_1 = 800 μC, Q_2 = 400 μC (3) C_{eq} = 3 μF

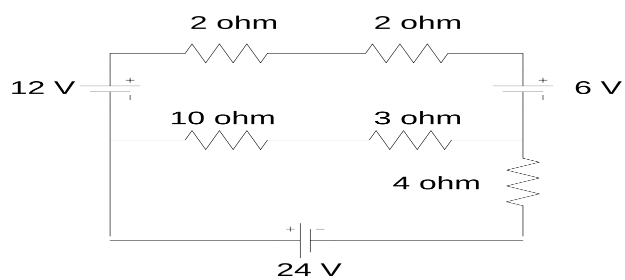
1.4 KIRCHHOFF'S LAWS:

1. Explain KVL and KCL.
2. Two batteries A and B are connected in parallel and combination is connected across 10 ohm resistance. Battery A has an internal resistance of 0.2 ohm and internal voltage is 110 V and corresponding value of battery B are 0.25 ohm and 110 V. calculate current in each battery and into 10 ohm resistance.

   ANS: 1) Current in battery of 110 V is = - 17.58 Amp

   2) Current in battery of 100 V = 28.02 Amp

   3) Current in 10 ohm resistor = 10.44 Amp
3. From the figure given below, find the current flowing through 10 ohm resistance, using Kirchhoff's law.



   ANS: Current in 10 ohm resistor = 0.6 Amp

1.5 METERS AND MEASUREMENTS:

1. Explain Ammeter, Voltmeter, and Multimeter. (GTU / SUMMER-2014)
2. Write short note on Instrument transformer.
3. Write short note on CRO or Oscilloscope.
4. Explain Wheatstone's bridge.

1.6 EXPLAIN ANALOGY BETWEEN ELECTRICAL AND NON-ELECTRICAL PHYSICAL SYSTEM:

1. Explain analogy between electrical and non-electrical physical system.

1.7 RESISTANCE STRAIN GAUGE:

1 Write short note on Resistance strain gauge.

CHAPTER-2 Circuit Analysis Techniques

2.1 Introduction:

- In this chapter we consider some circuit analysis techniques, since one needs not only basic knowledge but also practical and efficient techniques for solving problems associated with circuit operations.
- One simplifying technique often used in complex circuit problems is that of breaking the circuit into pieces of manageable size and analyzing individually the pieces that may be already familiar.
- Equivalent circuits are introduced which utilize Thevenin's and Norton's theorem to replace a voltage source by a current source or vice versa. Nodal and loop analysis methods are then presented. The principles of superposition and linearity are discussed.
- Also, wye-delta transformation is put forth as a tool for network reduction. Finally, computer- aided circuit analysis with SPICE and MATLAB are introduced. The chapter ends with a case study of practical application.

2.2 Basic Definitions for Circuit Analysis:

1) **Network:** The interconnection of two or more elements (sources, resistors etc.) is called an electric network.

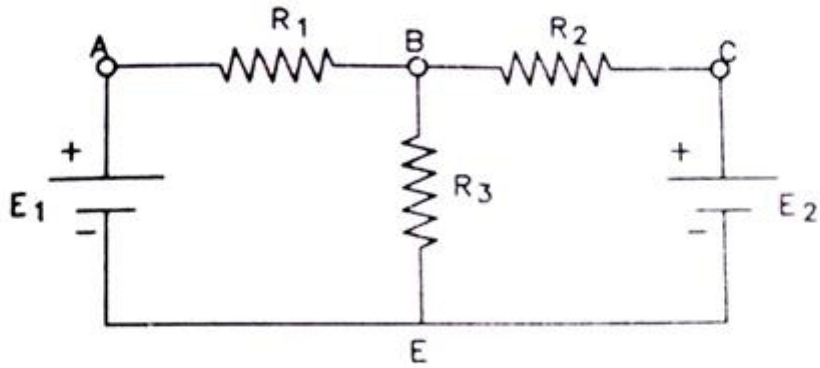


Figure 2.1 Simple Electrical Network

2) **Circuit:** If a network contains at least one closed path, it is called an electric circuit.
3) **Node:** The point at which two or more elements are connected together is generally called as node e.g. A, B, C and E, in figure.
4) **Junction:** It is a point where three or more elements are connected together e.g. B and E in figure-2.1.
5) **Branch:** A Section or portion of a network or circuit which lies between two junction points is called as branch. As shown in figure-2.1, there are three branches between the junctions B and E such as BE, BAE and BCE.
6) **Loop:** Any closed path in a network is called a loop e.g. ABEA, BCEB, ABCEA are the loops in figure-2.1.
7) **Mesh:** It is the most elementary form of a loop and cannot be further divided into other loops e.g. ABEA and BCEB are the meshes in figure-2.1 because they cannot be further

divided into other closed loops. But ABCEA cannot be called as mesh because it encloses two loops ABEA and BCEB.

2.3 Thevenin and Norton equivalent circuits:

2.3.1 Thevenin Equivalent Circuit:

Statement:

Thevenin's theorem states that any network having a number of energy sources and resistances, when viewed form open its output terminal A and B and can be replaced by simple equivalent network consisting of a single equivalent voltage source (V_{Th}) in series with a single equivalent resistance (R_{Th}).

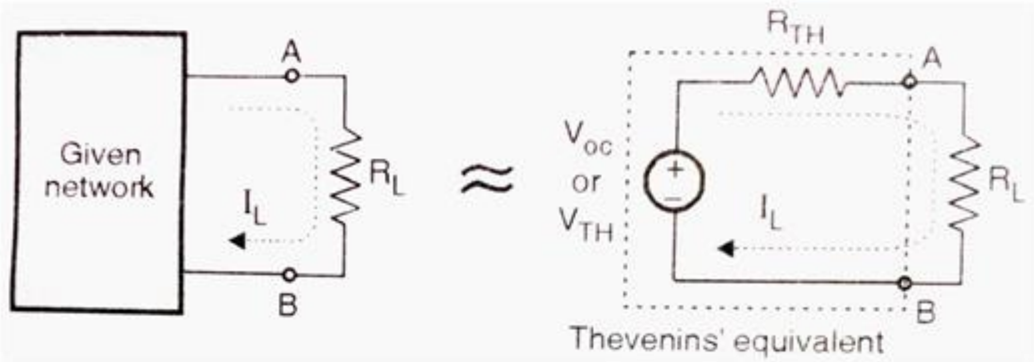


Figure 2.2 Thevenin Equivalent Circuit

Where,

V_{TH} = Thevenin's equivalent voltage source
= Open circuit voltage across AB terminals

R_{TH} = Thevenin's equivalent resistance
= Equivalent resistance across AB terminals when all the sources set to zero.

Example-1

Using Thevenin's theorem, calculate the current in 5 Ω resistor.

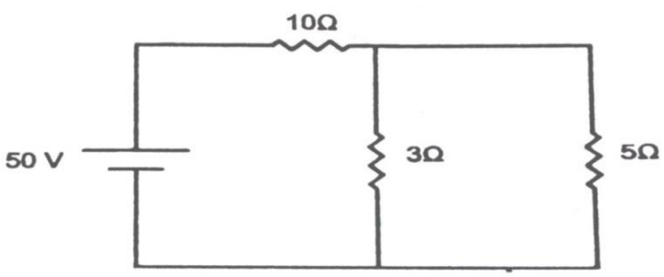


Figure-2.3

Solution:

Step-1: Disconnect the resistor from the network and redraw the remaining network.

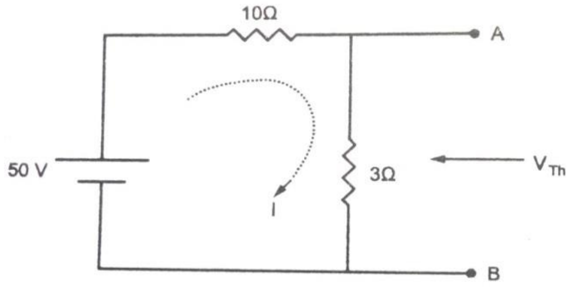


Figure-2.4

Step-2: Determine the Thevenin voltage (V_{th}) by using voltage divider method or Kirchhoff's Voltage Law.

Apply KVL in close loop,

$$50-10I-3I=0$$
$$I=\frac{50}{13}=3.84$$
$$V_{th}=I\times R_3 = 3.84\times 3$$
$$V_{th}=11.52\ Volt$$

Step-3: Determine the Thevenin resistance (R_{th}) by using voltage source is replaced by short circuit and current source is replaced by open circuit.

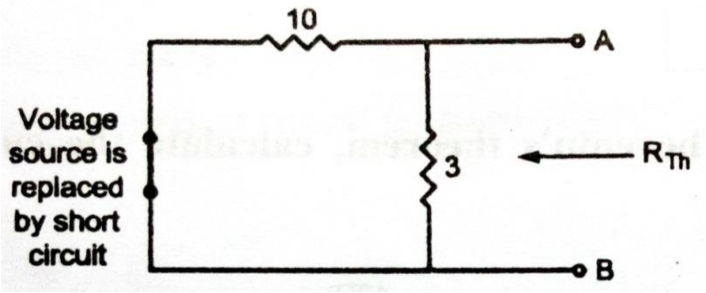


Figure-2.5

$$R_{th}=10\parallel 3$$

$$R_{th}=\frac{10\times 3}{10+3}=\frac{30}{13}=2.30\ \Omega$$

Step-4: Draw Thevenin's equivalent circuit and reconnect resistor between terminal A and B. and find out the load current by using KVL.

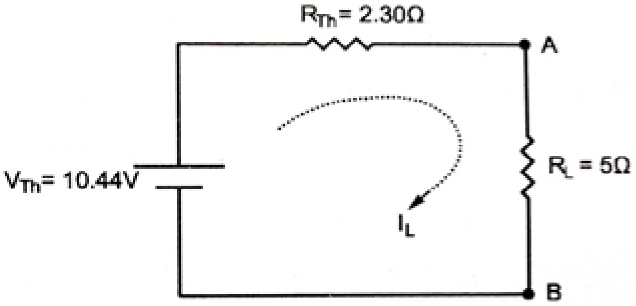


Figure-2.6

Apply KVL in close loop, 11.52 - 2.3I_L - 5I_L= 0

OR

$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{11.52}{2.3+5}$$
$$\boldsymbol{I_L = 1.58\ \text{Amp}}$$

Example-2
Apply Thevenin's theorem to calculate current flowing through 5 Ω resistor.

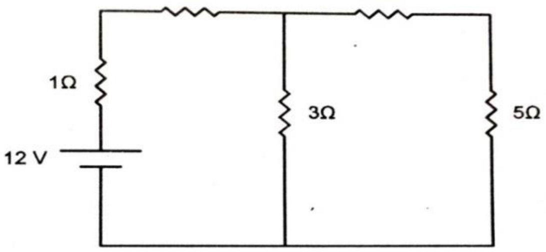


Figure-2.7

Solution:
Step-1: Disconnect the resistor from the network and redraw the remaining network.

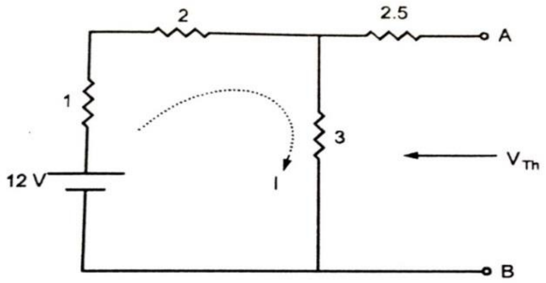


Figure-2.8

Step-2: Determine the Thevenin voltage (V_{th}) by using voltage divider method or Kirchhoff's Voltage Law.

Apply KVL in close loop
12-1I-2I-3I=0
I= $\frac{12}{6}$ = 2
$$V_{th} = I \times R_3 = 2 \times 3$$
$$V_{th} = 6\ Volt$$

Step-3: Determine the Thevenin resistance (R_{th}) by using voltage source is replaced by short circuit and current source is replaced by open circuit.

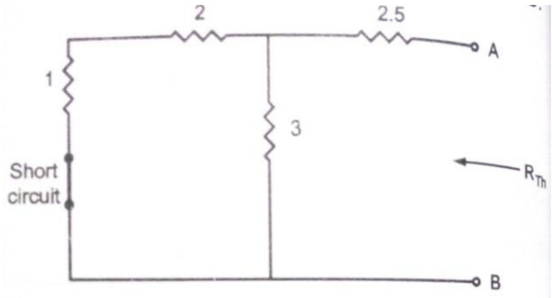


Figure-2.9

$$R_{th} = [(1+2) \parallel 3] + 2.5$$
$$= [3 \parallel 3] + 2.5$$
$$R_{th} = \frac{3\times3}{3+3} + 2.5 = 1.5 + 2.5 = 4\ \Omega$$

Step-4: Draw Thevenin's equivalent circuit and reconnect resistor between terminal A and B. and find out the load current by using KVL.

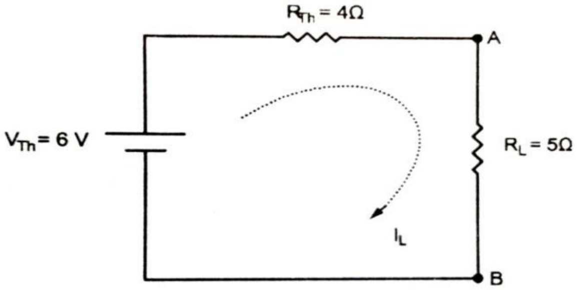


Figure-2.10

$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{6}{4+5}$$

$$\boldsymbol{I_L = 0.67\ \text{Amp}}$$

Example-3

Using Thevenin's theorem find the current in 8Ω resistor in the given circuit of figure-2.11.

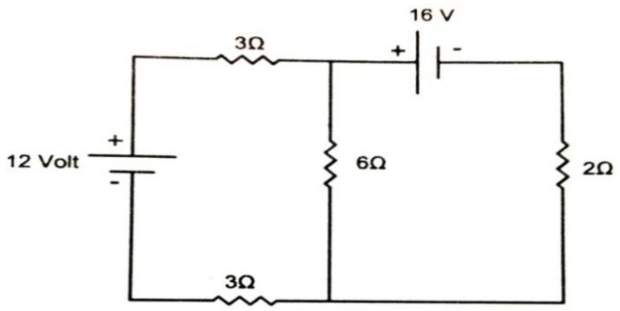


Figure-2.11

Solution:

Step-1: Disconnect the resistor from the network and redraw the remaining network.

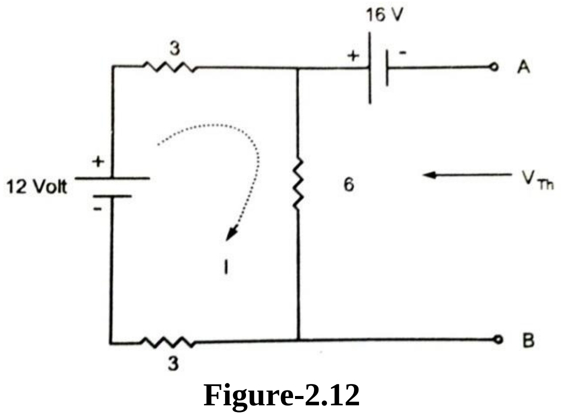


Figure-2.12

Apply KVL in close loop

$$12-3I-6I-3I=0$$
$$I= \frac{12}{6+3+3} = 1 \text{ Amp}$$
$$V_6 = I \times R_6 = 1 \times 6$$
$$V_6 = 6\,Volt$$

Step-2: Determine the thevenin voltage (V_{th}) by using voltage divider method or Kirchhoff's Voltage Law.

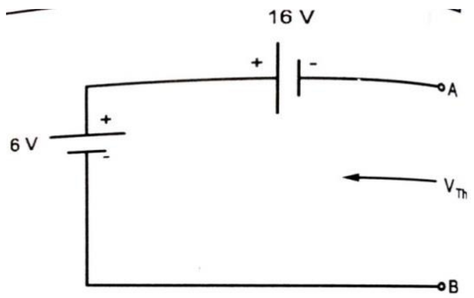


Figure-2.13

So

$$V_{TH} = 16 - 6 = 10\ V \text{ with B terminal is positive}$$

Step-3: Determine the Thevenin resistance (R_{th}) by using voltage source is replaced by short circuit and current source is replaced by open circuit.

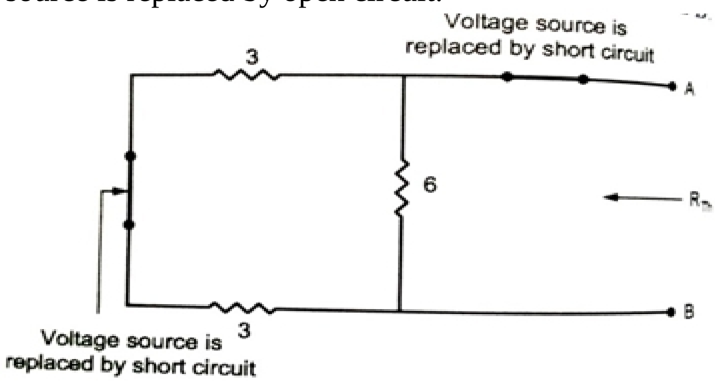


Figure-2.14

$$R_{th} = (3+3) \parallel 6 = 6 \parallel 6$$
$$R_{th} = \frac{6\times6}{6+6} = 3\ \Omega$$

Step-4: Draw Thevenin's equivalent circuit and reconnect resistor between terminal A and B. and find out the load current by using KVL.

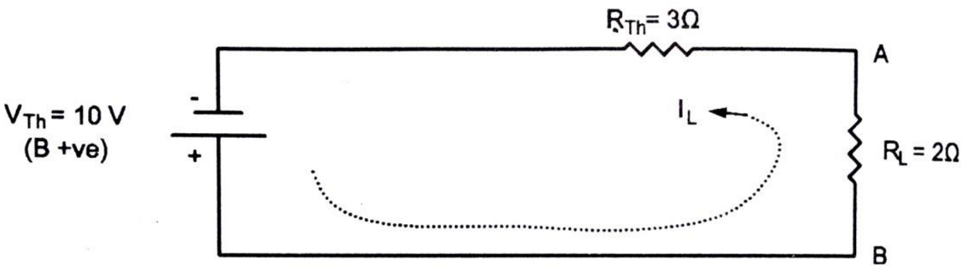


Figure-2.15

$$I_L = \frac{V_{TH}}{R_{TH}+R_L} = \frac{10}{3+2} = \frac{10}{5} = 2 \text{ Amp}$$

2.3.2 Norton's Equivalent Circuit:

Statement:

Norton's theorem states that any network having a number of energy sources and resistances, when viewed form open its output terminal A and B and can be replaced by simple equivalent network consisting of a single equivalent current source (I_{SC} *or* I_N) in parallel with a single equivalent resistance (R_{eq}).

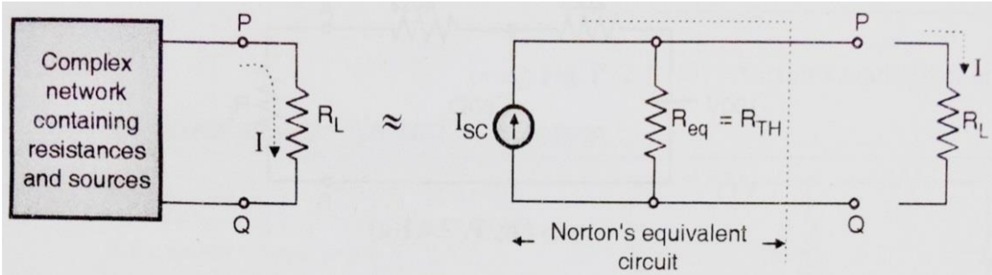


Figure-2.16 Norton equivalent circuit

Where,

I_N or I_{SC} = Norton's equivalent voltage source

= The value of current source is equal to the current passing through the short circuit applied at the open output terminal P and Q.

R_{eq} = Equivalent resistance across PQ terminals when all the sources set to zero.

Example-4

Using Norton's theorem, determine the current through 6 Ω resistor in figure-2.17

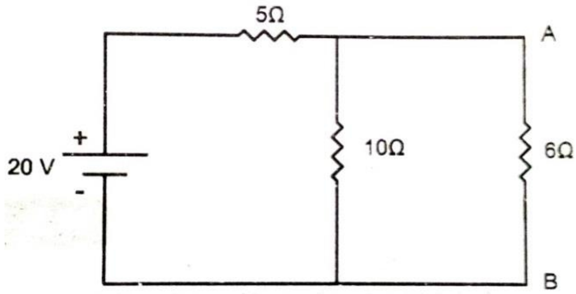


Figure-2.17

Solution

<u>Step-1:</u> Place a short circuit across the terminal A and B and calculate the short circuit current (i_{sc}) as follows.

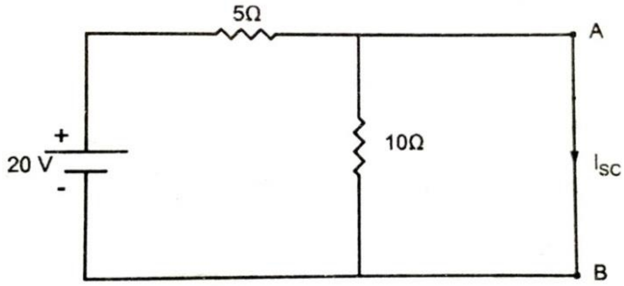


Figure-2.18

$$\mathrm{I_{SC}} = \frac{20}{5} = 4 \text{ Amp}$$

Step-3: Determine the Norton resistance or equivalent resistor or Thevenin resistor (R_{eq}).

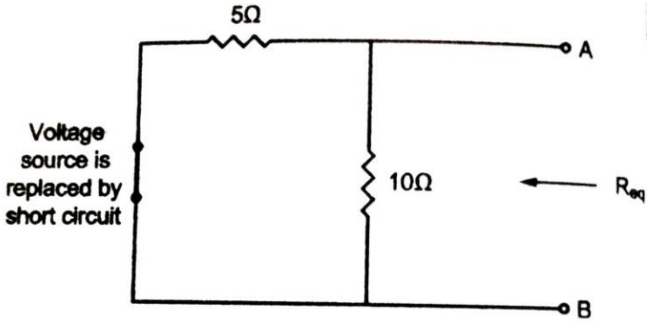


Figure-2.19

$$R_{eq} = 5 \parallel 10$$

$$R_{eq} = \frac{5\times10}{5+10} = 3.33\ \Omega$$

Step-4: Draw Norton's equivalent circuit and reconnect resistor between terminal A and B. and find out the load current.

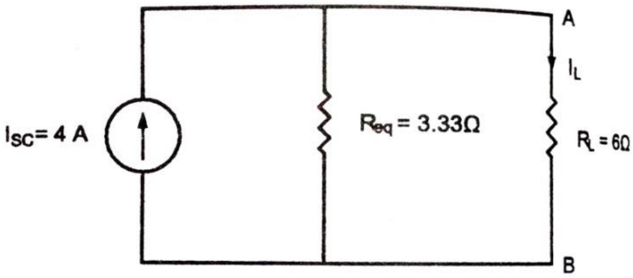


Figure-2.20

$$I_L = I_{SC} \times \frac{R_{eq}}{R_{eq}+R_L} = 4\times \frac{3.33}{3.33+6} = 1.427 \text{ Amp}$$

Example-5

Determine Norton's equivalent circuit at terminal AB for the circuit shown in figure-2.21

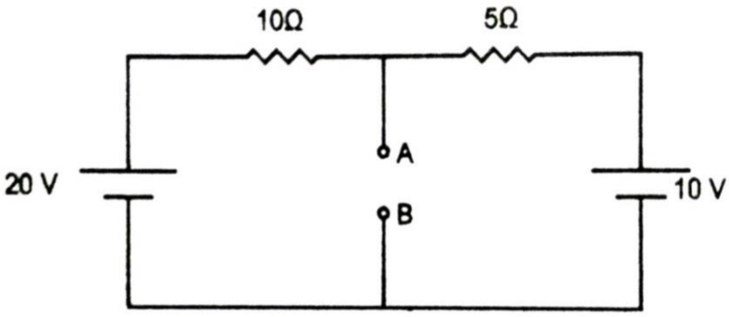


Figure-2.21

Solution

Step-1: Place a short circuit across the terminal A and B and calculate the short circuit current (i_{sc}) as follows.

$$I_{SC} = I_1 + I_2 = \frac{20}{10} + \frac{10}{5}$$
$$= 2 + 2 = 4 \text{ Amp}$$

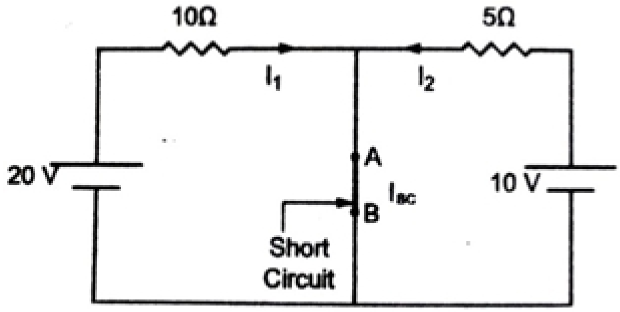


Figure-2.22

Step-3: Determine the Norton resistance or equivalent resistor or Thevenin resistor (R_{eq})

$$R_{eq} = 5 \parallel 10$$
$$R_{eq} = \frac{5 \times 10}{5 + 10} = 3.33\ \Omega$$

Step-4: Draw Norton's equivalent circuit

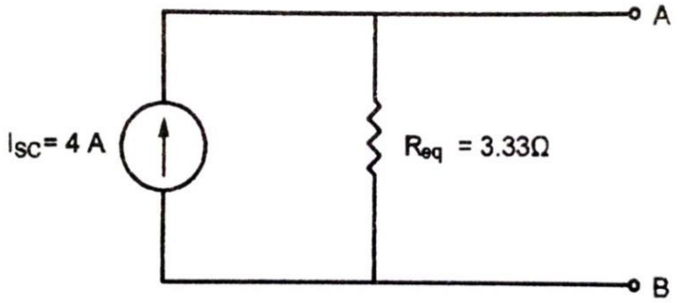


Figure-2.23

2.3.3 Thevenin's and Norton's Equivalent Circuit's Example:

Example-6

Consider the circuit shown in Figure 2.24(a). Reduce the portion of the circuit to the left of terminals *a–b* to (a) a Thevenin equivalent and (b) a Norton equivalent. Find the current through R = 16 Ω, and comment on whether resistance matching is accomplished for maximum power transfer.

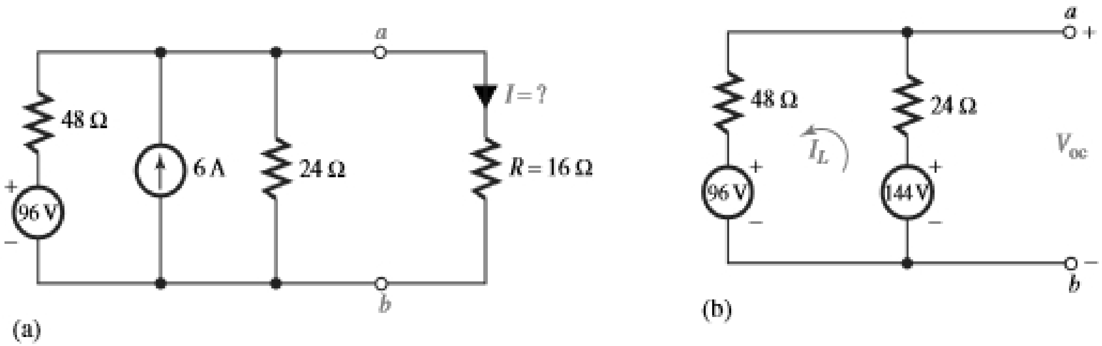


Figure-2.24 (a) & (b)

Solution:

(a) Thevenin equivalent

Step:1 first convert current source to voltage source

The 6-A source with 24 Ω in parallel can be replaced by a voltage source of 6 × 24 = 144 V with 24 in series using source transformation, in terms of voltage sources, the equivalent circuit to the left of terminals *a–b* is shown in Figure 2.24(b).

Apply KVL in loop I_L:

$$144 - 24I_L - 48I_L - 96 = 0,$$
$$72I_L = 48,$$
$$I_L = 2/3\ A$$
$$V_{oc} = 144 - 24(2/3) = 128V$$

Deactivating or zeroing all ideal sources, i.e., replacing voltage sources by short circuits in the present case, the circuit of Figure (b) reduces to that shown in Figure-2.24(c). Terminals *a–b*, the 48-Ω resistor and the 24- Ω resistor are in parallel, in figure-2.24(c)

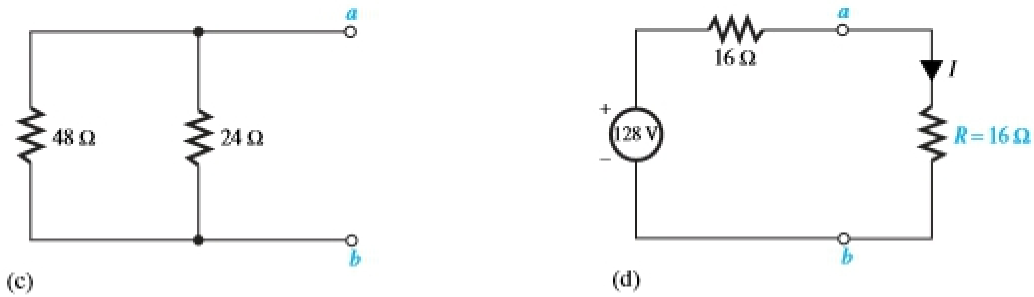


Figure-2.24 (c) & (d)

$$R_{Th} = 48 \parallel 24 = \frac{48 \times 24}{48 + 24} = 16\ \Omega$$

The Thevenin equivalent to the left of terminals a–b, attached with the 16- Ω resistor, is shown in Figure-2.24(d). Note that the Thevenin equivalent of any linear circuit consists of a single Thevenin voltage source in series with a single equivalent Thevenin resistance.

The current in the 16- Ω resistor to the right of terminals *a–b* can now be found,

$$I = 128/32 = 4\ A$$

Solution:

(b) Norton equivalent

The 96-V source with 48 Ω in series can be replaced by a current source of 96/48 = 2A with a parallel resistance of 48 Ω. Thus, by using source transformation, in terms of current sources, the equivalent circuit to the left of terminals *a–b* is given in Figure-2.24.

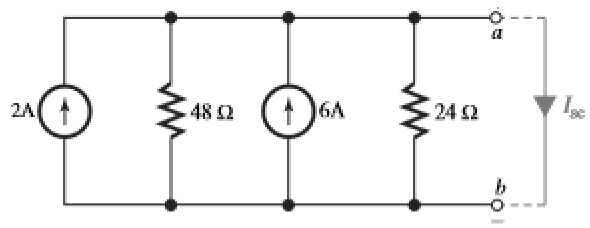


Figure-2.24(e)

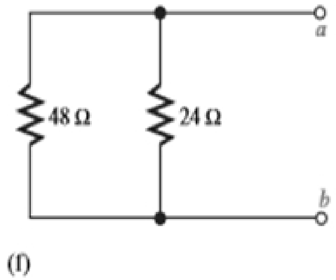
Shorting terminals *a–b*, find I_{sc} ,

$$I_{sc} = 6+2=8 \text{ A.}$$

Replacing current sources by open circuits, viewed from terminals *a–b*,

$$R_{Th} = 48 \parallel 24 = 16\ \Omega ,$$

which is the same as in part (a). The circuit of Figure-2.24(e) to the left of terminals *a–b* reduces to that shown in Figure-2.24(f).



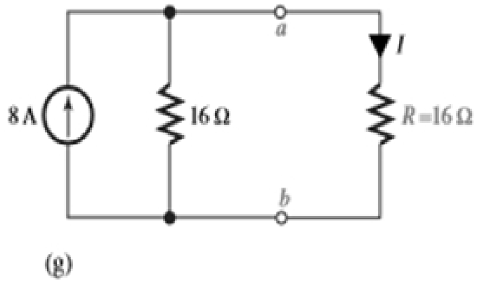


Figure-2.24 (f) & (g)

The Norton equivalent to the left of terminals *a–b*, attached with the 16-Ω resistor, is given in Figure-2.24 (g).

Note that the Norton equivalent of any linear circuit consists of a single current source in parallel with a single equivalent Thevenin resistance.

The current in the 16- Ω resistor to the right of terminals *a–b* can now be found. I= 4 A, which is the same as in part (a).

The equivalent source resistance, also known as the output resistance, is the same as the load resistance of 16 Ω in the present case. Hence, resistance matching is accomplished for maximum power transfer.

Example-7

Consider the circuit of Figure 2.25(a), including a dependent source. Obtain the Thevenin equivalent at terminals *a–b*.

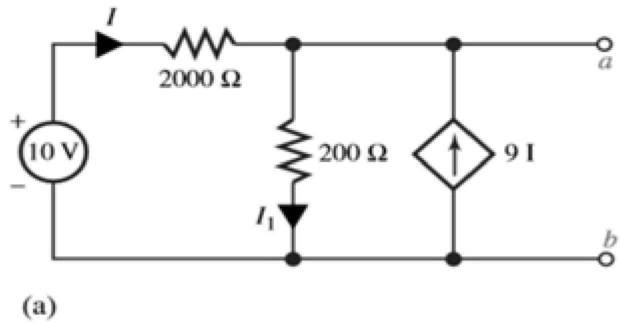


Figure-2.25 (a)

Solution:
First, the open-circuit voltage at terminals *a–b* is to be found.

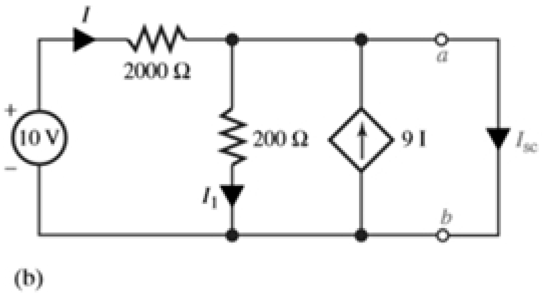
KCL at node *a*: $I + 9I = I_1$, or $I_1 = 10\ I$

KVL for the left-hand mesh:

$$2000\ I + 200\ I_1 = 10, \text{ or } 4000\ I = 10, \text{ or } I = 1/400\ A$$

$$Voc = 200\ I_1 = 200 \times 10\ I = 2000(1/400) = 5V$$

Because of the presence of a dependent source, in order to find R_{Th}, one needs to determine I_{sc} after shorting terminals *a–b,* as shown in Figure 2.25(b).



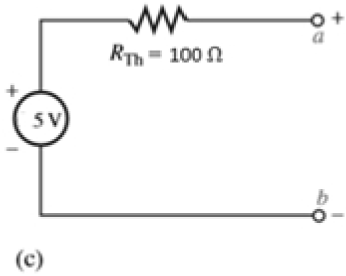


Figure-2.25 (b) & (c)

Note that $I_1 = 0$, since $V_{ab} = 0$.

KCL at node *a*: $I_{sc} = 9I + I = 10\ I$

KVL for the outer loop: $2000\ I = 10$, or $I = 1/200\ A$

$I_{sc} = 10(1/200) = 1/20\ A$

Hence the equivalent Thevenin resistance R_{Th} at, terminals a-b is

$$R_{Th} = \frac{V_{oc}}{I_{SC}} = \frac{5}{1/20} = 100\ \Omega$$

Thus, the Thevenin equivalent is given in figure-2.25(c)

2.4 Node-Voltage and Mesh-Current Analysis:

- The node-voltage and mesh-current methods, which complement each other, are well-ordered systematic methods of analysis for solving complicated network problems.
- The former is based on the KCL equations, whereas the KVL equations form the basis for the latter.

2.4.1 Nodal-Voltage Method

- A set of node-voltage variables that implicitly satisfy the KVL equations is selected in order to formulate circuit equations in this nodal method of analysis.
- A *reference* (datum) node is chosen arbitrarily based on convenience, and from each of the remaining nodes to the reference node, the voltage drops are defined as *node-voltage* variables.
- The circuit is then described completely by the necessary number of KCL equations whose solution yields the unknown nodal voltages from which the voltage and the current in every circuit element can be determined.

- Thus, the number of simultaneous equations to be solved will be equal to one less than the number of network nodes.
- All voltage sources in series with resistances are replaced by equivalent current sources with conductances in parallel.
- Note that the nodal-voltage method is a general method of network analysis that can be applied to any network.
- Figure-2.26 is redrawn as Figure-2.27, in which one can identify three nodes, *A*, *B*, and *O*.
- Notice that the voltages V_{AO}, V_{BO}, and V_{AB} satisfy the KVL relation:

$$V_{AB} + V_{BO} - V_{AO} = 0, \quad \text{or} \quad V_{AB} = V_{AO} - V_{BO} = V_A - V_B$$

where the node voltages V_A and V_B are the voltage drops from A to O and B to O, respectively. With node O as reference, and with V_A and V_B as the node-voltage unknown variables, one can write the two independent KCL equations:

$$\text{Node A: } V_A G_1 + (V_A - V_B)G_3 = I_1 \quad \text{or} \quad (G_1 + G_2)V_A - G_3 V_B = I_1 \qquad (1)$$
$$\text{Node B: } V_B G_2 - (V_A - V_B)G_3 = I_2 \text{ or } -G_3 V_A + (G_2 + G_3)V_B = I_2 \qquad (2)$$

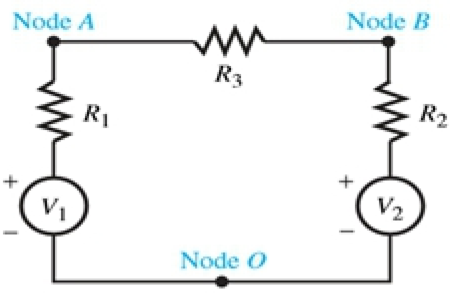


Figure-2.26 Circuit for illustration of nodal-voltage method

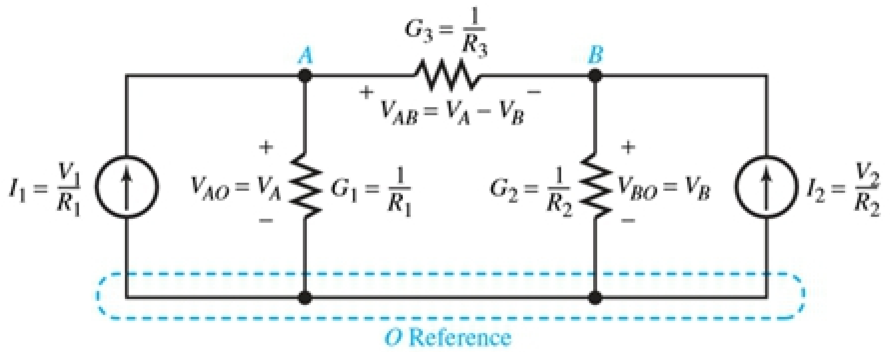


Figure-2.27 Redrawn Figure-2.26 for node-voltage method of analysis

An examination of these equations reveals a pattern that will allow nodal equations to be written directly by inspection by following the rules given here for a network containing no dependent sources.

1. For the equation of node A, the coefficient of V_A is the positive sum of the conductances connected to node A; the coefficient of V_B is the negative sum of the conductances connected between nodes A and B. The right-hand side of the equation is the sum of the current sources feeding into node A.

2. For the equation of node B, a similar situation exists. Notice the coefficient of V_B to be the positive sum of the conductances connected to node B; the coefficient of V_A is the negative sum of the conductances connected between B and A. The right-hand side of the equation is the sum of the current sources feeding into node B.

Example-8

Use nodal analysis to find the voltage across 5 Ω resistor, for the network of figure-2.28

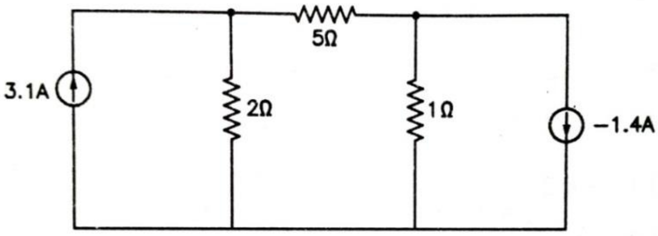


Figure-2.28

Solution:

<u>Step-1</u> Identify the no. of nodes in the given circuit.

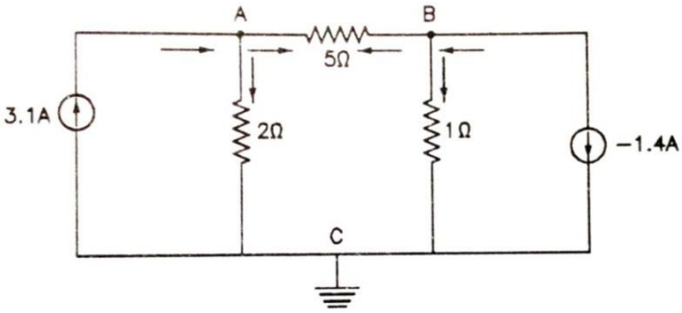


Figure-2.29

There are three nodes named A, B and C. Let us consider node C as reference node.
Let us assume potential at node A is V_A and potential at node B is V_B is as shown.

<u>Step-2</u> Applying KCL at node-A,

$$3.1 - \frac{V_A}{2} - \frac{V_A - V_B}{5} = 0$$
$$\frac{V_A}{2} + \frac{V_A}{5} - \frac{V_B}{5} = 3.1$$
$$0.5V_A + 0.2V_A + 0.2V_B = 3.1$$
$$0.7V_A - 0.2V_B = 3.1 \quad (1)$$

Applying KCL at node-B,

$$\frac{V_B - V_A}{5} + \frac{V_B}{1} - 1.4 = 0$$
$$\frac{V_B}{5} - \frac{V_A}{5} - \frac{V_B}{1} = 1.4$$
$$0.2V_B - 0.5V_A + V_B = 1.4$$
$$-0.2V_A + 1.2V_B = 1.4 \quad (2)$$

Solving equation (1) and (2) we get the value of V_A, V_B

$$\mathrm{V_A} = 5 \text{ Volts}, \mathrm{V_B} = 2 \text{ Volts}$$

Hence, the voltage across the 5 Ω resistor is

$$V_5 = V_A - V_B = 3\,Volts$$

Example-9

Find current through 2 Ω resistance using nodal analysis.

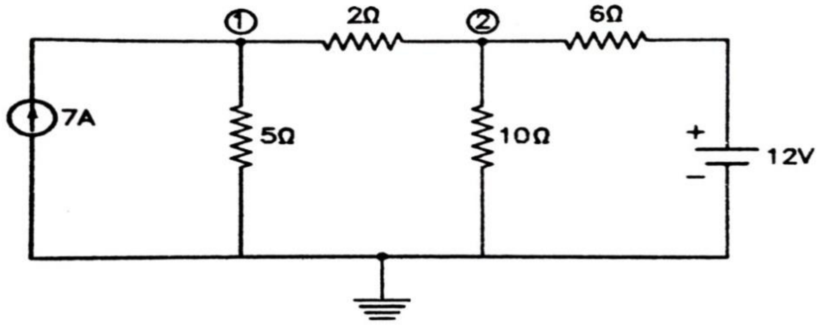


Figure-2.30

Solution:

Step:1 Convert voltage source into current source and write conductance value in place of resistance value

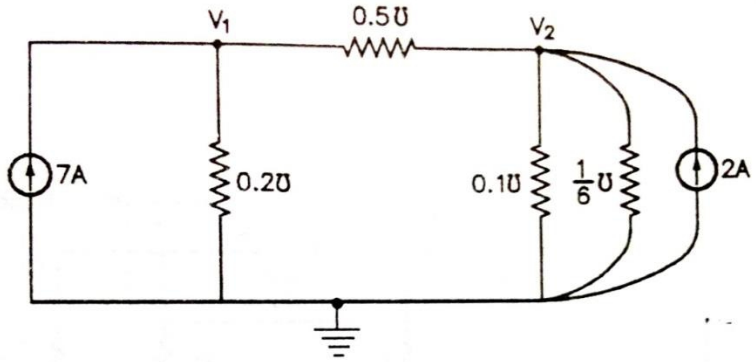


Figure-2.31

Step-2 Applying KCL at node-1

$$7 = 0.2V_1 + 0.5(V_1 - V_2)$$

$$7 = 0.7V_1 - 0.5V_2 \qquad (1)$$

Applying KCL at node-2

$$2 = 0.1V_2 + 0.166V_2 + 0.5(V_2 - V_1)$$

$$2 = 0.766V_2 - 0.5V_1 \qquad (2)$$

Now, multiply equation (1) by 5 and equation (2) by 7 and add

$$3.5V_1 - 2.5V_2 = 35$$

$$\underline{-3.5V_1 + 5.362V_2 = 14}$$

$$2.862V_2 = 21$$

$$V_2 = 7.337 \text{ V}$$

Substitute V_2 in equation (1), we get

$$7 = 0.7V_1 - 0.5 \times 7.337$$
$$V_1 = 15.24\ \text{V}$$

Current in 2 ohm resistor is

$$I = \frac{V_1 - V_2}{R} = \frac{15.24 - 7.337}{2}$$

$$I = 3.9525\ A$$

Example-10

By means of nodal analysis, find the current delivered by the 10-V source and the voltage across the 10- **Ω** Resistance in the circuit shown in Figure-2.32 (a)

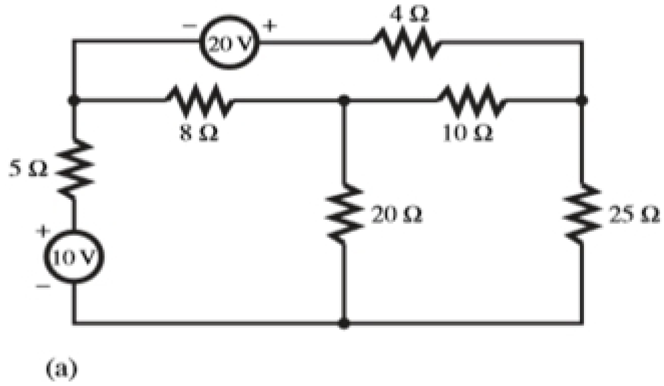


Figure-2.32 (a)

Solution:

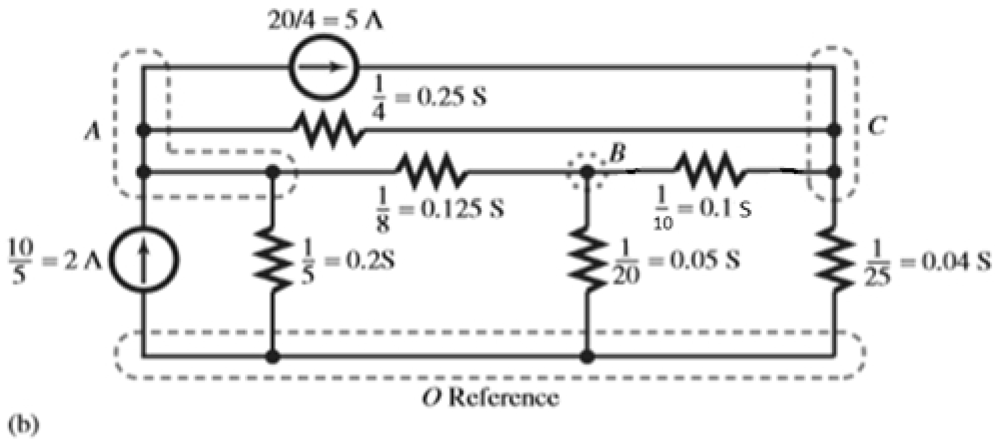


Figure-2.32 (b)

Step 1: Replace all voltage sources with series resistances by their corresponding Norton equivalents consisting of current sources with shunt conductances. The given circuit is redrawn in Figure 2.32(b) by replacing all resistors by their equivalent conductances.

Step 2: Identify the nodes and choose a convenient reference nodes O. this is also shown in figure 2.32(b).

Step 3: In terms of unknown node-voltage variables, write the KCL equations at all nodes (except, of course, the reference node) by following rules 1 and 2 for nodal equations given in this section.

$$\text{Node A:}\quad (0.2+0.125+0.25)V_A - 0.125V_B - 0.25V_C = 2-5 = -3$$
$$\text{Node B:}\quad -0.125V_A + (0.125+0.05+0.1)V_B - 0.1V_C = 0$$
$$\text{Node C:}\quad -0.25V_A - 0.1V_B + (0.25+0.1+0.04)V_C = 5$$

Rearranging, one gets

$$0.575V_A - 0.125V_B - 0.25V_C = -3$$
$$-0.125V_A + 0.275V_B - 0.1V_C = 0$$
$$-0.25V_A - 0.1V_B + 0.39V_C = 5$$

Step 4: Simultaneously solve the independent equations for the unknown nodal voltages by Gauss elimination or Cramer's rule. In our example, the solution yields

$V_A = 4.34$ V; $V_B = 8.43$V; $V_C = 17.77$ V

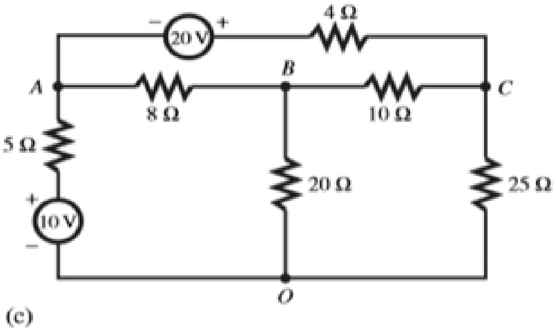


Figure-2.32 (c)

Step 5: Obtain the desired voltages and currents by the application of KVL and Ohm's law. To find the current *I* in the 10-V source, since it does not appear in Figure 3(b) redrawn for nodal analysis, one has to go back to the original circuit and identify the equivalence between nodes *A* and *O*, as shown in Figure-2.32(c).

$$V_A = 4.34 = -5I + 10 \quad \text{or} \quad I = \frac{5.66}{5} = 1.132 \text{ A}$$

Now one can solve for *I*, delivered by the 10-V source

The voltage across the 10-Ω resistance is $V_B - V_C = 8.43 - 17.77 = -9.34$ V. The negative sign indicates that node *C* is at a higher potential than node *B* with respect to the reference node *O*.

Example-11

For the network shown in figure, find the current in each resistor by means of nodal analysis

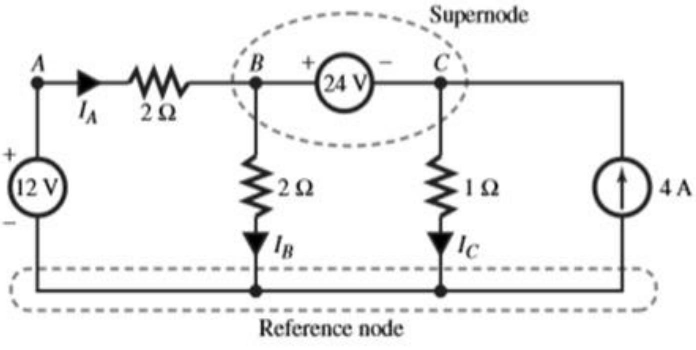


Figure-2.33

Note that the reference node is chosen at one end of an independent voltage source, so that the node voltage V_A is known at the start,

$$V_A = 12\text{ V}$$

Note that we cannot express the branch current in the voltage source as a function of V_B and V_C. Here we have constrained nodes *B* and *C*. Nodal voltages V_B and V_C are not independent. They are related by the constrained equation

$$V_B - V_C = 24\text{ V}$$

Let us now form a *supernode*, which includes the voltage source and the two nodes *B* and *C*, as shown in Figure-2.33. KCL must hold for this supernode, that is, the algebraic sum of the currents entering or leaving the supernode must be zero. Thus one valid equation for the network is given by

$$I_A - I_B - I_C + 4 = 0$$

OR

$$\frac{12-V_B}{2} - \frac{V_B}{2} - \frac{V_C}{1} + 4 = 0$$

which reduces to

$$V_B + V_C = 10\text{ V}$$

This equation together with the supernode constraint equation yields

$$V_B = 17\text{ V and } V_C = -7\text{ V}$$

The current in the resistors are thus given by

$$I_A = \frac{12 - V_B}{2} = \frac{12 - 17}{2} = -2.5\text{ A}$$

$$I_B = \frac{V_B}{2} = \frac{17}{2} = 8.5\text{ A}$$

$$I_C = \frac{V_C}{1} = \frac{-7}{1} = -7\text{ A}$$

2.4.2 Mesh-Current Method:

- This complements the nodal-voltage method of circuit analysis. A set of independent *mesh- current* variables that implicitly satisfy the KCL equations is selected in order to formulate circuit equations in this mesh analysis.
- An *elementary loop*, or a *mesh*, is easily identified as one of the “window panes” of the whole circuit.
- All current sources with shunt conductances will be replaced by their corresponding Thevenin equivalents consisting of voltage sources with series resistances.
- Replacing the current source with shunt resistance by the Thevenin equivalent, Figure-2.35 is redrawn as Figure-2.36, in which one can identify two elementary loops, or independent meshes.
- By assigning loop or mesh-current variables I_1 and I_2, as shown in Figure 2, both in the clockwise direction, one can write the KVL equations for the two closed paths (loops) ABDA and BCDB,

Loop ABDA: $I_1R_1 + (I_1 - I_2)R_2 = V_1 - V_2$ **or**

$(R_1 + R_2)I_1 - R_2I_2 = V_1 - V_2$ (1)

Loop BCDB: $I_2R_3 + (I_2 - I_1)R_2 = V_2 - V_3$ **or**

$-R_2I_1 + (R_2 + R_3)I_2 = V_2 - V_3$ (2)

- Notice that current I_1 exists in R_1 and R_2 in the direction indicated; I_2 exists in R_2 and R_3 in the direction indicated; hence, the net current in R_2 is $I_1 - I_2$ directed from B to D. An examination of Equations (1) and (2) reveals a pattern that will allow loop equations to be written directly by inspection by following rules:

1. In the first loop equation with mesh current I_1, the coefficient of I_1 is the sum of the resistances in that mesh; the coefficient of I_2 is the negative sum of the resistances common to both meshes. The right-hand side of the equation is the algebraic sum of the source voltage rises taken in the direction of I_1.
2. Similar statements can be made for the second loop with mesh current I_2. (See also the similarity in setting up the equations for the mesh-current and nodal-voltage methods of analysis.)

- Such a formal systematic procedure will yield a set of *N* independent equations of the following form for a network with *N* independent meshes containing no dependent sources:

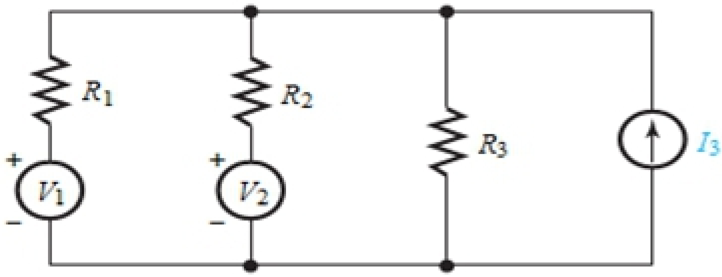


Figure-2.35 Illustration of mesh-current analysis

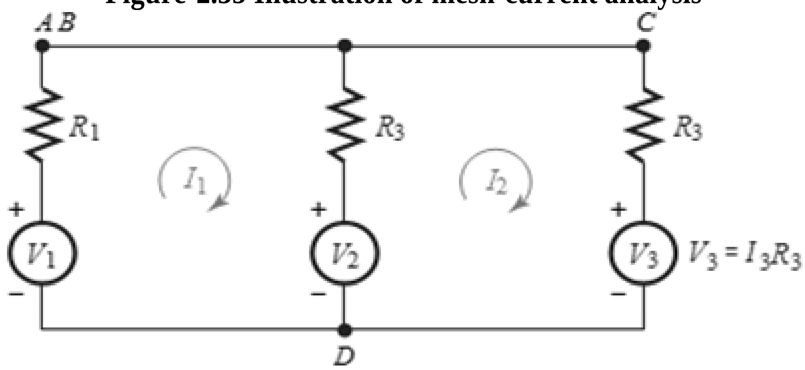


Figure-2.36 Redrawn figure-2.35 for mesh-current method of analysis

Example-12

For the given network, of figure write the mesh current equations and determine the currents.

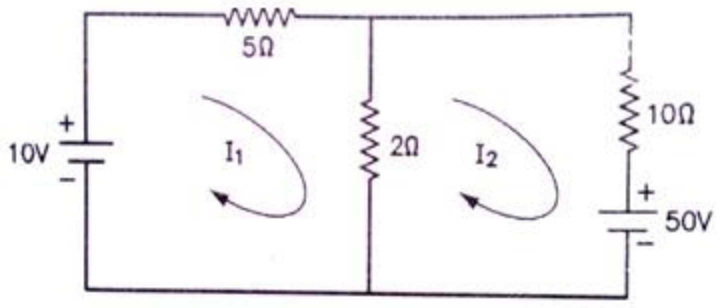


Figure-2.37

Solution:

Applying KVL to the above circuit,

1st loop equation,

$$10 - 5I_1 - 2(I_1 - I_2) = 0$$

$$7I_1 - 2I_2 = 10 \qquad (1)$$

2nd loop equation

$$-10I_2 - 50 - 2(I_2 - I_1) = 0$$

$$12I_2 - 2I_1 = -50 \qquad (2)$$

Now, multiply equation (1) by 6 and add to equation (2)

$$42I_1 - 12I_2 = 60$$

$$\underline{-2I_1 + 12I_2 = -50}$$

$$40I_1 = 10$$

So

$$I_1 = 10/40 = 0.25 \text{ A}$$

Put the value of I_1 in equation (1)

$$7 \times 0.25 - 2I_2 = 10$$

$$1.75 - 10 = 2I_2$$

So,

$$I_2 = -4.125\ A$$

Example-13

Determine the mesh currents I_1, I_2 and I_3 for network shown figure.

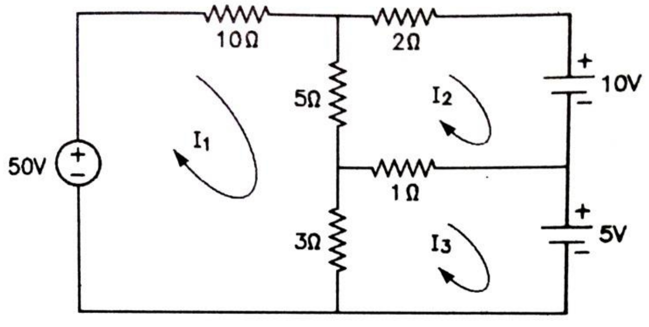


Figure-2.38

Solution:

Applying KVL to the above circuit

The mesh current equations are,

KVL for loop-1,

$$50 - 10I_1 - 5(I_1 - I_2) - 3(I_1 - I_3) = 0$$

$$50 = 18I_1 - 5I_2 - 3I_3 = 0 \qquad (1)$$

KVL for loop-2

$$-2I_2 - 10 - 1(I_2 - I_3) - 5(I_2 - I_1) = 0$$

$$-10 = -5I_1 + 8I_2 - I_3 \qquad (2)$$

KVL for loop-3

$$-5 - 3(I_3 - I_1) - 1(I_3 - I_2) = 0$$

$$-5 = -3I_1 - I_2 + 4I_3 \qquad (3)$$

From equation (1), (2) and (3)

$$\begin{bmatrix} 50 \\ -10 \\ -5 \end{bmatrix} = \begin{bmatrix} 18 & -5 & -3 \\ -5 & 8 & -1 \\ -3 & -1 & 4 \end{bmatrix} \begin{bmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \\ \mathrm{I}_3 \end{bmatrix}$$

Now,

$$\Delta\mathrm{R} = \begin{vmatrix} 18 & -5 & -3 \\ -5 & 8 & -1 \\ -3 & -1 & 4 \end{vmatrix}$$

ΔR = 18[(8×4) – (-1×-1)] – (-5)[(-5×4)-(-1×-3)] + (-3)[(-5×-1)-(8×-3)]

=18(32-1)+5(-20-3)-3(5+24)

=18(31)+5(-23)-3(29)

=558-115-87

ΔR =356 Ω

That's way $\Delta\mathrm{R}_1, \Delta\mathrm{R}_2$ *and* $\Delta\mathrm{R}_3$

$$\Delta R_1 = \begin{vmatrix} 50 & -5 & -3 \\ -10 & 8 & -1 \\ -5 & -1 & 4 \end{vmatrix} = 1175\ \Omega$$

$$\Delta R_2 = \begin{vmatrix} 18 & 50 & -3 \\ -5 & -10 & -1 \\ -3 & -5 & 4 \end{vmatrix} = 355\ \Omega$$

$$\Delta R_3 = \begin{vmatrix} 18 & -5 & 50 \\ -5 & 8 & -10 \\ -3 & -1 & -5 \end{vmatrix} = 525\ \Omega$$

Using Cramer's rule,

$$I_1 = \frac{\Delta R_1}{\Delta \mathrm{R}} = \frac{1175}{356} = 3.30\ \mathrm{A}$$

$$I_2 = \frac{\Delta R_2}{\Delta \mathrm{R}} = \frac{355}{356} = 0.99\ \mathrm{A}$$

$$I_3 = \frac{\Delta R_3}{\Delta \mathrm{R}} = \frac{525}{356} = 1.47 \text{ A}$$

Example-14

By means of mesh-current analysis, obtain the current in the 10-V source and the voltage across the 10-Ω resistor in the circuit.

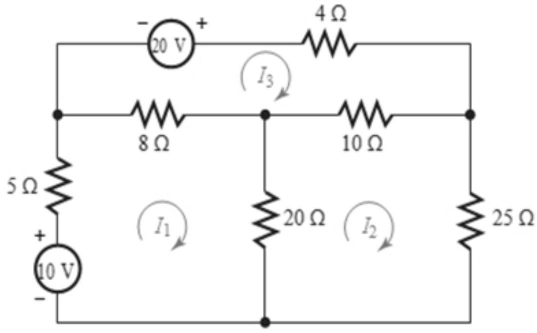


Figure-2.39

Solution:

Step-1: Replace all current sources with shunt resistances by their corresponding Thevenin equivalents consisting of voltage sources with series resistances. Conductances included in the circuit are replaced by their equivalent resistances.

In this example, since there are no current sources and conductances, the circuit of Figure-2.39 is redrawn as Figure 2 for convenience.

Step-2: Identify elementary loops (meshes) and choose a mesh-current variable for each elementary loop, with all loop currents in the same clockwise direction. Mesh currents I_1, I_2, and I_3 are shown in Figure 3.

Step-3: In terms of unknown mesh-current variables, write the KVL equations for all meshes by following the rules for mesh analysis.

Loop 1 with mesh current I_1: $(5 + 8 + 20)\, I_1 - 20I_2 - 8I_3 = 10$

Loop 2 with mesh current I_2: $-20I_1 + (20 + 10 + 25)I_2 - 10I_3 = 0$

Loop 3 with mesh current I_3: $-8I_1 - 10I_2 + (4 + 10 + 8)I_3 = 20$

Rearranging, one gets

$$33I_1 - 20I_2 - 8I_3 = 10$$
$$-20I_1 + 45I_2 - 10I_3 = 0$$
$$-8I_1 - 10I_2 + 22I_3 = 20$$

Step-4: Simultaneously solve the independent equations for the unknown mesh currents by Gauss Elimination or Cramer's rule.

In this example the solution yields

$I_1 = 1.132$ A; $I_2 = 0.711$ A; $I_3 = 1.645$ A

The current through the 10-V source is $I_1 = 1.132$ A, The voltage across the 10- Ω resistor is

$$V_{BC} = 10(I_2 - I_3) = 10(0.711 - 1.645) = -9.34 \text{ V}$$

Example-15

For the network shown in Figure 4, find the current delivered by the 10-V source and the voltage across the 3-Ω resistor by means of mesh-current analysis

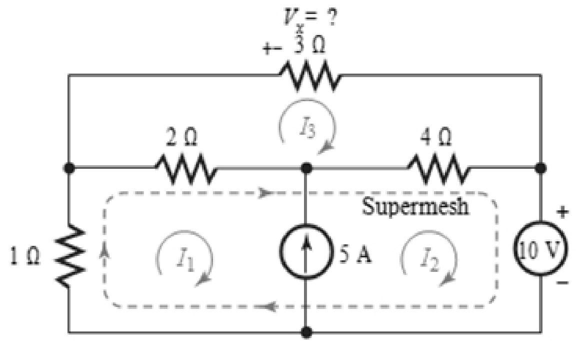


Figure-2.40

Solution:

Note that we cannot express the voltage across the current source in terms of the mesh currents I_1 and I_2 . The current source does, however, *constrain* the mesh currents by the following equation:

$$I_2 - I_1 = 5$$

Let us now form a *supermesh*, which includes meshes 1 and 2, as shown in Figure. We now write a KVL equation around the periphery of meshes 1 and 2 combined. These yields

$$1I_1 + 2(I_1 - I_3) + 4(I_2 - I_3) + 4(I_2 - I_3) + 10 = 0$$

Next we write a KVL equation for mesh 3,

$$3I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$$

Simplify all this equations, and rewrite

$$3I_1 + 8I_2 - 10I_3 = -10$$
$$-2I_1 - 4I_2 + 9I_3 = 0$$
$$I_2 - I_1 = 5$$

Now we have the three linearly independent equations needed to find the three mesh currents I_1, I_2, and I_3

$$I_1 = \frac{-25}{9}\text{A}, \; I_2 = \frac{20}{9}\text{A}, \; I_3 = \frac{70}{27}\text{A}$$

The current delivered by the 10-V source is $-I_2$, or −20/9 A. That is to say, the 10-V source is absorbing the current 20/9 A.

The voltage across the 3-Ω resistor is $V_x = 3I_3 = 3(70/27) = 70/9 =$ **7.78 V.**

2.4.3 Node-Voltage and Mesh-Current Equations with Controlled Sources

- Since a controlled source acts at its terminals in the same manner as does an independent source, source conversion and application of KCL and KVL relations are treated identically for both types of sources.

- Because the strength of a controlled source depends on the value of a voltage or current elsewhere in the network, a *constraint* equation is written for each controlled source.
- After combining the constraint equations with the loop or nodal equations based on treating all sources as independent sources, the resultant set of equations are solved for the unknown current or voltage variables.

Example-16

Consider the circuit in Figure-2.41, which include a controlled source, and find the current in the 5-V source and the voltage across the 5-Ω resistor by using (a) The loop-current method and (b) The node-voltage method.

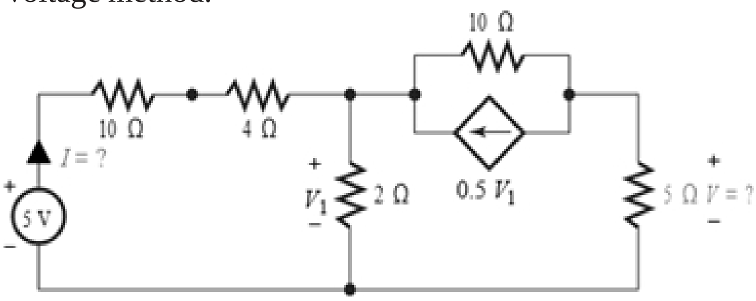


Figure-2.41

Solution:

(a) Loop-Current Method:

The voltage-controlled current source and its parallel resistance are converted into a voltage-controlled voltage source and series resistance. When you are source transforming dependent sources, note that the identity of the control variable (i.e., the location in the circuit) must be retained.

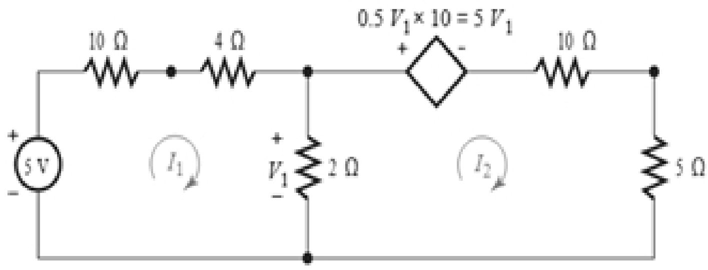


Figure-2.42

The converted circuit is shown in Figure-2.42 with the chosen loop currents I_1 and I_2

The KVL equations are

For loop carrying I_1: $(10 + 4 + 2)I_1 - 2I_2 = 5$

For loop carrying I_2: $-2I_1 + (2 + 10 + 5)I_2 = -5V_1$

The constraint equation is

$$V_1 = (I_1 - I_2)2$$

Combining the constraint equation with the loop equations, one gets

$$16I_1 - 2I_2 = 5; -2I_1 + 17I_2 = -10(I_1 - I_2), \qquad \textbf{or}$$

$$8I_1 + 7I_2 = 0$$

from which $I_1 = 35/128\,A$; $I_2 = -5/16\,A$

Thus, the current through the 5-V source is $I = I_1 = 35/128 = 0.273\,A$, and the voltage across the 5- Ω resistor is $V = 5I_2 = 5(-5/16) = -1.563\,V$.

(b) Node-Voltage Method:

The 5-V voltage source with its 10-Ω series resistor is replaced by its Norton equivalent. Resistances are converted into conductances and the circuit is redrawn in Figure-2.43 with the nodes shown.

The nodal equations are

$$A : (0.1 + 0.25)V_A - 0.25V_B = 0.5$$
$$B : -0.25V_A + (0.25 + 0.5 + 0.1)V_B - 0.1V_C = 0.5V_1$$
$$C : -0.1V_B + (0.1 + 0.2)V_C = -0.$$

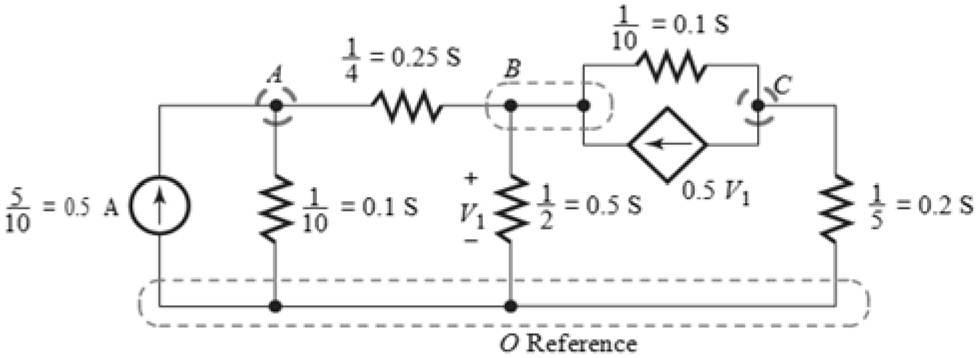


Figure-2.43

The constraint equation is $V_1 = V_B$

Combining these with the nodal equations already written, one has

$$0.35V_A - 0.25V_B = 0.5$$
$$-0.25V_A + 0.35V_B - 0.1V_C = 0$$
$$0.4V_B + 0.3V_C = 0$$

Solving, one gets

$V_A = 2.266V$; $V_B = 1.173V$; $V_C = -1.564$ V

Notice that $V_C = -1.564$ V is the voltage V across the 5-Ω resistor, which is almost the same as that found in part (a).

In order to find the current *I* through the 5-V source, one needs to go back to the original circuit and recognize that

$$5 - 10I = V_A = 2.266 \quad \text{or} \quad I = 0.273\,A$$

Which is the same as that found in part (a)

1.4 Superposition & Linearity:

- Mathematically a function is said to be linear if it satisfies two properties: *homogeneity* (*proportionality* or *scaling*) and *additivity* (*superposition*),

  $f(Kx) = Kf(x)$ (Homogeneity), Where K is a scalar constant, and

  $f(x_1 + x_2) = f(x_1) + f(x_2)$ (additivity)

  Linearity requires both additivity and homogeneity.
- For a linear circuit or system in which excitations x_1 and x_2 produce responses y_1 and y_2, respectively, the application of K_1x_1 and K_2x_2 together (i.e., $K_1x_1 + K_2x_2$) results in a response of $(K_1y_1 + K_2y_2)$, where K_1 and K_2 are constants. With the cause-and-effect relation between the excitation and the response, all *linear* systems satisfy the principle of *superposition*.

- Superposition theorem is used to solve electrical circuits which have more than one energy source.
- This theorem is applicable to only linear circuit.
- Therefore, it is not applicable to circuit which have non-linear elements such as diodes, transistors etc.

Example-17

To explain the superposition theorem let us find the current 9 Ω resistor as shown in the circuit.

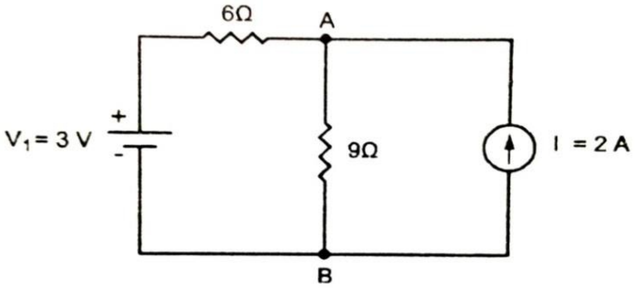


Figure-2.44

Solution:

Step-1 Let us first consider voltage V_1 only and reduce the current source to zero (open circuit)

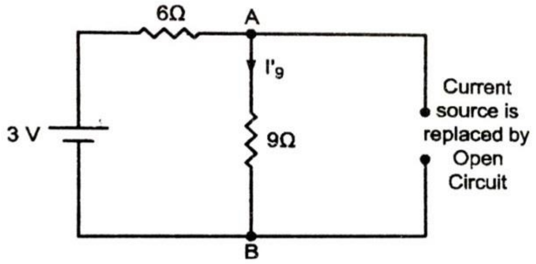


Figure-2.45

Therefore,

$$I' = \frac{3}{6+9} = \frac{3}{15} = 0.2 \text{ Amp} \quad \text{from A to B}$$

Step-2 Let us now consider current source I_2 alone reducing voltage source, V_1 to zero(replacing it by short circuit)

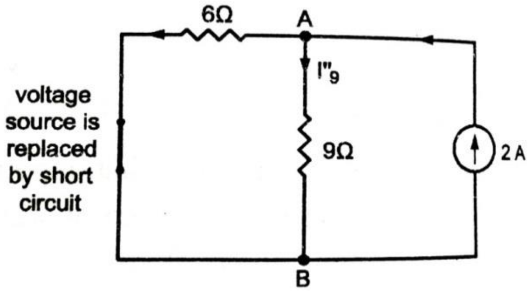


Figure-2.46

$$I'' = 2 \times \frac{6}{6+9} = \frac{12}{15} = 0.8 \text{ Amp} \quad \text{from A to B}$$

Step-3 Calculation of I_9

Therefore, current in 9 Ω resistor when both the sources are acting,

I_9 = algebraic sum of the two current

$$I_9 = I'_9 + I''_9 = 0.2 + 0.8 = \mathbf{1\ Amp}$$

Example-18

Determine the current in each resistor of figure using the superposition theorem.

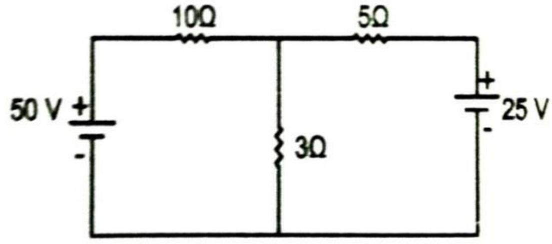


Figure-2.47

Solution:

<u>Step-1</u>: Consider 50 V source alone

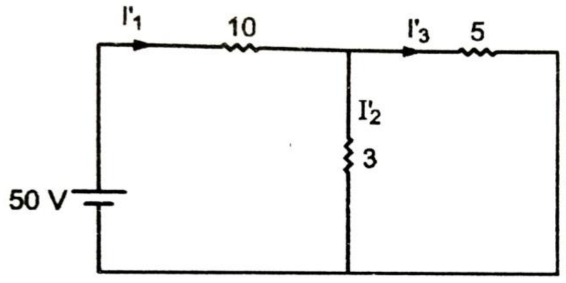


Figure-2.48

The total resistance across 50 V = $10 + 3 \| 5 = 10 + \frac{3 \times 5}{3+5} = 11.9\ \Omega$

Current supplied by 50 V source, $I_1 = \frac{50}{11.9} = 4.2$ A

Current in 10 Ω resistor, $I'_1 = 4.2$ A

The current I'_1 divides in parallel paths formed by resistors 3 Ω and 5 Ω

Current in 3Ω resistor, $I'_2 = 4.2 \times \frac{5}{3+5} = 2.63$ A

Current in 5Ω resistor, $I'_3 = 4.2 \times \frac{3}{3+5} = 1.58$ A

<u>Step-2</u>: Consider 25 V source alone

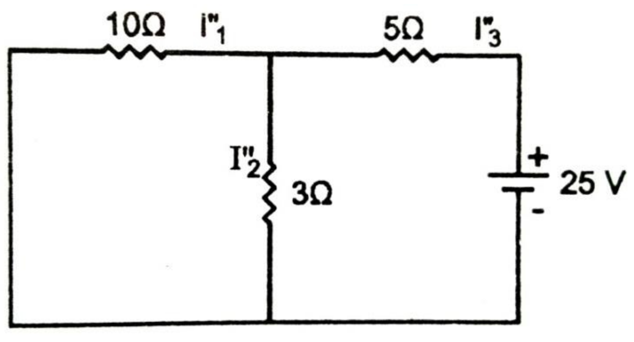


Figure-2.49

The total resistance across 25 = $5 + 3 \| 10$

$$= 5 + \frac{3 \times 10}{3 + 10}$$

$$= 7.31\ \Omega$$

Current supplied by battery = current through 5 Ω resistor

$$I''_3 = \frac{25}{7.31} = 3.42\ \text{A}$$

Current in 3 Ω resistor, $I''_2 = 3.42 \times \frac{10}{3+10} = 2.63\ \text{A}$

Current in 10 Ω resistor, $I''_1 = 3.42 \times \frac{10}{3+10} = 0.78\ \text{A}$

Step-3 Calculate the total current I_1, I_2 and I_3.

According to superposition theorem

Current in 10 Ω Resistor, $\mathrm{I}_1 = \mathrm{I}'_1 - \mathrm{I}''_1 = 4.2 - 0.78 = 3.42\ \text{A}$

Current in 3 Ω Resistor, $\mathrm{I}_2 = \mathrm{I}'_2 - \mathrm{I}''_2 = 2.63 + 2.63 = 5.26\ \text{A}$

Current in 5 Ω Resistor, $\mathrm{I}_3 = \mathrm{I}'_3 - \mathrm{I}''_3 = 3.42 - 1.58 = 1.84\ \text{A}$

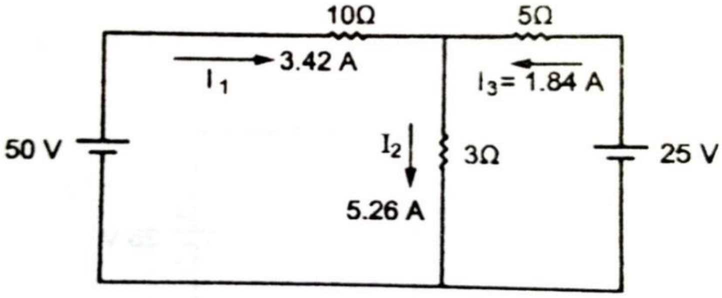


Figure-2.50

Example-19

Determine the voltage across the 20-Ω resistor in the following circuit of Figure 2.51 with the application of superposition. **(W-2014)** **(7)**

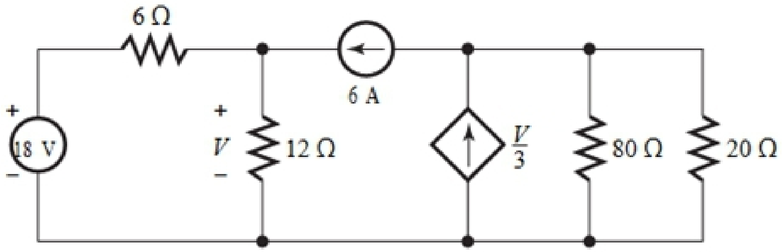


Figure-2.51

Solution:

Let us suppress the independent sources in turn, recognizing that there are two independent sources.

First, by replacing the independent current source with an open circuit, the circuit is drawn in Figure-2.52.

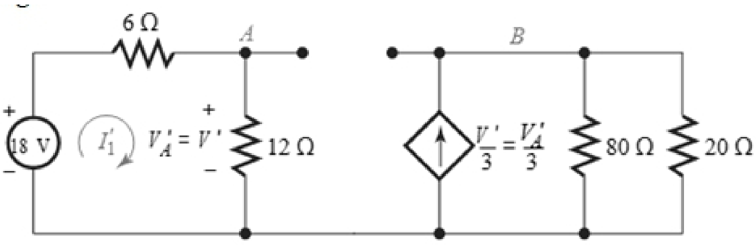


Figure-2.52

Notice that the designation of V' across the 12-Ω resistor and V'/3 as the dependent current source for this case. At node B,

$$\left(\frac{1}{80}+\frac{1}{20}\right)V'_B=\frac{V'_A}{3} \text{ or } V'_B=\frac{20V'_A}{3.75}$$

For the mesh on the left-hand side, $(6+12)\,I'_1=18$, or $I'_1=1$ A. but, $I'_1=V'_A/12$, or $V'_A=12$ V.
The voltage across the 20-Ω resistor form this part of the solution is

$$V'_B=\frac{20V'_A}{3.75}=\frac{20\times 12}{3.75}=64 \text{ V}$$

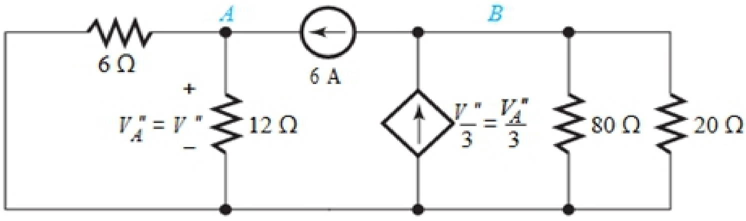


Figure-2.53

Next, by replacing the independent voltage source with short circuit, the circuit is shown in figure-2.53. Notice the designation of V'' across the 12-Ω resistor and V''/3 as the dependent current source for this case. At node A,

$$\left(\frac{1}{6}+\frac{1}{12}\right)V''_A=6 \text{ or } V''_A=24 \text{ V}$$

and at node B,

$$\left(\frac{1}{80}+\frac{1}{20}\right)V''_B=\frac{V''_A}{3}-6=\frac{24}{3}-6=2 \text{ or } V''_B=32 \text{ V}$$

Thus, the voltage across the 20-Ω resistor for this part of the solution is

$$V''_B=32 \text{ V}$$

Then the total net response, by superposition, is

$$V_B=V'_B+V''_B=64+32=96 \text{ V}$$

The principle of superposition is indeed powerful tool for analyzing a wide range of linear systems in electrical, mechanical, civil or industrial engineering.

2.6 Wye-Delta Transformation

(Explain WYE-DELTA transformation in brief with necessary equations and circuit diagrams.**) (W-2014)** **(3)**

2.6.1 Delta-Wye Transformation

- It is the replacement of delta connected resistance into equivalent star connected system.
- Suppose three resistances R_{12}, R_{23} and R_{31} are connected in delta fashion between the terminal 1, 2 and 3 as shown in figure-2.54(a).

- So far as the respective terminals are concerned, these three resistance (R_{12}, R_{23} and R_{31}) can be replaced by the three resistance R_1, R_2 and R_3 connected in star system as shown in figure-2.54(b)

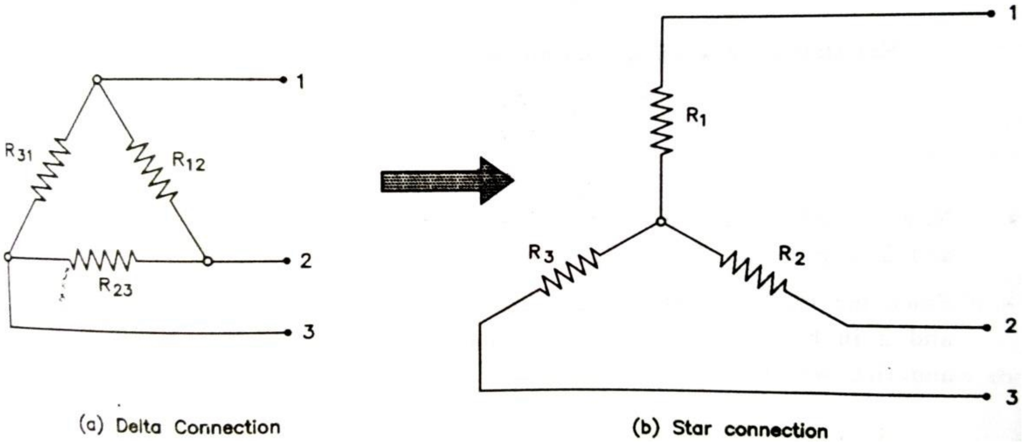


Figure-2.54 (a) & (b)

- These two systems will be electrically equivalent if the resistances as measured between any pair of terminals is the same in both the arrangements when the third terminals open circuited.
- First consider delta connection. Between the terminal 1 and 2, there are two parallel paths, one having a resistance of R_{12} and the other having a resistance of $(R_{23} + R_{31})$.

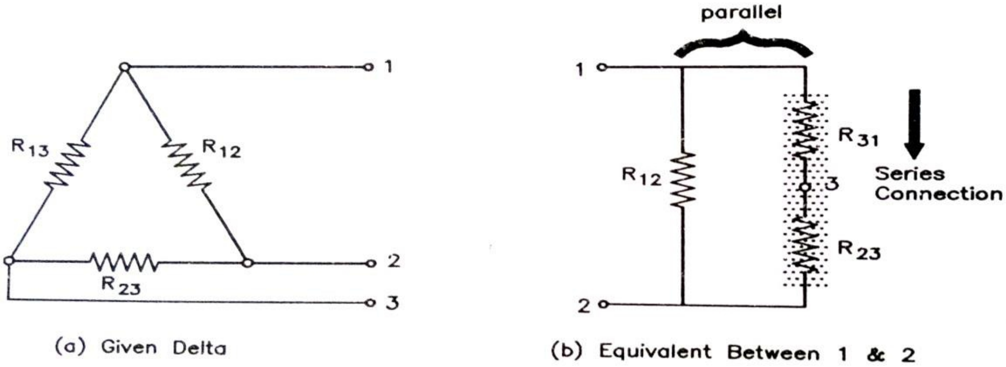


Figure-2.55 (a) & (b)

- Resistance between the terminal 1 and 2 = $R_{12} \| (R_{23} + R_{31})$.

$$= \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad (1)$$

- Now consider start connection. The resistance between the same terminals 1 and 2 (figure-2.55(b)) is $(R_{31} + R_{23})$. (2)
- Since the two systems are identical, resistance measured between terminals 1 and 2 in both systems must be equal. Therefore equating the equation (1) and (2), we get.

$$R_1 + R_2 = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad (3)$$

- Similarly for terminals 2 and 3 and terminals 3 and 1, we get

$$R_2 + R_3 = \frac{R_{23} \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \qquad (4)$$

And

$$R_3 + R_1 = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \qquad (5)$$

Now subtracting (4) from (3) and adding the result to (5) we get,

$$(R_1 + R_2) - (R_2 + R_3) + (R_3 + R_1) = \frac{R_{12} \times (R_{23} + R_{31}) - R_{23} \times (R_{31} + R_{12}) + R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}$$

$$2R_1 = \frac{R_{12}R_{23} + R_{12}R_{31} - R_{23}R_{31} + R_{23}R_{12} + R_{31}R_{12} + R_{31}R_{23}}{R_{12} + R_{23} + R_{31}}$$

$$2R_1 = \frac{2R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

$$R_1 = \frac{R_{12}R_{31}}{R_{12} + R_{23} + R_{31}}$$

Similarly,

$$R_2 = \frac{R_{23}R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_3 = \frac{R_{31}R_{23}}{R_{12} + R_{23} + R_{31}}$$

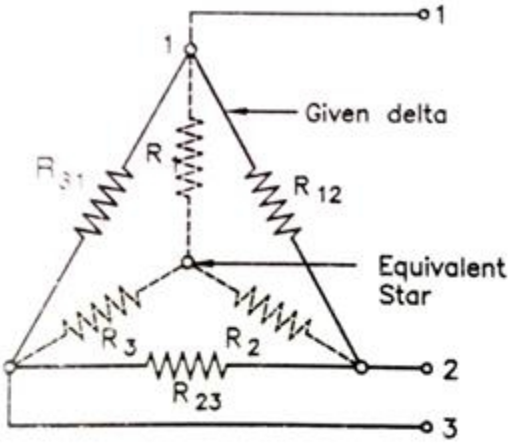


Figure-2.56 Delta and Equivalent Star

2.6.2 Star-Delta Transformation:

- It is replacement of star connected resistances into equivalent delta connected system.
- Let us consider the three resistances R_1, R_2 and R_3 connected in star as shown in figure-2.57
- Now we wants to finding out values of R_{12}, R_{23} and R_{31} interms of R_1, R_2 and R_3.

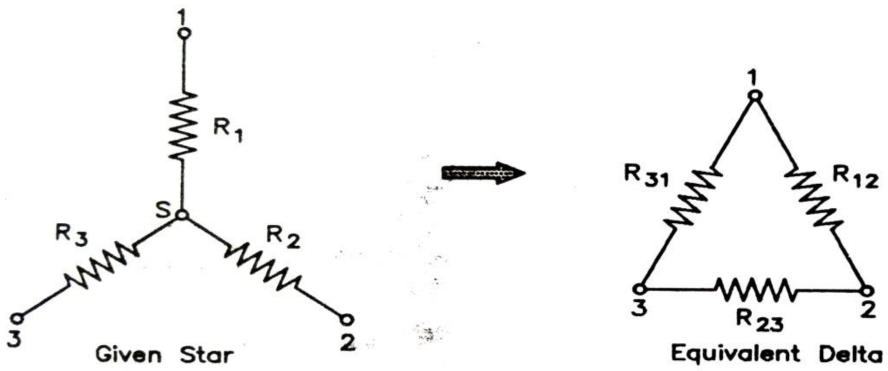


Figure-2.57

- From result of delta-star transformation we know that

$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}} \qquad (1)$$

$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}} \qquad (2)$$

$$R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}} \qquad (3)$$

- Multiplying equation (1) to (2), (2) to (3) and (3) to (1), we get

$$R_1R_2 = \frac{R_{12}^2R_{23}R_{31}}{(R_{12}+R_{23}+R_{31})^2} \qquad (4)$$

$$R_2R_3 = \frac{R_{12}R_{23}^2R_{31}}{(R_{12}+R_{23}+R_{31})^2} \qquad (5)$$

$$\text{and} \quad R_3R_1 = \frac{R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2} \qquad (6)$$

- Adding the equation (4),(5) and (6), we get

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2R_{23}R_{31} + R_{12}R_{23}^2R_{31} + R_{12}R_{23}R_{31}^2}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{(R_{12}R_{23}R_{31})(R_{12}+R_{23}+R_{31})}{(R_{12}+R_{23}+R_{31})^2} = \frac{(R_{12}R_{23}R_{31})}{R_{12}+R_{23}+R_{31}} \qquad (7)$$

- Equation (7) can be written as

$$R_1R_2 + R_2R_3 + R_3R_1 = R_{12} \times (\frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}})$$

$$R_1R_2 + R_2R_3 + R_3R_1 = R_{12} \times R_3 \text{ from equation (3)}$$

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

Similarly from equation (7), we can write

$$R_1R_2 + R_2R_3 + R_3R_1 = R_{23} \times \left(\frac{R_{31}R_{12}}{R_{12} + R_{23} + R_{31}}\right) = R_{31}R_1$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

Similarly,

$$R_{31} = R_3 + R_1 + \frac{R_3R_1}{R_2}$$

$$R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

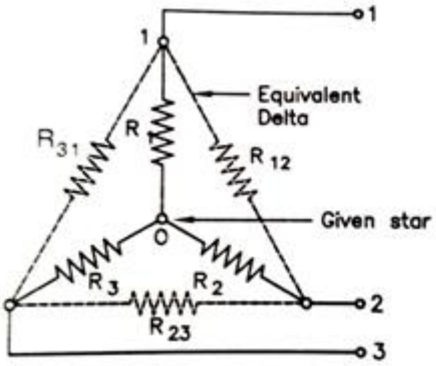


Figure-2.58

- For the simple case when $R_1 = R_2 = R_3 = R_Y$, and $R_{12} = R_{23} = R_{31} = R_\Delta$,Equation becomes

$$R_Y = \frac{R_\Delta}{3}$$

$$R_\Delta = 3R_Y$$

Example-20
Find out resistance between terminals A and B.

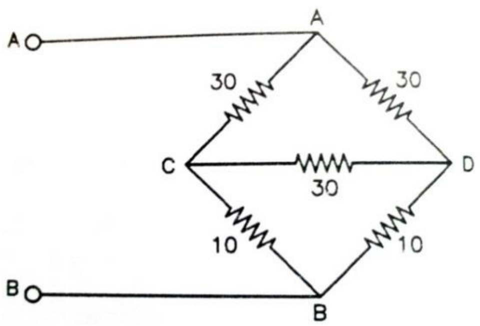


Figure-2.59

Solution:
First convert delta Δ ACD into star.

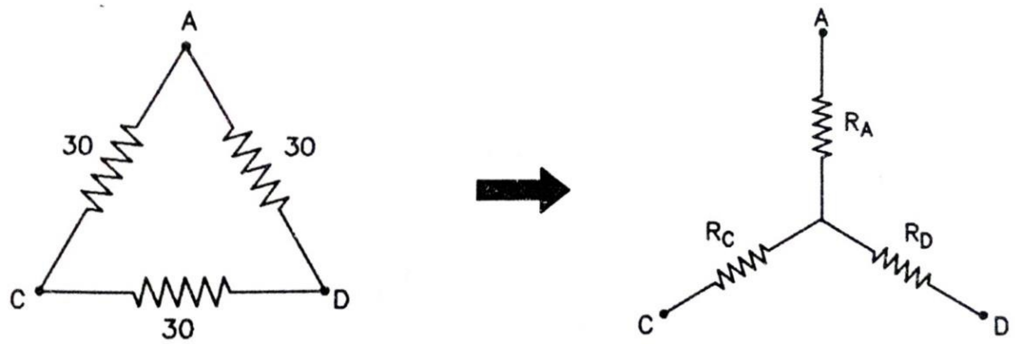


Figure-2.60

$$R_A = \frac{R_{AC}R_{DA}}{R_{AC} + R_{CD} + R_{DA}}$$

$$R_A = R_C = R_D = \frac{30 \times 30}{30 + 30 + 30} = 10\ \Omega$$

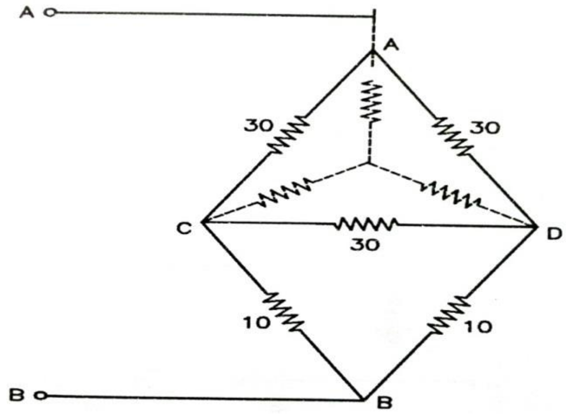
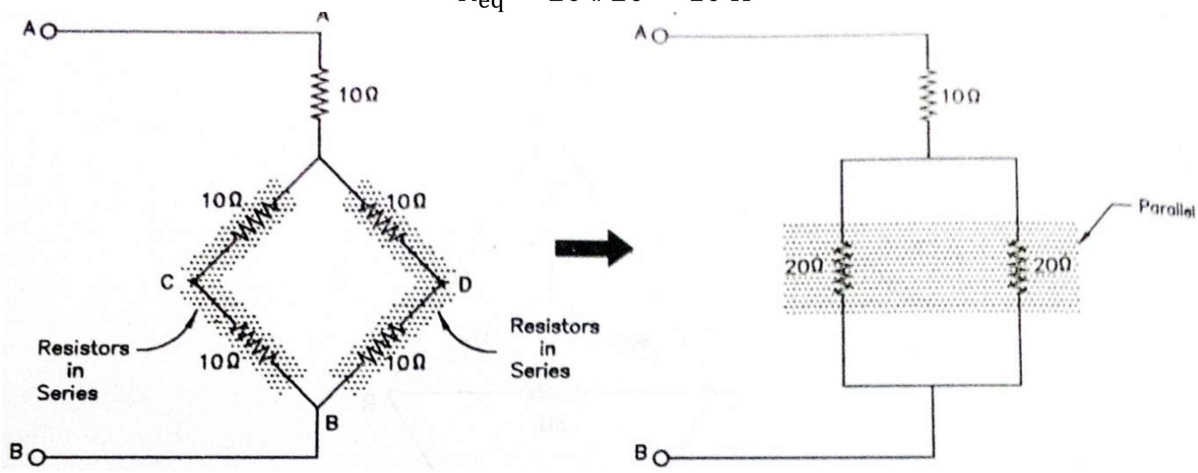


Figure-2.61

The resistance 10 Ω and 10 Ω are in series. Again these series combinations are in parallel. Thus equivalent resistance is given by

$$R_{eq} = 20 \parallel 20 = 10\ \Omega$$



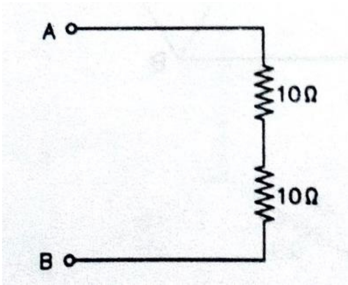


Figure-2.62

Now 10 **Ω** resistor is in series with 10 **Ω** resistor. So

$$R_{AB} = 10 + 10 = 20\Omega$$

Example-21

Find the resistance between the terminal A and B in the network shown in the figure-2.63.

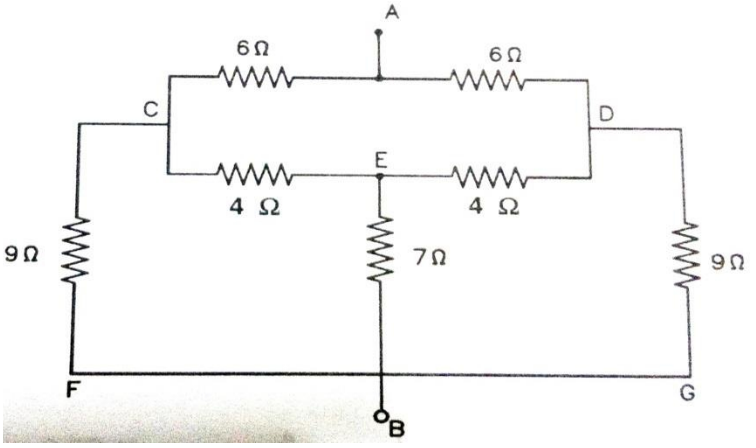


Figure-2.63

Solution:

Convert star to delta equivalent:

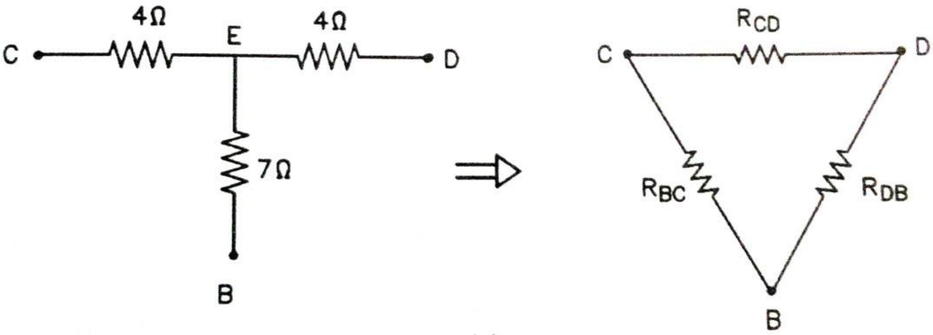


Figure-2.64

$$R_{CD} = \frac{R_C R_B + R_B R_D + R_C R_D}{R_B}$$

$$R_{AB} = \frac{4 \times 7 + 7 \times 4 + 4 \times 4}{7} = 10.29\ \Omega$$

$$R_{DB} = \frac{R_C R_D + R_B R_C + R_B R_D}{R_C}$$

$$R_{DB} = \frac{4 \times 4 + 7 \times 4 + 4 \times 7}{4} = 18\ \Omega$$

$$R_{BC} = \frac{R_D R_B + R_C R_D + R_B R_C}{R_D}$$

$$R_{BC} = \frac{7 \times 4 + 4 \times 4 + 4 \times 7}{4} = 18\ \Omega$$

The equivalent circuit is as under.

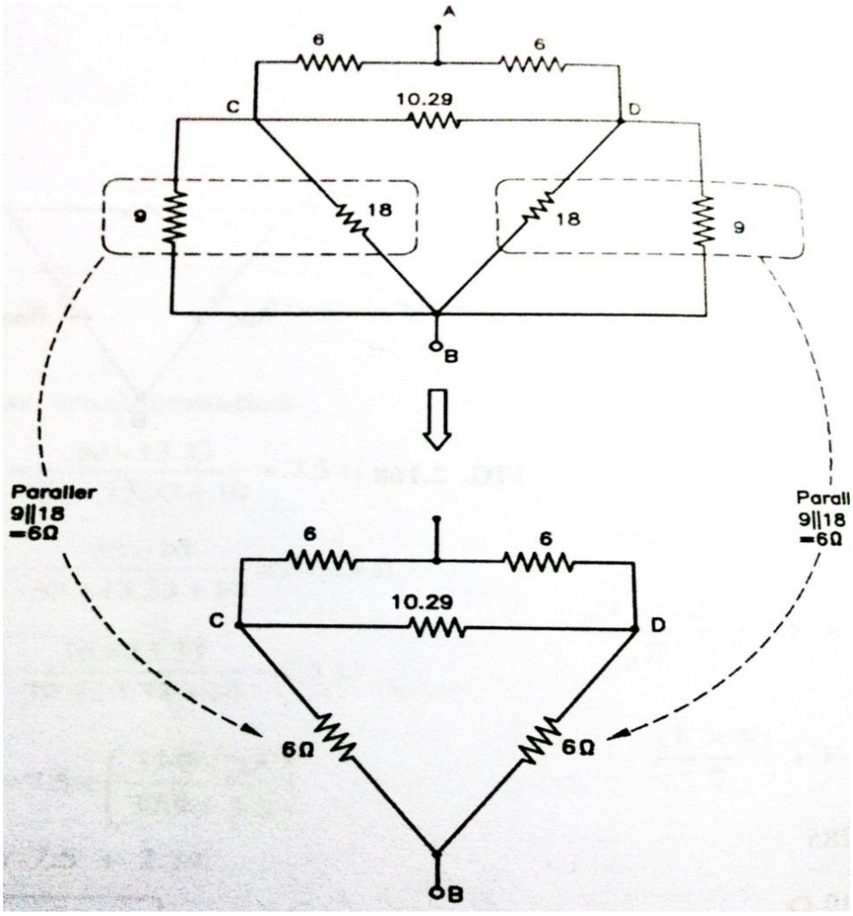


Figure-2.65

Convert delta CBD to star:

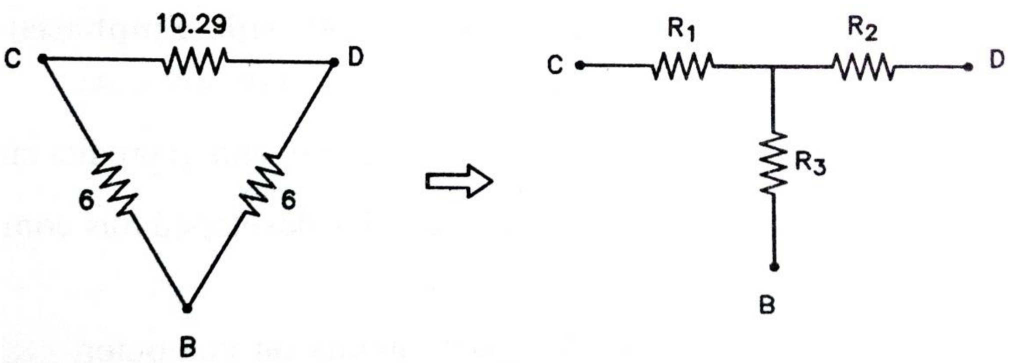


Figure-2.66

$$R_1 = \frac{6 \times 10.29}{6 + 6 + 10.29} = 2.77\Omega$$

$$R_2 = \frac{6 \times 10.29}{6 + 6 + 10.29} = 2.77\Omega$$

$$R_3 = \frac{6 \times 6}{6 + 6 + 10.29} = 1.62\ \Omega$$

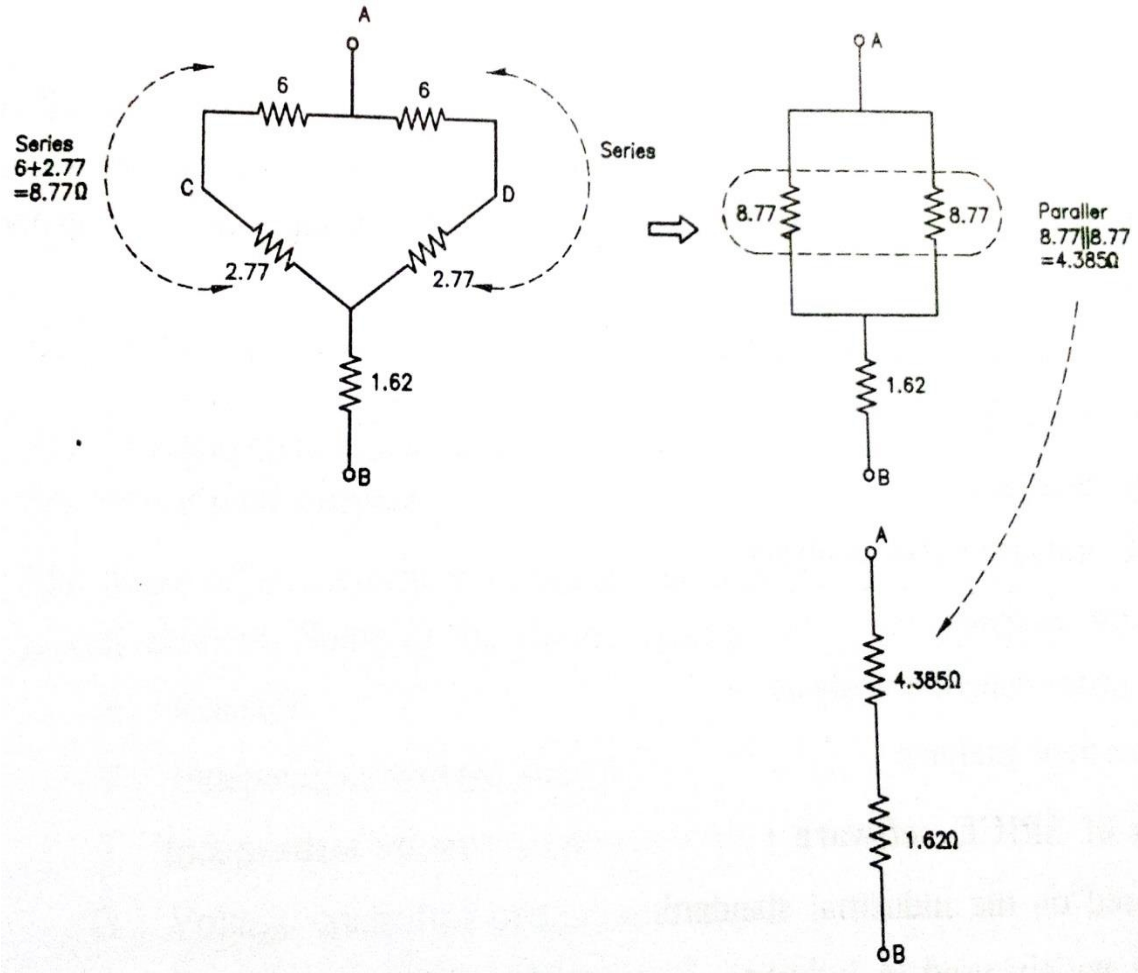


Figure-2.67

$$R_{AB} = 4.385 + 1.62 = 6.005\ \Omega$$

Example-22
Use delta–wye transformation for network reduction and determine the current through the 12-**Ω** resistor in the circuit of Figure-2.68

The delta-connected portion between terminals *A–B–C* is replaced by an equivalent wye connection with this equation

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

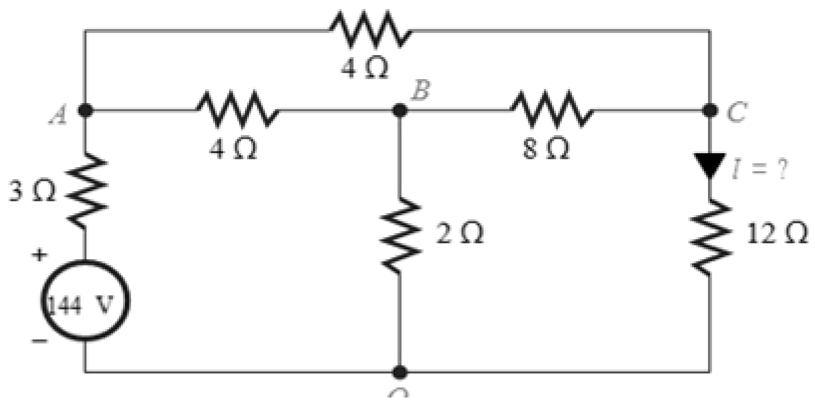


Figure-2.68 (a)

$$R_1 = \frac{4 \times 4}{4+8+4} = 1\ \Omega;\ R_2 = \frac{4 \times 8}{4+8+4} = 2\ \Omega;\ R_1 = \frac{4 \times 8}{4+8+4} = 2\ \Omega$$

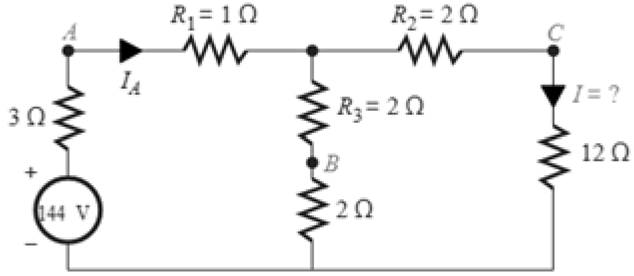


Figure-2.68 (b)

The circuit redrawn in Figure-2.69
Using the KVL equation,

$$I_A = \frac{144}{(3+1)+(4 \| 14)} = \frac{81}{4}\ A$$

By current division,

$$I_A = \frac{81}{4} \times \frac{4}{18} = \frac{9}{2} = 4.5\ A$$

2.7 COMPUTER – AIDED CIRCUIT ANALYSIS : SPICE

- **SPICE (Simulation Program with Integrated Circuit Emphasis)** is a general purpose, open source analog electronic circuit simulator software.
- It is a powerful program that is used in integrated circuit and to predict circuit behavior.
- Researchers at the University of California, Berkeley developed this computer program during 1970.
- SPICE is program that simulates electronic circuits on computer.
- We can view any voltage or current waveform in circuit. SPICE calculates these voltages and currents versus time (Transient Analysis) or versus frequency (AC Analysis).

Why Use SPICE?

- SPICE is a great tool for learning electronics. We can increase our understanding of circuit's behaviors during experiment and also modify the circuit and see what happen. How long does it take? Change a resistor value and see the effect on a circuit in seconds.

- Ideally, we would actually build and test actual circuits to understand all of its behaviors. However, we would need breadboard, components, connecting wires etc. actual circuits also require expensive equipment like power supplies, signal generators and oscilloscopes.
- SPICE included following analysis:
  1) AC analysis
  2) DC analysis
  3) DC transfer curve analysis
  4) Noise analysis
  5) Transfer function analysis
  6) Transient analysis

Advantages of SPICE software:

- It is based on the industrial standard.
- Spice is widely used in industry.
- It allows the mixing of digital and analog parts without any problems.
- Learning the spice software is quite easy.
- Figure-2.69(a) shows the block diagram which summarizes the major features of the SPICE based circuit simulation program.
- The circuit to be analyzed is described using statement with a separate statement written for each circuit element.
- The name of an element must begin with a particular letter identifying the type of circuit element. Some of the circuit elements are as listed below:
  R: Resistor
  V: Independent voltage source
  I: Independent current source
  G: Voltage controlled current source
  E: Voltage controlled voltage source
  F: Current controlled current source
  H: Current controlled voltage source
- SPICE is case insensitive. It does not recognize subscripts. So R_1 will be represented as R1 etc.
- The name of each element has to be unique. The scale factor designations used for scale factors are as follows:
- Comment statement are identified by an asterisk (*) in the first column. It is also possible to insert comments on any line by starting the comment with a semicolon.

$P = \text{IE}-12 = 1 \times 10^{-12}$ $\qquad$ $K = \text{IE3} = 1 \times 10^{3}$

$N = \text{IE}-9 = 1 \times 10^{-9}$ $\qquad$ $MEG = \text{1E6} = 1 \times 10^{6}$

$U = \text{IE}-6 = 1 \times 10^{-6}$ $\qquad$ $G = \text{1E9} = 1 \times 10^{9}$

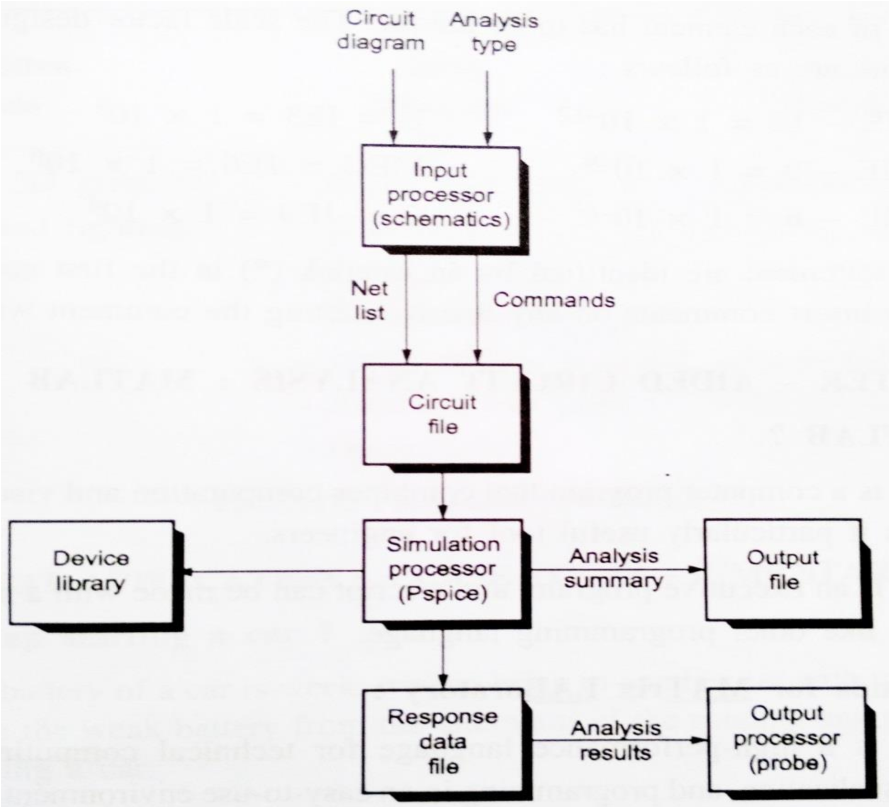


Figure-2.69 (a)

2.8 COMPUTER – AIDED CIRCUIT ANALYSIS: MATLAB

What is MATLAB?

- MATLAB is computer program that combines computation and visualization power that makes it particularly useful tool for engineers.
- MATLAB is an executive program, and a script can be made with a list of MATLAB commands like other programming language.

MATLAB stands for <u>MAT</u>rix <u>LAB</u>oratory :

MATLAB is high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familier mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modeling, simulation and prototyping
- Data analysis, exploration and visualization
- Scientific and engineering graphics
- Application development, including graphical user interface building
  The user's program is written, edited, created and saved in the edit window. Most of the programs written in MATLAB are saved in the M- files.

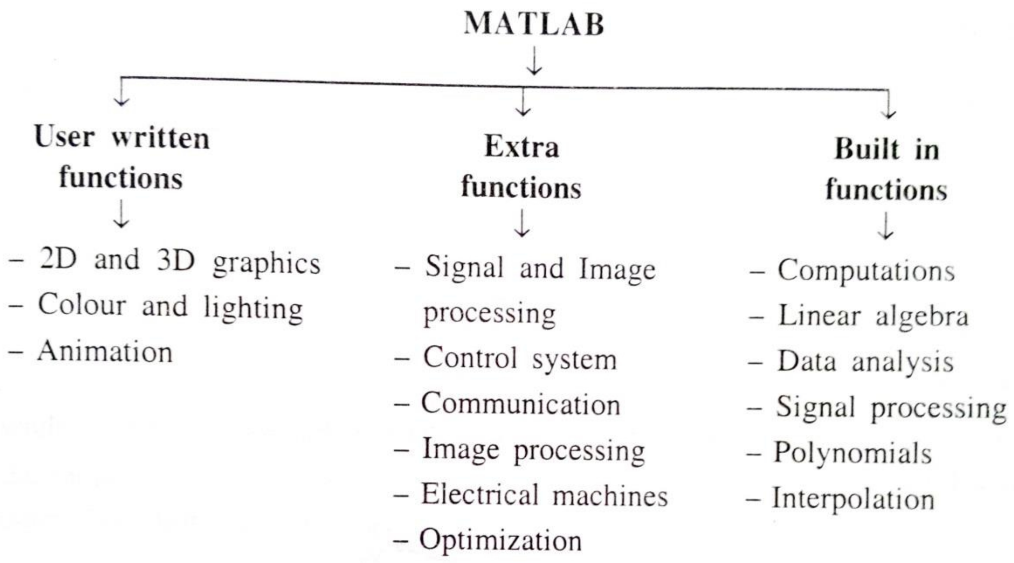


Figure-2.69(b) MATLAB Features

2.9 A case study: Jump Starting a Car

- Voltage and current in an electric network are easily measured. They obey Kirchoff's laws, KCL and KVL, and facilitate the monitoring of energy flow. For these reasons, voltage and current are used by engineers in order to describe the state of an electric network.
- When a car battery is weak, say 11 V in a 12-V system, in order to jump-start that car, we bring in another car with its engine running and its alternator charging its battery.
- Let the healthy and strong battery have a voltage of 13 V. According to the recommended practice, one should first connect the positive terminals with the red jumper cable, as shown in Figure-2.70, and then complete the circuit between the negative terminals with the aid of the black jumper cable.
- Note that the negative terminal of any car battery is always connected to its auto chasis.

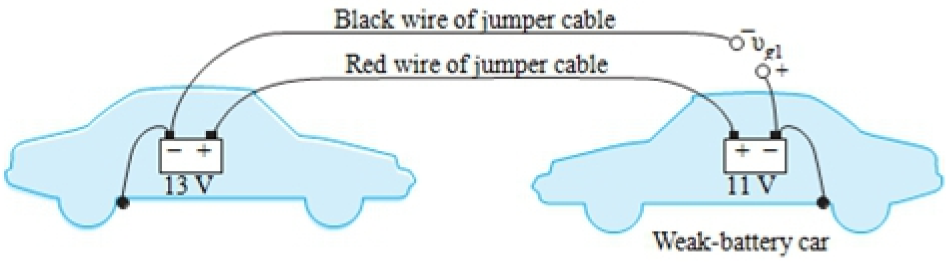


Figure-2.70 Jumper cable connections for jump starting a car with a weak battery.

Applying KVL in Figure-2.70, we have

$$v_{g1} - 13 + 11 = 0 \qquad \text{or} \qquad v_{g1} = 2\text{ V}$$

where v_{g1} is the voltage across the airgap, or the voltage existing between the black jumper cable and the negative terminal of the weak battery.

- Now suppose one makes, by mistake, incorrect connections, as shown in Figure-2.71. Note that the red jumper cable is connected between the positive terminal of the strong battery and the negative terminal of the weak battery. Application of the KVL now fields.

$$v_{g2} - 13 - 11 = 0 \quad \text{or} \quad v_{g2} = 24\ \text{V}$$

where v_{g2} is the gap voltage with incorrect connections. With such a large voltage difference, when one tries to complete the black jumper cable connection, it presents a danger to both batteries and to the person making the connections.

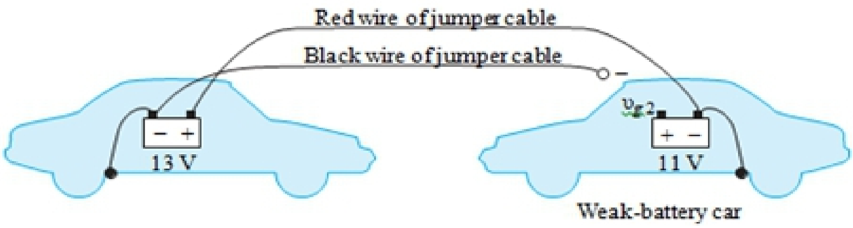


Figure-2.71 Incorrect connections for jump starting a car with a weak battery.

Energy to Start an Engine

- A simplified circuit model for an automotive starter circuit is shown in Figure-2.72. Let the car battery voltage be 12.5 V and let the automobile starter motor draw 60 A when turning over the engine. If the engine starts after 10 seconds, we can easily calculate the power to the starter motor, which is the same as the power out of the battery,

$$P = VI = 12.5 \times 60 = 750\ \text{W}$$

The energy required to start the engine can be computed as

$$W = 750 \times 10 = 7500\ \text{J}$$

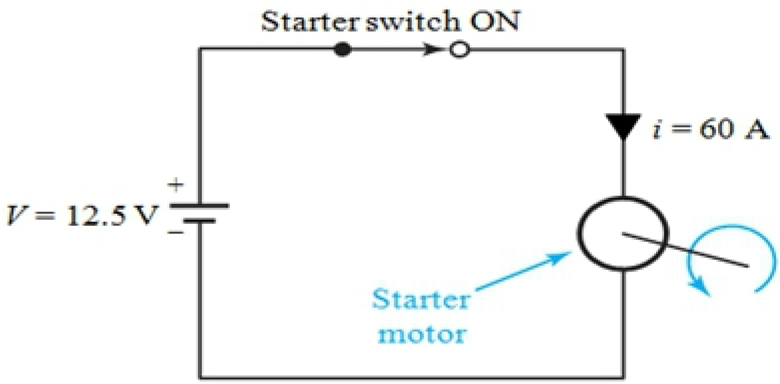


Figure-2.72 Simplified circuit model for the automotive starter circuit

- Thus, simple circuit models can be used to simulate various physical phenomena of practical interest. They can then be analyzed by circuit-analysis techniques to yield meaningful solutions rather easily.

Problems:

1) Reduce the circuit of figure-2.73 to a Thevenin and a Norton equivalent circuit.
Ans.$(v_{TH} = 8\text{ V}, R_{TH} = 4\ \Omega)(I_N = 2\text{ A}, R_{TH} = 4\ \Omega)$

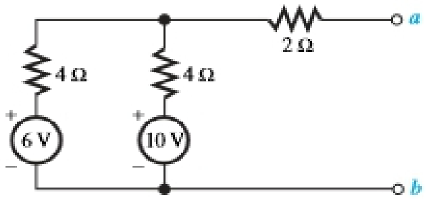


Figure-2.73

2) Find the Thevenin and Norton equivalent circuits for the configuration of figure-2.74 as viewed from terminal a-b. **Ans.**$(v_{TH} = 72\text{ V}, R_{TH} = 4\ \Omega)(I_N = 18\text{ A}, R_{TH} = 4\ \Omega)$

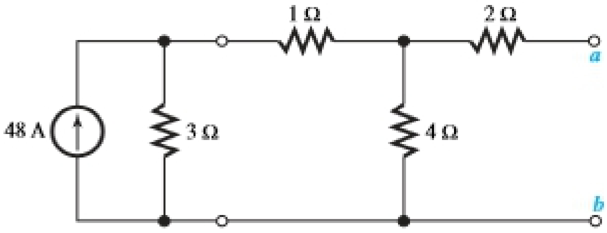


Figure-2.74

3) Use the node-voltage method to find the current I through the 5-Ω resistor of the circuit of figure-2.75 **Ans.**$(I = 1A)$

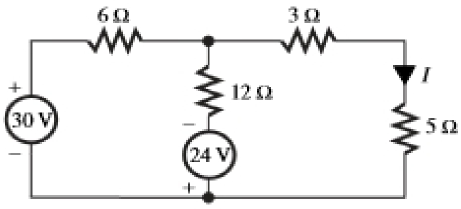


Figure-2.75

4) Use the node-voltage method to determine the voltage across the 12- Ω resistor of the circuit given figure-2.76. Verify by mesh analysis. **Ans.**$(v = 54V)$

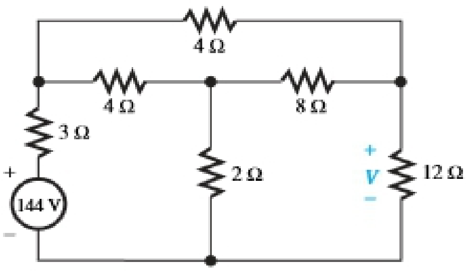


Figure-2.76

5) (a) Find the voltage across the 8-A current source in the circuit of figure 2.77 with the use of nodal analysis. **Ans.**$(v = 6\text{ V})$
(b) determine the current in the 0.5 resistor of the circuit by mesh analysis.
Ans.$(I = -4\text{ A})$

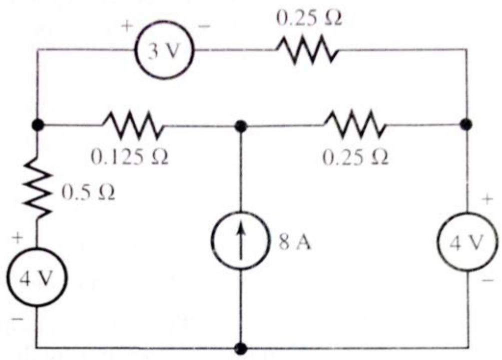


Figure-2.77

6) Determine the current I through the 10- Ω resistor of the circuit of figure-2.78 by the application of superposition. **Ans.**$(\mathrm{I} = 0.5\mathrm{A})$

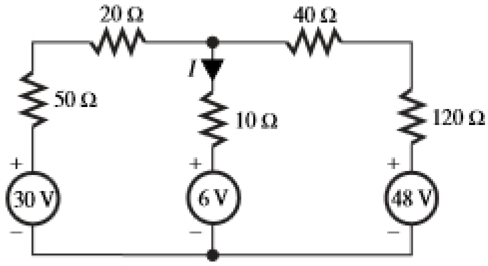


Figure-2.78

7) Find the current I_1 through the 20- Ω resistor of the circuit of figure-2.79 by the application of superposition. **Ans.**$(\mathrm{I}_1 = -1\mathrm{A})$

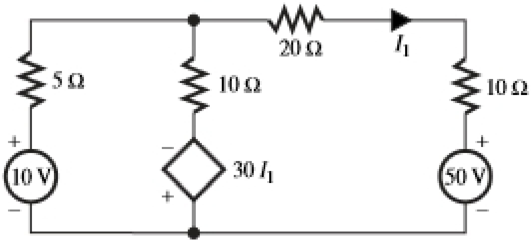


Figure-2.79

8) Find the power delivered by the source in the circuit given in figure-2.80. use network reduction by wye-delta transformation. **Ans.**$(\mathrm{P} = 10.93\ \mathrm{W})$

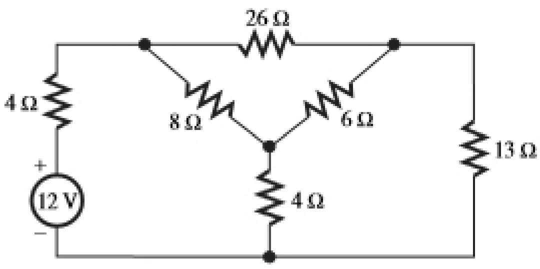


Figure-2.80

CHAPTER 5

ANALOG BUILDING BLOCKS

TOPICS:

- **THE AMPLIFIER BLOCK**
- **OPERATIONAL AMPLIFIER (OP-AMP)**
- **OP-AMP PARAMETERS**
- **IDEAL OP-AMP**
- **PRACTICAL OP-AMP**
- **OP-AMP APPLICATIONS**
- **PRACTICAL APPLICATION: A CASE STUDY**

AMPLIFICATION

- Amplification is process of adding strength to the input signal without changing its shape.
- The circuit which amplifies a small input signal is called as amplifier.
- An amplifier receives a signal from transducer or sensor or other input source and provides a larger version of the signal to some output device or to another amplifier stage.
- An input signal is from transducer generally small in few millivolts from a cassette or CD input, or a few microvolts from an antenna, microphone. Its needs to be amplified sufficiently to operate an output device like speaker, relay or other power-handling device which works at large power.

THE AMPLIFIER BLOCK

- An amplifier can be modeled as a two-port device, that is, a box with two pairs of terminals designated as input and output, as shown in Figure 1.1

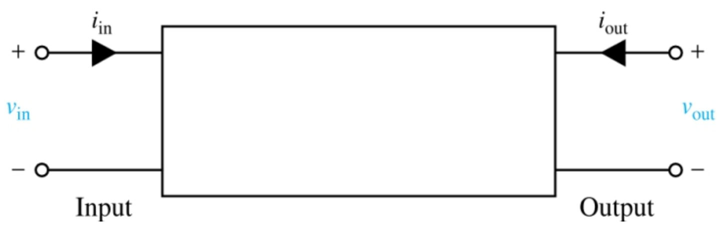


Figure 1

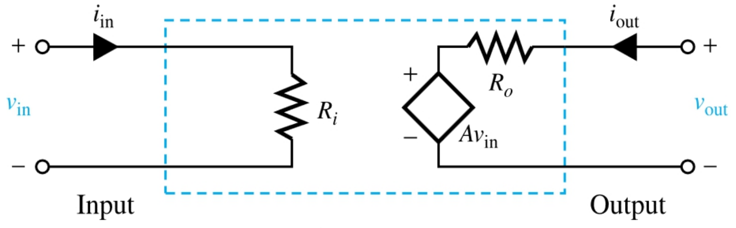


Figure 1.1

- The circuit model of the amplifier block shown in Figure 1.1 is developed on the basis of the following considerations:
  1) For most amplifiers, the input current is proportional to the input voltage. The input terminals in the model are connected by a resistance Ri, known as the input resistance of the amplifier.
  2) An amplifier delivers electric power (to a speaker, for example), the output current can be represented by its Thevenin-source model. The Thevenin resistance Ro is known as the output resistance and the Thevenin voltage is a dependent voltage source AVin, where A is called the open-circuit voltage amplification.

- Thus, the amplifier block is a linear circuit block in which the output is proportional to the input, and the amplifier is characterized by the three constants Ri, Ro, and A.

- The input and output resistances may be generalized to input and output impedances in ac systems.

- Since Ri and R_S are connected in parallel,

$$V_{in} = I_S \frac{R_S R_i}{R_S + R_i}$$

Where, I_S = Source current

- Using the voltage-divider formula, one has

$$V_{out} = A\, V_{in} \frac{R_L}{R_o + R_L} = \frac{A R_L R_S R_i\, I_S}{(R_o + R_L)(R_S + R_i)}$$

A is the open-circuit voltage amplification.

a) Voltage amplification (Av)

Circuit shown in Figure 2 explains voltage amplification, or voltage gain.

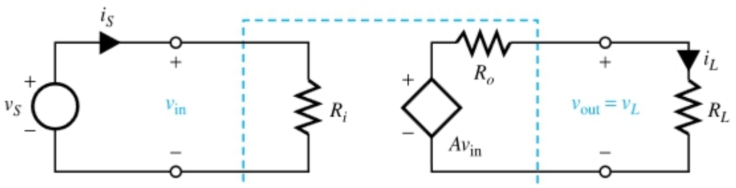


Figure 2

- In this circuit a signal voltage Vs is applied to the input of the amplifier block, whereas the output terminals are connected to a load resistance RL.

- Evaluate the ratio of the voltage across the load to the signal voltage VL/Vs, which is known as voltage gain Av,

$$A_v = \frac{V_L}{V_S} = \frac{I_L R_L}{V_S} = \frac{A\, V_{in}}{(R_o + R_L) V_S} R_L = \frac{A R_L}{(R_o + R_L)}$$

b) Current gain (AI)

➢ By defining the current gain A_I to be the ratio of the current through R_L to the current through V_S, one gets

$$A_I = \frac{i_L}{i_S} = \frac{A v_{in}/(R_o + R_L)}{v_{in}/R_i} = \frac{A R_i}{R_o + R_L}$$

c) Power gain (A_P)

➢ The power gain A_P, defined by the ratio of the power delivered to the load to the power given out by the signal source, is obtained as

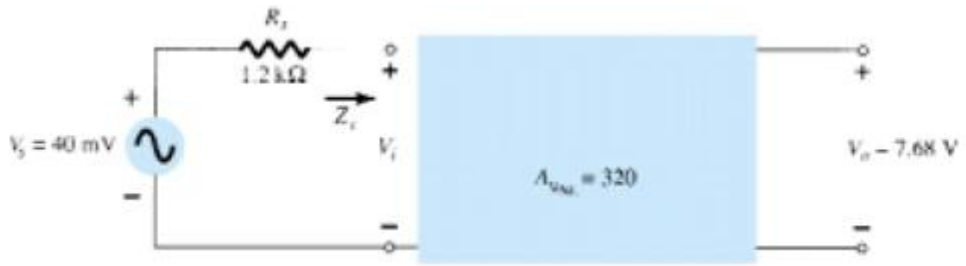
$$A_P = \frac{v_L^2/R_L}{v_S^2/R_i} = \frac{A_V^2 R_i}{R_L} = \frac{A^2 R_L R_i}{(R_o + R_L)^2} = A_V A_I$$

For fixed values of Ro and Ri, A_P is maximized when R_L is chosen equal to Ro, and this corresponds to maximum power transfer to the load.

EXAMPLE

For given two port block determine:

a) Vi.
b) Ii.
c) Zi.
d) Av.



SOLUTION

$$\text{(a)}\ A_{v_{NL}} = \frac{V_o}{V_i} \text{ and } V_i = \frac{V_o}{A_{v_{NL}}} = \frac{7.68\ \text{V}}{320} = \mathbf{24\ mV}$$

$$\text{(b)}\ A_i = \frac{V_s - V_i}{R_s} = \frac{40\ \text{mV} - 24\ \text{mV}}{1.2\ \text{k}\Omega} = \mathbf{13.33\ \mu A}$$

$$\text{(c)}\ Z_i = \frac{V_i}{I_i} = \frac{24\ \text{mV}}{13.33\ \mu\text{A}} = \mathbf{1.8\ k\Omega}$$

$$\text{(d)}\ A_v = \frac{Z_i}{Z_i + R_s} A_{v_{NL}}$$

$$= \frac{1.8\ \text{k}\Omega}{1.8\ \text{k}\Omega + 1.2\ \text{k}\Omega}(320)$$

$$= \mathbf{192}$$

OPERATIONAL AMPLIFIER (OP-AMP)

- The operational amplifier (OP-AMP) is a fundamental active element of analog circuit design.
- It is most commonly used in amplifier and analog signal processing circuits in the frequency band from 0 to 100 kHz. High-frequency OP-AMPs are used in applications that require a bandwidth into the MHz range.
- The first OP-AMPs were vacuum-tube circuits which were developed for use in analog computers. Modern OPAMPS are fabricated as integrated circuits (IC) so having all advantage of monolithic IC such as small in size, inexpensive, compact, less power consumption and versatile used in a variety of simple circuits.
- Operational amplifiers are precision, high-gain, differential amplifiers. They were originally designed to perform mathematical operations in computers. But now a days widely used in many other applications
- An OP-AMP can be built from individual transistors and resistors but practically all OP-AMP are manufactured as integrated circuits. Dozens of different types of OP-AMPs are available with various combinations of characteristics.
- In OP-AMP, internal amplifier stages use BJT or FET as amplifying devices along with resistors and capacitors.
- The first OPAMP in IC manufactured by Fairchild Company. They named OP-AMP as μA 741 which became extremely popular as it was used in variety of application. OP-AMP is linear and more accurately an analog integrated circuit.

Symbol and terminals:

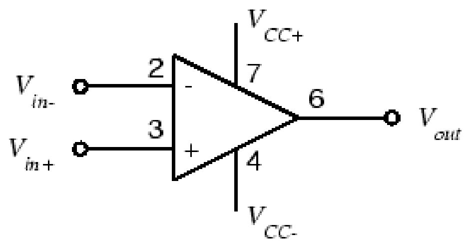


Figure 3

- The symbol for the OP-AMP is shown in Figure having two terminals labelled Vin_+ (non inverting terminal) and Vin_- (inverting terminal) are available for inputs. The voltages of these terminals are labelled with respect to a ground.
- The output voltage is related to the difference between the two input voltages as

$$V_O = A_V\ (Vin_+ - Vin_-)$$

Where Av is the open-loop voltages gain.

- Thus, the OP-AMP is basically a form of differential amplifier, in which the difference (Vin_+ - Vin) is amplified.

OP-AMP BLOCK DIAGRAM

- An operational amplifier is a direct coupled high gain amplifier consisting of one or more differential (OPAMP) amplifiers and followed by a level translator and an output stage.
- An operational amplifier is available as a single integrated circuit package.

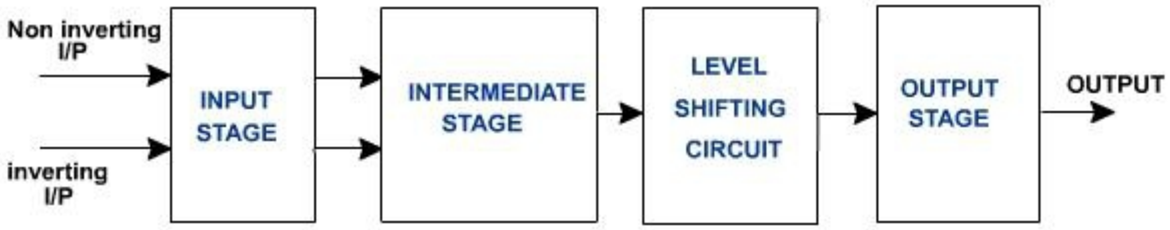


Figure 4

Input stage

- It is a dual input balanced output differential amplifier. This stage provides most of the voltage gain of the amplifier and also establishes the input resistance(R_i) of the OPAMP.

Intermediate stage

- It is another differential amplifier which is driven by the output of the first stage. This is usually dual input unbalanced output.

Level Shifting Stage

- Because direct coupling is used, the dc voltage level at the output of intermediate stage is well above ground potential. Therefore level shifting circuit is used to shift the dc level at the output downward to zero with respect to ground.

Output stage

- It is generally a push pull complementary amplifier. The output stage increases the output voltage swing and raises the current supplying capability of the OPAMP. It also provides low output resistance (R_O).

OP-AMP INPUT MODES

- OPAMP has two input terminals and one output terminal. Input mode can be determined by how you connect input signal to the both input terminals of OPAMP.
- Different OP-AMP input mode listed below.
  a) Single input
  b) Double-ended or differential input
  c) Common-mode operation

- Each mode explained in detail below.

a) Single input

- In Single-ended input operation the input signal is connected to one input terminal and the other input terminal connected to ground. Figure 5.1 shows the signals connected for this operation.

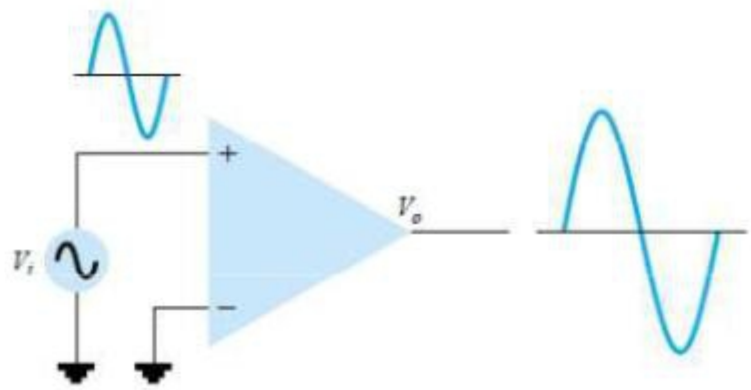


Figure 5.1

- In Figure 5.1 given below the input is applied to the plus input (with minus input at ground), which results in an output having the same polarity as the applied input signal.
- Figure 5.2 below shows an input signal applied to the minus input, the output then being opposite in phase to the applied signal.

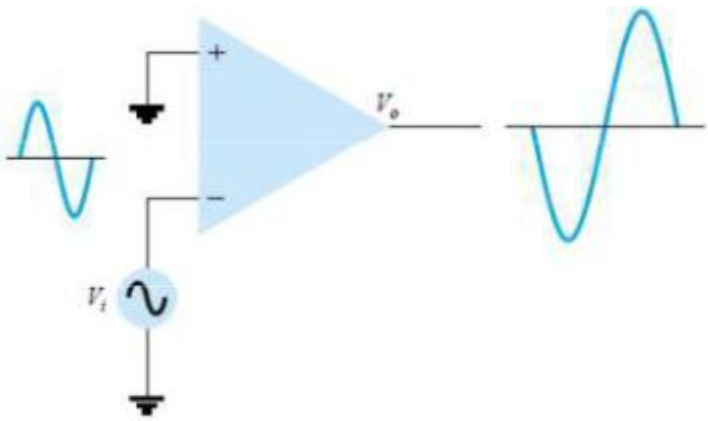


Figure 5.2

b) Double-ended or differential input

- In addition to using only one input, it is possible to apply signals at each input it's called a double-ended operation.
- Figure 5.3 given below shows an input Vd applied between the two input terminals (recall that neither input is at ground), with the resulting amplified output in phase with that applied between the plus and minus inputs.

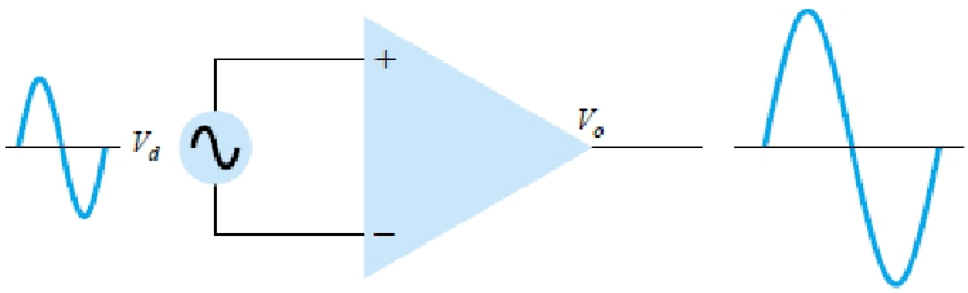


Figure 5.3

- Figure 5.4 below shows the same when two separate signals are applied to the inputs. The difference signal being (V_1 - V_2).

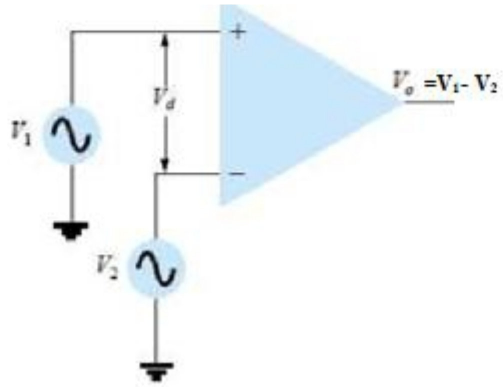


Figure 5.4

c) Common-mode operation

- When the same input signals are applied to both inputs, common-mode operation results, as shown in Figure 5.5.

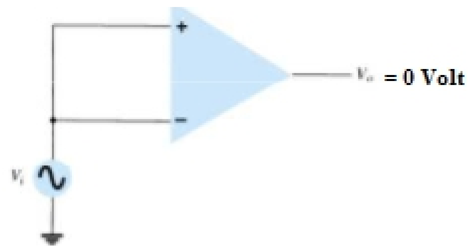


Figure 5.5

- Ideally, the two inputs are equally amplified, and since they result in opposite polarity signals at the output, these signals cancel, resulting in 0-V output. Practically, a small output signal is present.
- Producing a zero output voltage for common mode signal is called as common mode rejection.

OP-AMP PARAMETERS

- OP-AMP characteristics are important in practice because we use them to identify the performance of various OP-AMP ICs and select the best suitable from them for the required application.
- OP-AMP characteristics can be classified into two categories.
  (i) DC characteristics and
  (ii)AC characteristics.
- DC characteristics include input bias current , input offset current, input offset voltage and thermal drift whereas AC characteristics include the frequency response, slew rate, stability of OPAMP, frequency compensation etc.

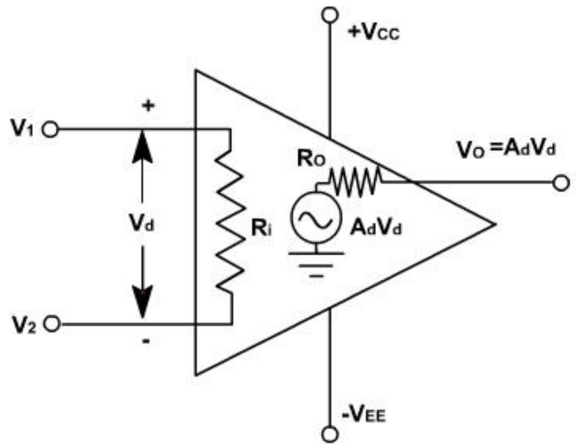


Figure 5.6

1. **OPEN-LOOP VOLTAGE GAIN (A_V)**

➢ It is internal voltage gain of the device and represents by the ratio of output voltage to input voltage when there are no external components.

$$V_O = A_V \times V_d$$

➢ For Ideal OP-AMP, it is infinite (∞).
➢ A typical/practical value for an OP-AMP 741 is 2×10^5

2. **INPUT RESISTANCE (Ri)**

➢ It is defined as the resistance looking into the two input terminals of OP-AMP.
➢ For Ideal OP-AMP, it is infinite (∞).
➢ A typical value for an OP-AMP 741 is 2 MΩ

3. **OUTPUT RESISTANCE (Ro)**

➢ It is defined as resistance looking from the output terminals of OP-AMP.
➢ For Ideal OP-AMP, it is zero.
➢ A typical value for an OP-AMP 741 is 60Ω - 70Ω

4. **BANDWIDTH**

➢ It is the difference between the upper and lower frequencies in a continuous set of frequencies over which OP-AMP can work.
➢ For Ideal OP-AMP, it is infinite (∞).
➢ A typical value for an OP-AMP 741 is 1 MHz

5. **COMMON-MODE REJECTION RATIO (CMRR)**

➢ The ability of an Op-Amp to suppress common signals is expressed in terms of its Common-Mode-Rejection-ratio (CMRR).

$$\text{CMRR} = \frac{A_d}{A_c}$$

- The higher value of the CMRR is better. A very high value of CMRR means that the differential gain A_V is high and the common-mode gain A_C is low.

- The CMRR is often expressed in decibels (dB) as

$$\text{CMRR(dB)} = 20 \log \left(\frac{A_d}{A_c}\right)$$

- For Ideal OP-AMP, it is infinite (∞).
- A typical value for an OP-AMP 741 is 90 db or 31622.

6. INPUT OFFSET VOLTAGE (V_{io})

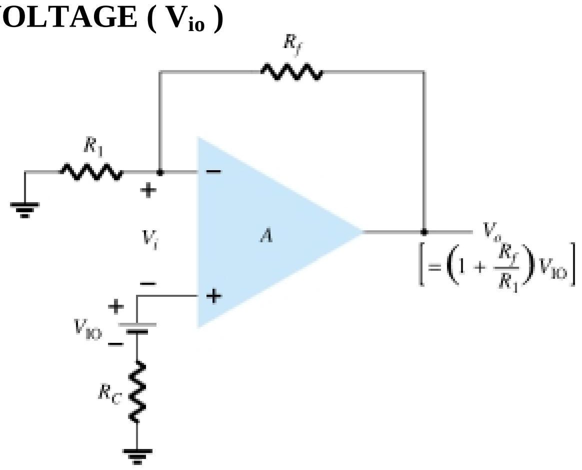


Figure 5.8

- When both inputs are tied to ground the output voltage should be zero. In practice there will be some voltage due to mismatches in amplifier components which produce significant output voltage. The input offset voltage Vios is the differential input voltage required to make the output zero.
- Ideally it is zero.
- A typical value for an OP-AMP 741 is few Milivolt range. It is temperature dependent.

7. INPUT BIAS CURRENT (I_B)

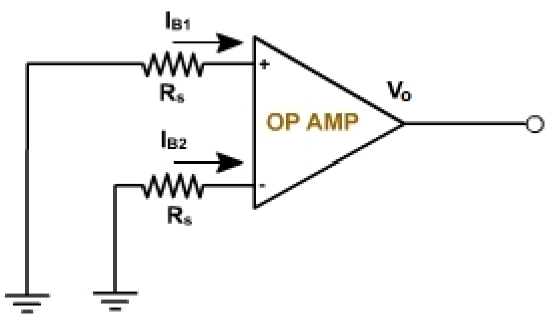


Figure 5.9

- This is the average of the currents that flow into the inverting(I_B-) and non-inverting (I_B+)input terminals of the op-amp is define as input bias current.

$$I_B = \frac{I_{B1}+I_{B2}}{2}$$

- Ideally it is zero.

- A typical value for an OP-AMP 741 is near about 50 nA;

8. INPUT OFFSET CURRENT (I_{ios})

- The algebraic difference between the currents into the inverting and non-inverting terminals when they connected to ground is referred to as input offset current I_{ios}.

$$I_{io} = |I_{B1} - I_{B2}|$$

- Ideally it is zero.
- A typical value for an OP-AMP 741 is 6 nA;

9. POWER-SUPPLY REJECTION RATIO (PSRR)

- PSRR is defined as the ratio of change in the input offset voltage V_{io} with a change in one of the bias power supplying V_{CC}, when the other power supply is held constant. The term is also called the **Supply Voltage Rejection Ratio (SVRR)** or **Power Supply Sensitivity (PSS).**

$$PSRR = \frac{\Delta V_{io}}{\Delta V_{CC}}$$

- It is expressed in microvolt/volt.
- Ideally it is zero.
- A typical value for an OP-AMP 741 is 150 μV/ V

10. SLEW RATE (S)

- Slew (or slewing) rate is defined as maximum rate of change of output voltage with respect to time.

$$SR = \left.\frac{dV_O}{dt}\right| \text{maximum Volts / μS}$$

- It is expressed in Volt / microseconds.
- Ideally it is infinite (∞).
- A typical value for an OP-AMP 741 is 0.5 V/ μS

11. INPUT CAPACITANCE (C_i)

- It is equivalent capacitance measured at either inverting or with other terminal connected to ground.
- Ideally it is zero.
- A typical value for an OP-AMP 741 is 1.4 pF.

12. GAIN BANDWIDTH PRODUCT (GB)

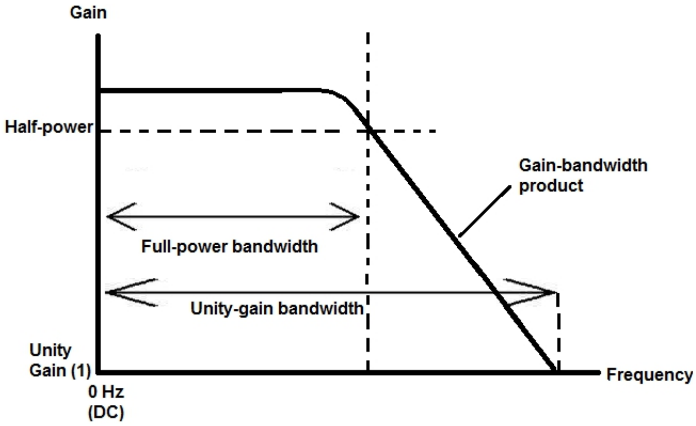


Figure 5.10

- It is defined as bandwidth of OP-AMP when voltage gain is 1. Other terms used for it is close loop bandwidth, unity gain bandwidth and **small signal bandwidth.**
- A typical value for an OP-AMP 741 is 1 MHz.

13. OUTPUT VOLTAGE SWING

- It is defined as difference between the maximum positive & negative output voltage by OP-AMP.
- It is generally $\pm 0.9V_{CC}$ (supply voltage) for typical OP-AMP. So its voltage swing from $+0.9V_{CC}$ to $-0.9V_{CC}$.

Example

An operational amplifier has a slew rate of 2 V / µs. If the peak output is 12 V, what is the power bandwidth?

Solution:

The slew rate of an operational amplifier is

$$\text{Slew rate} = 2\pi f_{max} V_P$$

$$f_{max} = \frac{Slew\ rate}{2\pi V_p}$$

As for output free of distribution, the slews determine the maximum frequency of operation f_{max} for a desired output swing.

$$f_{max} = \frac{2 X 10^6}{2\pi X 12}$$

$$f_{max} = \frac{1}{12\pi \times 10^{-6}} = 26.5 \text{ kHz}$$

So bandwidth = 26.5 kHz.

Example

For the circuit given in figure 5.11 $I_{in(off)}$ = 20 nA. If $V_{in(off)}$ = 0, what is the differential input voltage?. If A = 10^5, what does the output offset voltage equal?

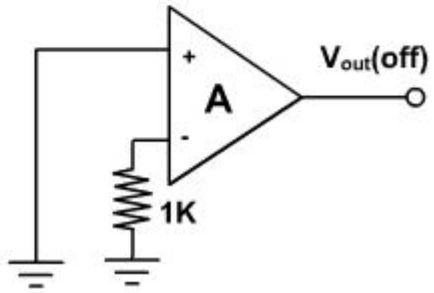


Figure 5.11

Solution:

$I_{in(off)}$ = 20 nA
$V_{in(off)}$ = 0

(i) The differential input voltage = $I_{in(off)}$ x 1k = 20 nA x 1 k = 20μ V

(ii) If A = 105 then the output offset voltage $V_{in(off)}$ = 20 μ V x 10^5 = 2 volt

Output offset voltage = 2 volts.

<u>Example</u>

Calculate the CMRR for the circuit measurements shown in Figure.

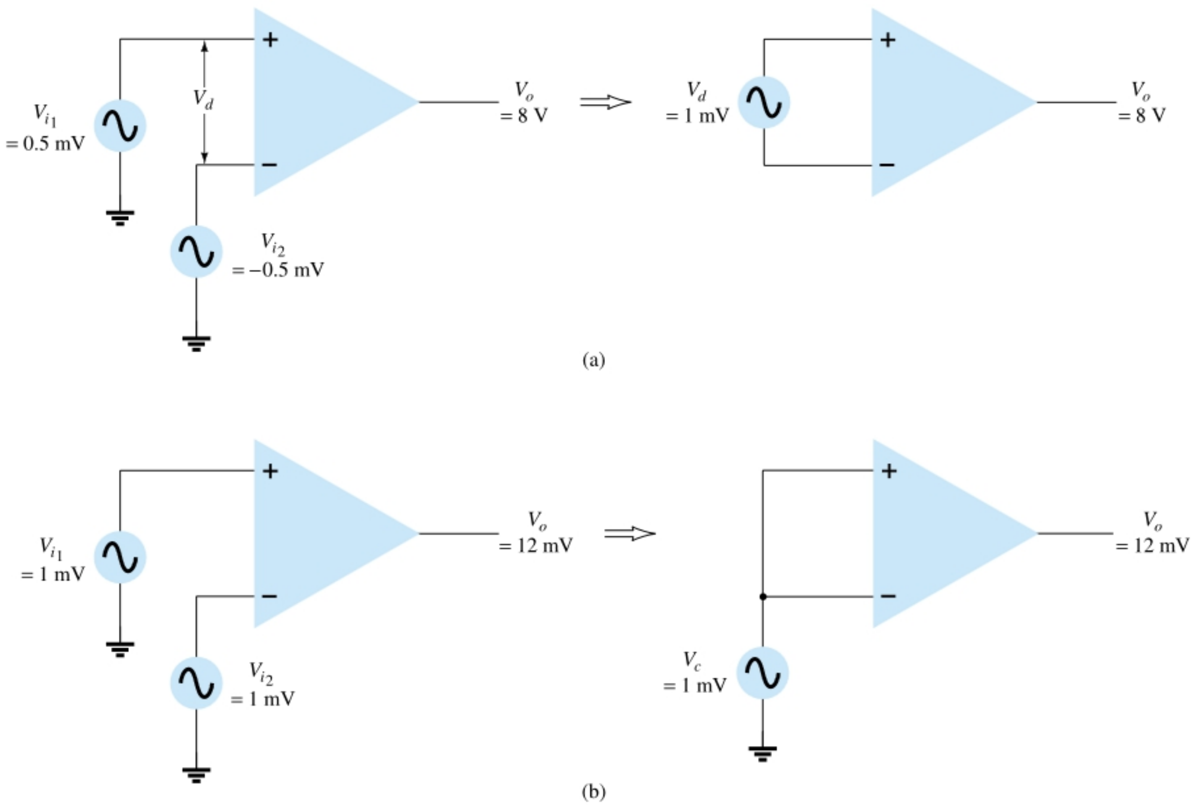


Figure 5.12

Solution
From the measurement shown in Fig. 5.12 a, using the procedure in step 1 above, we obtain

$$A_d = \frac{V_o}{V_d} = \frac{8\text{ V}}{1\text{ mV}} = 8000$$

The measurement shown in Figure 5.12 b, using the procedure in step 2 above, gives us

$$A_c = \frac{V_o}{V_c} = \frac{12\text{ mV}}{1\text{ mV}} = 12$$

The value of CMRR is

$$\text{CMRR} = \frac{A_d}{A_c} = \frac{8000}{12} = \mathbf{666.7}$$

which can also be expressed as

$$\text{CMRR} = 20\log_{10}\frac{A_d}{A_c} = 20\log_{10} 666.7 = \mathbf{56.48\ dB}$$

Example

Determine the output voltage of an op-amp for input voltages of V_{i1}=150μV, V_{i2}= 140μV. The amplifier has a differential gain of Ad=4000 and the value of CMRR is:
(a) 100.
(b) 105.

Solution

$$V_d = V_{i_1} - V_{i_2} = (150 - 140)\ \mu\text{V} = 10\ \mu\text{V}$$
$$V_c = \frac{1}{2}(V_{i_1} + V_{i_2}) = \frac{150\ \mu\text{V} + 140\ \mu\text{V}}{2} = 145\ \mu\text{V}$$

(A)

$$\begin{aligned} V_o &= A_d V_d\left(1 + \frac{1}{\text{CMRR}}\frac{V_c}{V_d}\right) \\ &= (4000)(10\ \mu\text{V})\left(1 + \frac{1}{100}\frac{145\ \mu\text{V}}{10\ \mu\text{V}}\right) \\ &= 40\text{ mV}(1.145) = \mathbf{45.8\ mV} \end{aligned}$$

(B)

$$V_o = (4000)(10\ \mu\text{V})\left(1 + \frac{1}{10^5}\frac{145\ \mu\text{V}}{10\ \mu\text{V}}\right) = 40\text{ mV}(1.000145) = \mathbf{40.006\ mV}$$

<u>IDEAL OP-AMP</u>

An ideal OP-AMP would exhibit the following electrical characteristic.

1. Infinite voltage gain A_d
2. Infinite input resistance R_i, so that almost any signal source can drive it and there is no loading of the input source.

3. Zero output resistance R_O, so that output can drive an infinite number of other devices.
4. Zero output voltage when input voltage is zero.
5. Infinite bandwidth so that any frequency signals from 0 to infinite Hz can be amplified without attenuation.
6. Infinite common mode rejection ratio so that the output common mode noise voltage is zero.
7. Infinite slew rate, so that output voltage changes occur simultaneously with input voltage changes.

Equivalent circuit

- Figure 5.13 shows an equivalent circuit of an OP-AMP.

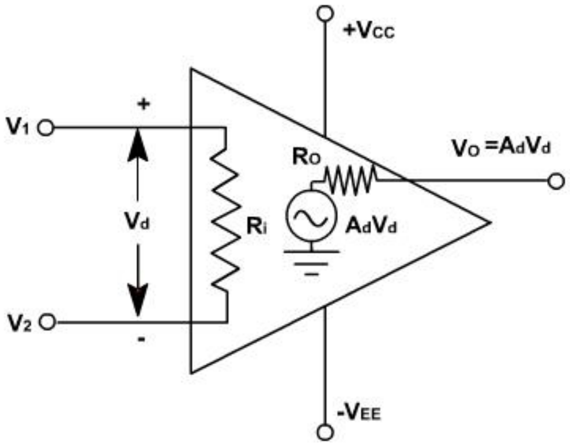


Figure 5.13

- V_1 and V_2 are the two input voltage voltages. R_i is the input impedance of OP-AMP. $A_d \cdot V_d$ is an equivalent Thevenin voltage source and R_O is the Thevenin equivalent impedance looking back into the terminal of an OP-AMP.

- This equivalent circuit is useful in analyzing the basic operating principles of OP-AMP and in observing the effects of standard feedback arrangements

$$V_O = A_d (V_1 - V_2) = A_d \cdot V_d$$

- This equation indicates that the output voltage V_O is directly proportional to the algebraic difference between the two input voltages.
- In other words the OP-AMP amplifies the difference between the two input voltages. It does not amplify the input voltages themselves.
- The polarity of the output voltage depends on the polarity of the difference voltage V_d.

Ideal Voltage Transfer Curve:

- The graphic representation of the output equation is shown in figure in which the output voltage V_O is plotted against differential input voltage V_d, keeping gain Ad constant.

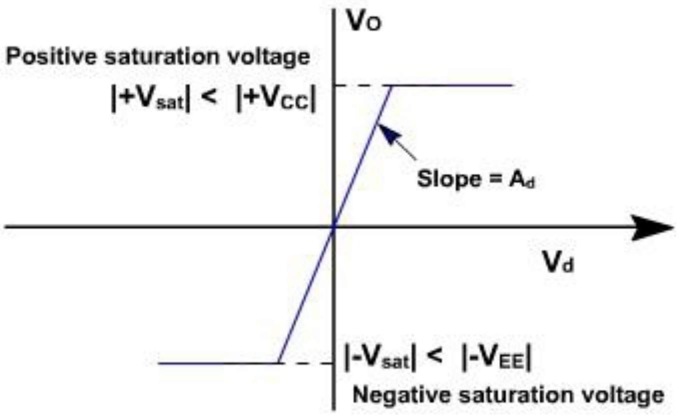


Figure 5.14

- The output voltage cannot exceed the positive and negative saturation voltages.
- These saturation voltages are specified for given values of supply voltages. This means that the output voltage is directly proportional to the input difference voltage only until it reaches the saturation voltages and thereafter the output voltage remains constant.

- Thus curve is called an ideal voltage transfer curve, ideal because output offset voltage is assumed to be zero. If the curve is drawn to scale, the curve would be almost vertical because of very large values of Ad.

PRACTICAL OP-AMP

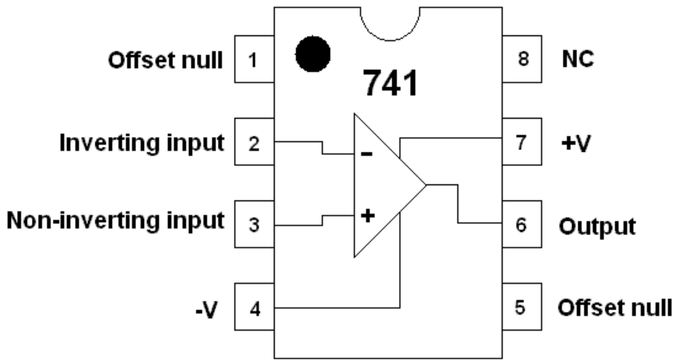


Figure 5.15a

Figure 5.15b

Characteristics:

- Practical OP-AMPs that can be made to approximate some of ideal characteristics using a negative feedback arrangement.

For example, a popular **741 OP-AMP** has the following characteristics:

1. Very high voltage gain. The gain without feedback (known as the open-loop gain) is of the order of 200,000.
2. Gain falls with frequency. It is constant up to about 10 kHz then falls until it reaches 1 at the transition frequency, f_T. Typically, f_T is 1 MHz, but is much higher in some OP-AMPs.
3. High input resistance. This is usually at least 2 MΩ, often much more.

4. Low output resistance. Typically 60-75 Ω.
5. Output takes a finite time to reach its correct value and may take additional time to settle to a steady value. Typically 0.5V/µs.
6. High value of CMRR. Typically 90db or 31622.
7. The output voltage swings to within a few volts of the supply voltages (typically ±13 V for an amplifier run on ±15 V).
8. Input voltage offset is a few millivolts.

- **SUMMERY OF OP-AMP CHARACTERISTICS**

Sr. No.	Characteristics	Value for IC 741	Ideal Value
1	Input resistance(R_i)	2 MΩ	∞
2	Output Resistance(R_O)	60-75 Ω	0
3	Voltage gain(A_V)	$2X10^5$	∞
4	Bandwidth(BW)	1 MH_Z	∞
5	CMRR	90 db	∞
6	Slew Rate	0.5 µV/V	∞
7	PSRR	150 µV/V	0
8	Input Bias Current(I_B)	50 nA	0
9	Input Offset Current(I_{ios})	6 nA	0
10	Input Offset Voltage(V_{ios})	2mV	0

VIRTUAL SHORT AND VIRTUAL GROUND

- In analysis of different circuits that includes OP-AMP, need to use an important concepts called virtual short and virtual ground.
- According to voltage short circuit concept, the potential difference between two input terminals of an OP-AMP is almost zero
- In other words both the input terminals are approximately at the same potential.

Virtual Short

- The concept of virtual short circuit explained below.

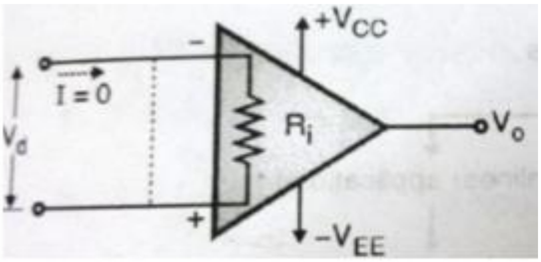


Figure 5.16

- As shown in figure 5.16, input impedance (R_i) of ideal OP-AMP is infinite. Hence current I flowing from one input terminal to another input terminal is zero.
- Thus the voltage drop across Ri will be zero and both the input terminals will be at same potential.
- In other words they are virtual shorted (not actually).

- Another way to explain this concept mathematically as follow:
- The output voltage of OP-AMP is given by

$$V_O = A_V \cdot V_d$$

Where, A_V = Open loop voltage gain and

V_d = Differential input voltage

$$\therefore V_d = \frac{V_O}{A_V}$$

- But for ideal OP-AMP, $A_V = \infty$ so, $V_d = 0$

$$V_d = \text{Differential input voltage} = V_{1(\text{non-inverting})} - V_{2(\text{inverting})}$$

$$\therefore V_1 = V_2$$

- Now both at same potential so input terminal of OP-AMP at equal potential will be consider as virtual short circuit.

Virtual Ground

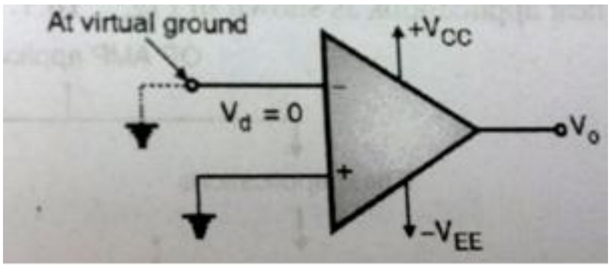


Figure 5.16

- If the non inverting terminal of OP-AMP is connected to ground as shown in figure 5.16 then as per virtual short circuit concept inverting terminal also at ground potential.
- Hence it is said to be at "virtual ground" potential.
- If the inverting terminal of OP-AMP is connected to ground then non inverting terminal also at "virtual ground" potential.

FEEDBACK IN OP-AMP

- Feedback means part of the output given to the input.
- Thus at the input of amplifier with feedback two input signal will be simultaneously present .one is original input signal and second is feedback signal.
- Feedback signal can be inphase or out of phase with input signal. Based on that feedback can be classified as positive feedback and negative feedback.
- In positive feedback, feedback signal is inphase and it is generally used for oscillation and triggering.
- In negative feedback, feedback signal is out of phase (180̊) and generally used in amplifier circuits.

NEGATIVE FEEDBACK IN OP-AMP

- OP-AMP used negative feedback in most of applications.

- Any input signal slightly greater than zero drive the OP-AMP output to saturation level because of very high gain.
- Thus when operated in open-loop, the output of the OP-AMP is either negative or positive saturation or switches between positive and negative saturation levels. Therefore open loop op-amp is not used in linear applications.
- With negative feedback, the voltage gain (A_{CL}) can be reduced and controlled so that op-amp can function as a linear amplifier.
- In addition to provide a control and stable voltage gain, negative feedback provides control of input & output impedance and amplifier bandwidth.
- In OP-AMP amplifier circuit with negative feedback, a feedback resistor R_F is connected between the output and inverting terminal as shown in figure 5.17.

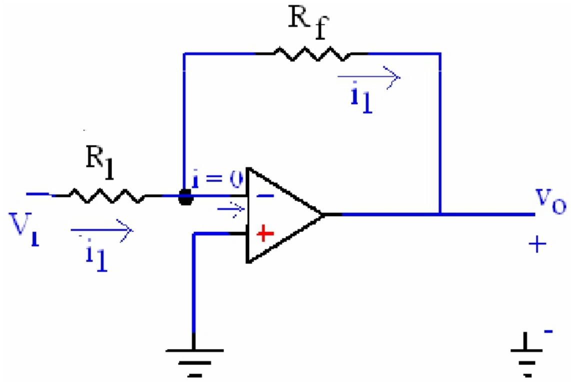


Figure 5.17

OP-AMP APPLICATIONS

1) INVERTING AMPLIFIER

- Circuit diagram of OP-AMP as inverting amplifier is shown in figure 5.18
- Signal which is to be amplified connected to inverting terminal and non-inverting terminal connected to ground.
- In this configuration output voltage signal will be 180º out of phase with input signal.

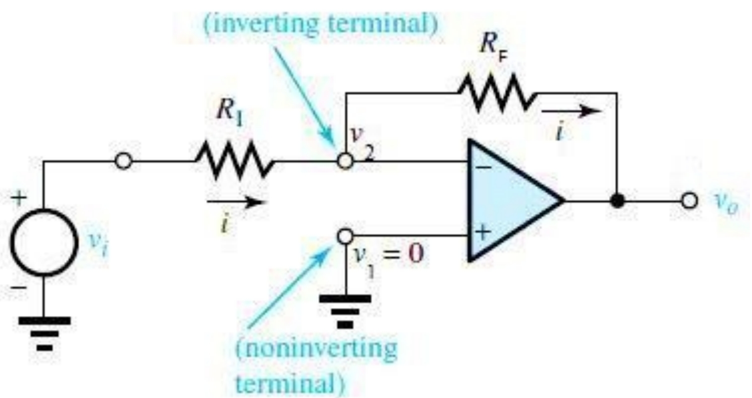


Figure 5.18

- As per ideal characteristics of OP-AMP, input resistance = ∞ so current going into op-amp is zero. Therefore current I will pass through R_1 will also passed through R_F as shown in figure.

- Since V_2= 0, because terminal 2 is grounded and because of that V_1=0 (virtual ground)
- As V_2 = 0, so input voltage is voltage across R_1 and voltage across R_F is output voltage.

Expression for gain:

Input voltage V_S given by $V_S = IR_1$
Output voltage $V_O = - IR_1$

- Closed loop gain $A_{VF} = \frac{V_O}{V_S}$

Substitute the value of Vo and Vs.

$$A_{VF} = -\frac{IR_F}{IR_1} = -\frac{R_F}{R_1}$$

$$A_{VF} = -\frac{R_F}{R_1}$$

- An output voltage can be

$$Vo = A_{VF} \times V_S$$

Waveform:

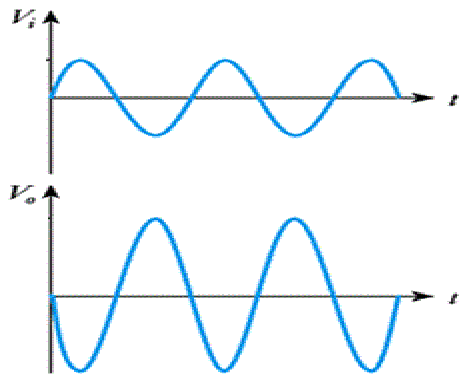


Figure 5.18b

Example:

An OP-AMP is used in inverting mode with R = 1KΩ, R_F = 15KΩ, V_{CC} = ±15V.Calculate the output voltage for following inputs.1) V_{in} = 150mV 2) V_{in} = 2 V

Solution:

Gain of the amplifier:

$$A_{VF} = -\frac{R_F}{R_1} = -\frac{15K\Omega}{1K\Omega} = -15$$

Output voltage for V_{in} = 150 mV

$$V_O = A_{VF} \text{ X } V_{in} = -15 \text{ X } 150 \text{ X} 10^{-3} = -2.25 \text{ Volts}$$

Output voltage for V_{in} = 2V

$$V_O = A_{VF} \times V_{in} = -15 \times 2 = -30 \text{ Volts}$$

But Vo can never be higher than ±V sat hence Vo will be restricted to -Vsat.
Assuming $-V_{o(sat)} = 0.9$ VCC

So, $V_O = 0.9 \times -15 = -13.5V$

Example

Find the closed loop gain of the following inverting amplifier circuit.

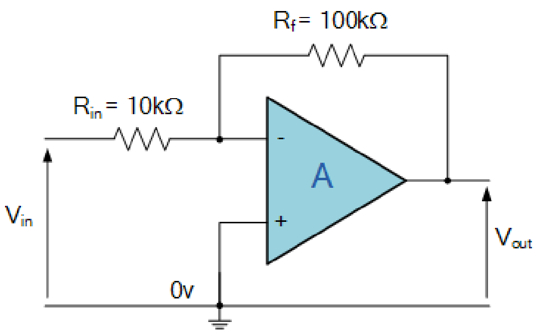


Figure 5.19

Solution:
Inverting op-amp gain

$$\text{Gain (Av)} = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

we can now substitute the values of the resistors in the circuit as follows,

Rin = 10 kΩ and Rf = 100kΩ.

and the gain of the circuit is calculated as

$$-\frac{Rf}{Rin} = -\frac{100k}{10k} = -10.$$

therefore, the closed loop gain of the inverting amplifier circuit above is given -10 or 20dB (20log(10)).

2) NON-INVERTING AMPLIFIER

- Circuit diagram of OP-AMP as non-inverting amplifier is shown in figure 5.19.
- Signal which is to be amplified connected to non-inverting terminal and inverting terminal connected to ground.

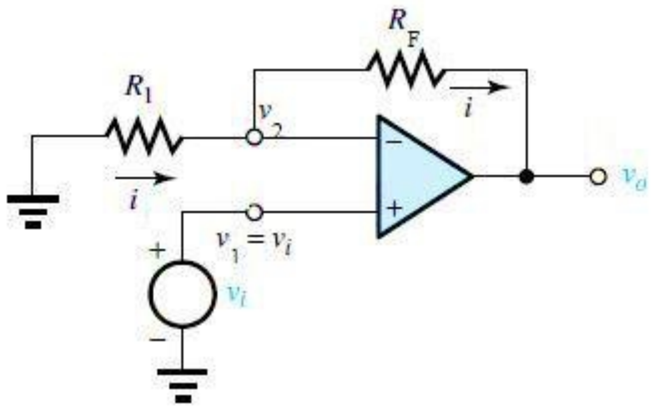


Figure 5.19

- As per ideal characteristics of OP-AMP, input resistance = ∞ so current going into op-amp is zero.
- In this configuration output voltage signal will be in phase with input signal.
- As we Consider ideal OP-AMP, input resistance(Rin) = ∞ so current going into both input terminals of op-amp is zero.($I_{B1} = I_{B2} = 0$)

  Therefore voltage across R1 given by,

$$V_2 = \left(\frac{R_1}{R_F+R_1}\right) Vo$$

- As per virtual short circuit concept

$$V_2 = V_1 = Vs$$

$$\text{So,} \quad Vs = \left(\frac{R_1}{R_F+R_1}\right) Vo$$

- Closed loop gain $A_{VF} = \frac{V_O}{V_S} = \frac{R_F+R_1}{R_1} = 1+ \frac{R_F}{R_1}$
- An output voltage can be

$$V_o = A_{VF} \times V_S$$

Waveform:

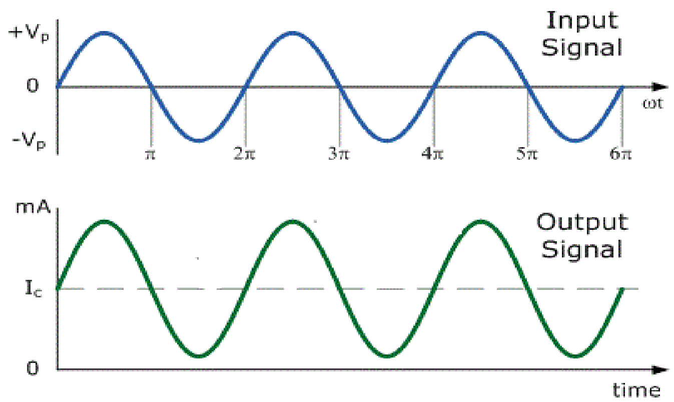


Figure 5.19b

Example

Calculate the output voltage of a non-inverting amplifier for values of V1= 2 V, R_F =500 KΩ, and R1=100 KΩ.

Solution

$$V_o = \left(1 + \frac{R_f}{R_1}\right)V_1 = \left(1 + \frac{500\ \text{k}\Omega}{100\ \text{k}\Omega}\right)(2\ \text{V}) = 6(2\ \text{V}) = \mathbf{+12\ V}$$

3) VOLTAGE FOLLOWER (UNITY GAIN BUFFER)

- In non-inverting amplifier configuration, When $R_1 = \infty$ and $R_F = 0$ it converts to voltage follower or unity gain amplifier.
- This amplifier has gain unity so it's called as unity gain amplifier.

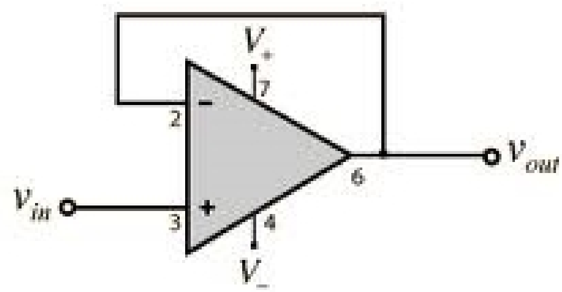


Figure 5.20

- This configured can be obtaining by short circuit R_F and open circuit R_1 connected in non-inverting amplifier configuration.
- Consider expression for closed loop gain of non-inverting amplifier that is,

$$A_{VF} = 1 + \frac{R_F}{R_1}$$

Substitute value $R_F = 0$ and $R_1 = \infty$

$$A_{VF} = 1$$

- Therefore output voltage will be equal to input voltage and in phase of input voltage. It is not behave like conventional amplifier but can be used as resistance transformer. It has also useful feature which are given below.

Feature:

- Very high input impedance
- Very low output impedance
- Large bandwidth

Example

Figure 5.21 shows a source connected to a load with a voltage follower. It is given that $R_S = 10\ k\Omega$ and $R_L = 100\Omega$. (a) Calculate V_O. (b) Calculate V_O if the voltage follower is removed and the source connected to the load.

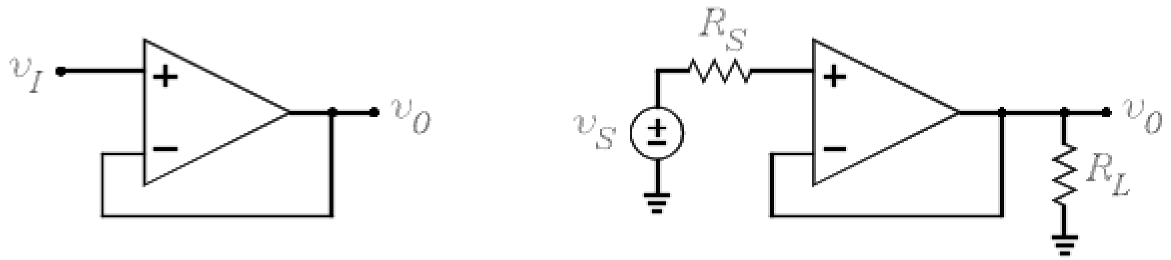


Figure 5.21

Solution

a) With the voltage follower, there is no current through RS so that the voltage at the op-amp input is V_S. It follows that $V_O = V_S$.
b) If the voltage follower is removed and the source is connected directly to the load, V_O is given by $V_O = V_S R_L/ (R_S + R_L) = V_S /101$. This is a decrease in output of 20 log 101 = 40.1 dB. This example illustrates how a unity gain amplifier can increase the gain of a circuit.

4) SUMMING AMPLIFIER or ADDER

- It is possible to apply more than one input signals to an inverting amplifier. This circuit will then add all these input signals to produce their addition at the output. Such circuit called adder or summing amplifier.
- Polarity or sign of output voltage the summing or adder circuits will be negative because we made inverting summing or adder configuration.
- Inverting adder or inverting summing amplifier:

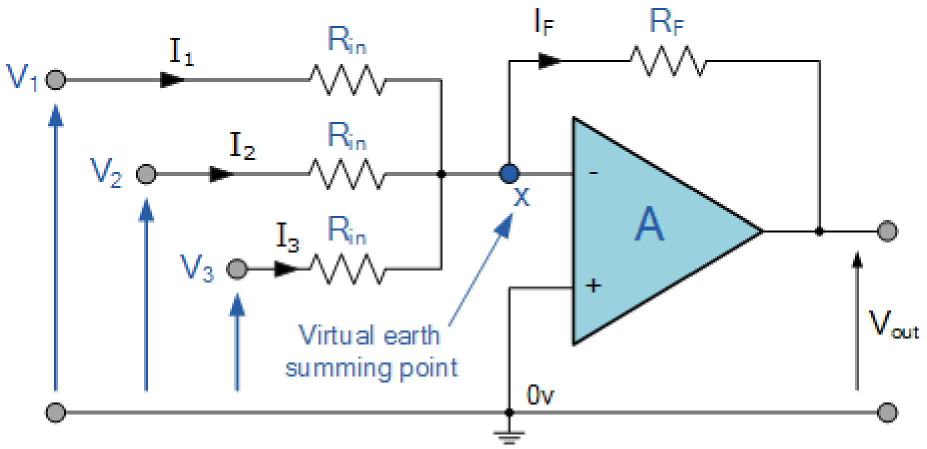


Figure 5.22

- Figure 5.22 shows inverting summing amplifier configuration with three input V1, V2 and V3.

- Depending upon the relation between feedback resistor RF and three resistors R1, R2 and R3 the same circuit can be used as summing amplifier, average amplifier or scaling amplifier.
- For analysis we assumed that OP-AMP is ideal so input resistance = ∞ so current flowing into input terminals of OP-AMP is zero ($I_{B1}=I_{B2}=0$). Addition to this, Node A is at virtual ground potential.
- Expression for output voltage

$$I_1 + I_2 + I_3 = I_{B2} + I_F \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

But Ri = ∞ so, $I_{B2}=0$ and $V_A=V_B= 0$ due to virtual ground concept.

$$\text{Hence, } I_1 + I_2 + I_3 = I_F \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

From the input side, $I_1 = \frac{V_1 - V_A}{R_1} = \frac{V_1}{R_1}$ as $V_A=0$

Likewise, $I_2 = \frac{V_2}{R_2}$ and $I_3 = \frac{V_3}{R_3}$

And from the output side,

$$I_F = \frac{V_A - V_O}{R_F} = -\frac{V_O}{R_F}$$

As per equation (2)

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_O}{R_F}$$

$$V_O = -\left[\frac{R_F}{R_1} V_1 + \frac{R_F}{R_2} V_2 + \frac{R_F}{R_3} V_3\right]$$

If we substitute $R_F = R_1 = R_2 = R_3$ then we get

$$\mathbf{V_O = -(V_1 + V_2 + V_3)}$$

❖ SCALING OR WEIGHTED AMPLIFIER

- Above given circuit can also used as a scaling amplifier. This is possible if each input amplified by a different factor.

$$V_O = -\left[\frac{R_F}{R_1} V_1 + \frac{R_F}{R_2} V_2 + \frac{R_F}{R_3} V_3\right] \text{ where } R_1 \neq R_2 \neq R_3$$

❖ AVARAGE CIRCUIT

- Inverting adder circuit can also be used as averaging circuit by setting $R_1=R_2=R_3=R$ and $R_F = R/3$

$$V_O = -\frac{[V1 + V2 + V3]}{3}$$

- Magnitude of output voltage is equal to the average of input three input voltages.

Example

Find the output voltage of the following Summing Amplifier circuit.

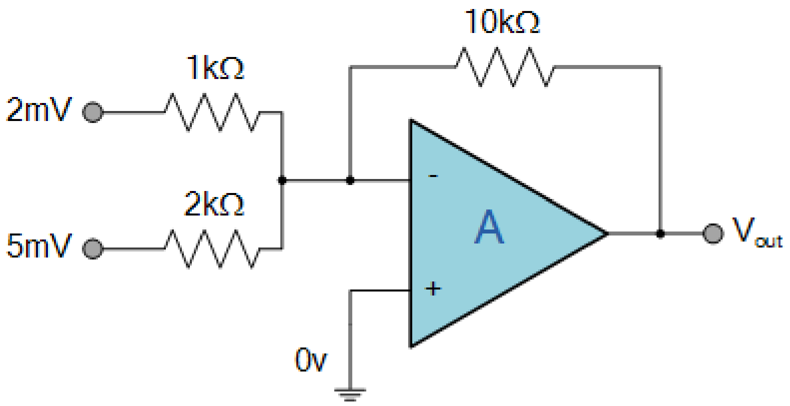


Figure 5.23

Solution:
Using the previously found formula for the gain of the circuit

$$\text{Gain (Av)} = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

we can now substitute the values of the resistors in the circuit as follows,

$$A_1 = \frac{10k\Omega}{1k\Omega} = -10$$

$$A_2 = \frac{10k\Omega}{2k\Omega} = -5$$

we know that the output voltage is the sum of the two amplified input signals and is calculated as:

$$Vout = (A_1 \times V_1) + (A_2 \times V_2)$$

$$Vout = (-10(2mV)) + (-5(5mV)) = -45mV$$

then the output voltage of the Summing Amplifier circuit above is given as -45 mV and is negative as it's an inverting amplifier.

5) DIFFERENCE AMPLIFIER OR SUBSTRACTOR

- The difference amplifier or substractor circuits used to obtain the subtraction of the two input signal.
- Circuit diagram of difference amplifier or substractor given below. For analysis we assumed that OP-AMP is ideal so input resistance = ∞ so current flowing into input terminals of OP-AMP is zero ($I_{B1}=I_{B2}=0$).

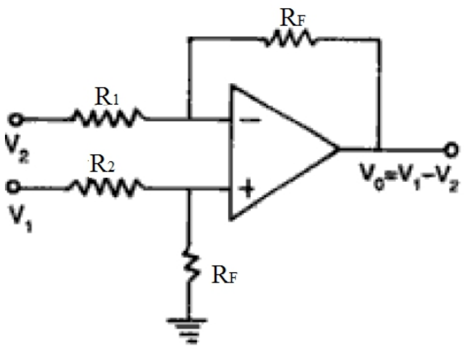


Figure 5.24

- Expression for output voltage of difference amplifier:

$$V_O = \frac{R_F}{R_1}(V_1 - V_2)$$

- Output voltage will be amplified voltage of Difference of input voltage.

❖ SUBSTRACTOR:

- Same circuit can be used as substractor by placing resistor $R_F = R_1 = R$.
- Equation for output voltage.

$$V_O = (V_1 - V_2)$$

EXAMPLE

For the diff amp circuit of Figure 5.25, it is given that $R_1 = R_3 = 10\ k\Omega$ and $R_2 = R_F = 20\ k\Omega$. Solve for the output voltage, the input resistance to the V_{I1} terminal, and the input resistance to the V_{I2} terminal for the three cases: $V_{I1} = 0$, $V_{I1} = -V_{I2}$, and $V_{I1} = +V_{I2}$.

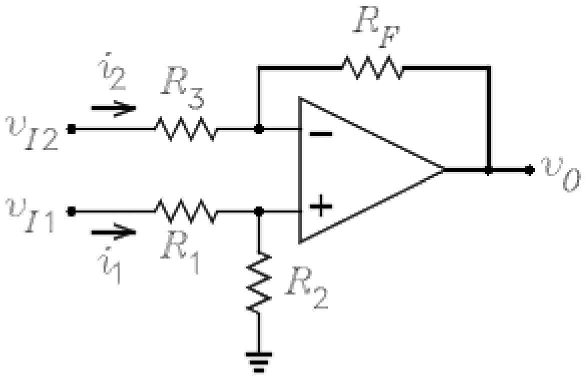


Figure 5.25

Solution
Because RF /R3 = R2/R1, It follows that $V_O = 2\,(V_{I1} - V_{I2})$.

The input resistance to the V_{I1} node is 30 kΩ.

For $V_{I1} = 0$, it is 10 kΩ.
For $V_{I1} = -V_{I2}$, it is 6 kΩ.
For $V_{I1} = +V_{I2}$, it is 40 kΩ.

6) CURRENT-TO-VOLTAGE AMPLIFIER

- Circuit diagram of current to voltage convertor is shown below.

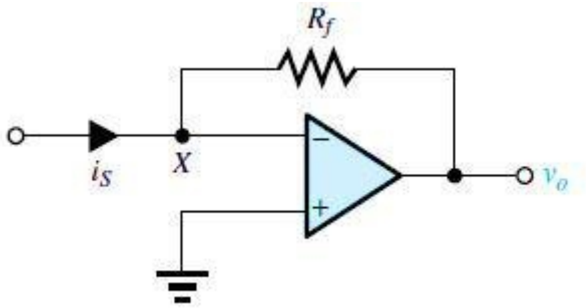


Figure 5.26

- For analysis we assumed that OP-AMP is ideal so input resistance = ∞ so current flowing into input terminals of OP-AMP is zero ($I_{B1}=I_{B2}=0$).
- Expression for output voltage

$$V_O = -\,R_F\,I_s$$

7) CURRENT-TO-CURRENT AMPLIFIER

- Circuit diagram of current to current convertor is shown below.

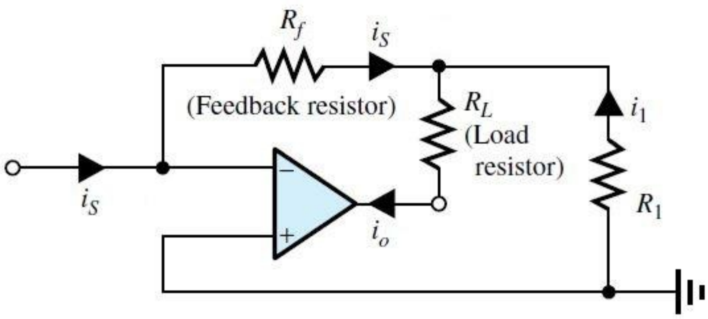


Figure 5.27

- For analysis we assumed that OP-AMP is ideal so input resistance = ∞ so current flowing into input terminals of OP-AMP is zero ($I_{B1}=I_{B2}=0$).
- Expression for output current in the form of current gain

$$\frac{I_O}{I_S} = 1 + \frac{R_F}{R_1}$$

8) CHARGE-TO-CHARGE AMPLIFIER

- Circuit diagram of charge to charge convertor is shown below.

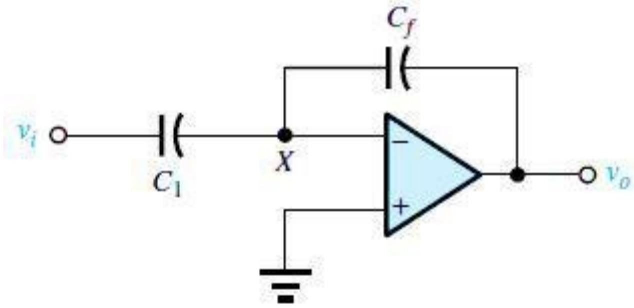


Figure 5.28

- For analysis we assumed that OP-AMP is ideal so input resistance = ∞ so current flowing into input terminals of OP-AMP is zero ($I_{B1}=I_{B2}=0$).
- Expression for output voltage

$$\frac{V_O}{V_i} = \frac{C_1}{C_F}$$

9) INTEGRATORS

- Figure of integrator is shown below which is obtained by placing capacitor C in the feedback branch and resistor R on inverting terminal. Assuming OP-AMP has ideal characteristics.

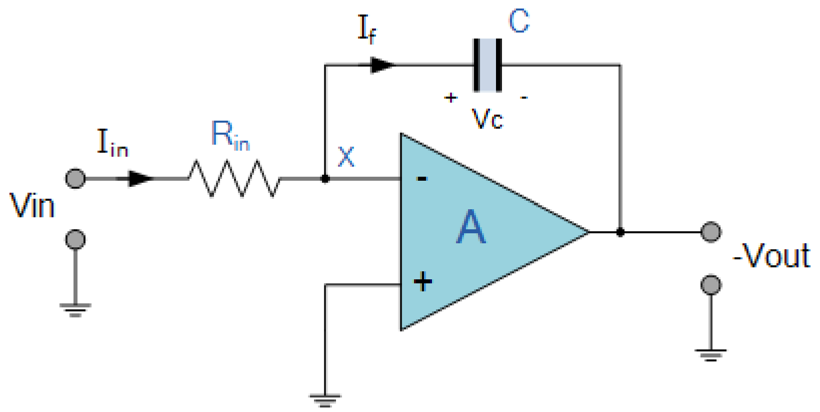


Figure 5.29

- For analysis we assumed that OP-AMP is ideal so input resistance = ∞ so current flowing into input terminals of OP-AMP is zero ($I_{B1}=I_{B2}=0$).
- Expression for output voltage of integration amplifier:

$$V_O = \frac{1}{R_1 C_F} \int_0^t V_{in} \; dt + C$$

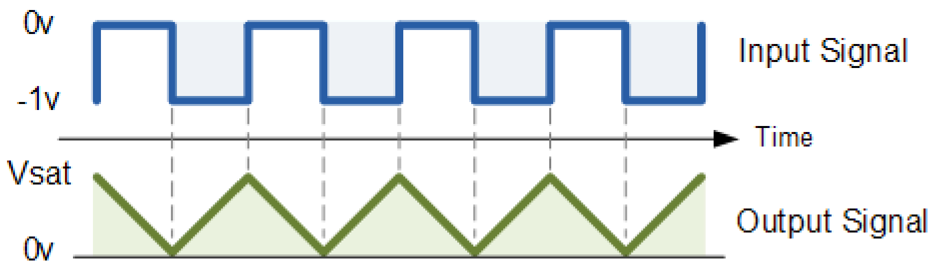


Figure 5.29 b

10) DIFFERENTIAL AMPLIFIER

- Figure of differentiator is shown below which is obtained by placing R_F in the feedback branch and capacitor C on inverting terminal. Assuming OP-AMP has ideal characteristics.

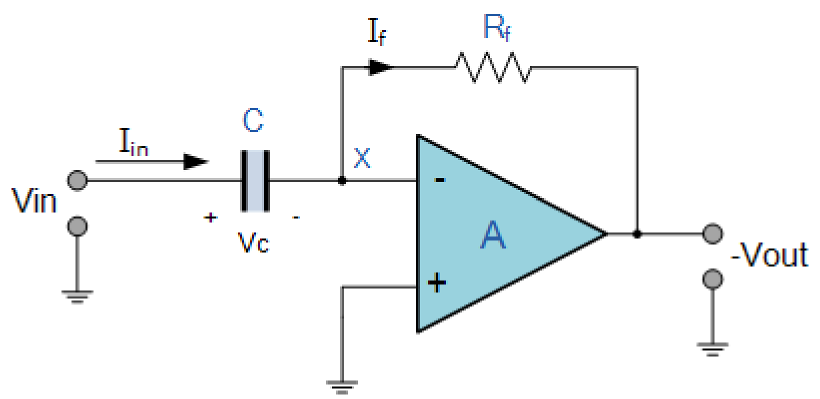


Figure 5.30

- For analysis we assumed that OP-AMP is ideal so input resistance = ∞ so current flowing into input terminals of OP-AMP is zero ($I_{B1}=I_{B2}=0$).
- Expression for output voltage of difference amplifier:

$$V_O = -R_F C_1 \frac{d}{dt}(V_{in})$$

Which corresponds to a differentiator with a gain of $-R_F C_1$

- In practice, Differentiators are normally avoided because of high-frequency noise and stability problems (which make them oscillate).

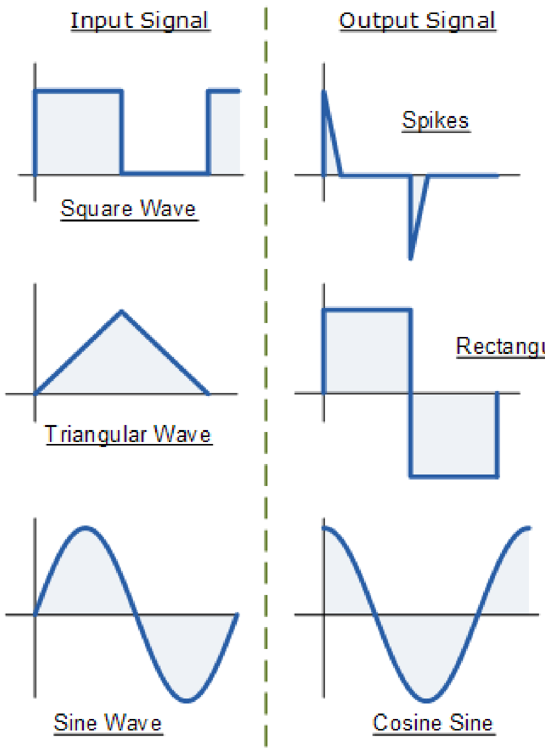


Figure 5.30b

11) FILTER

- Filters designed to pass certain frequency band or frequencies and attenuate or block signal of frequencies outside this band.
- Mainly two type of filter available named active filters and passive filter.

INDUCTORLESS (ACTIVE) FILTERS

- Filters made from active components such BJT, FET & OP-AMP along with the passive components known as ACTIVE FILTERS.
- Inductor is not possible to fabricate on IC so in active filter inductor is not present.

1. ACTIVE LOW PASS FILTERS

- Circuit diagram and frequency response of active low pass filter given below in figure.
- This circuits use R-C network for filtering and OP-AMP in non inverting mode as amplifier.
- R-C value decides the cut-off frequency of the filter.
- Resistor R1 and RF decide gain of the amplifier.

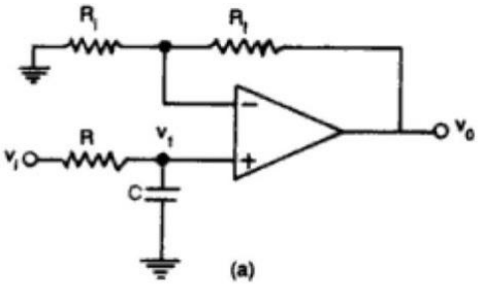


Figure 5.31

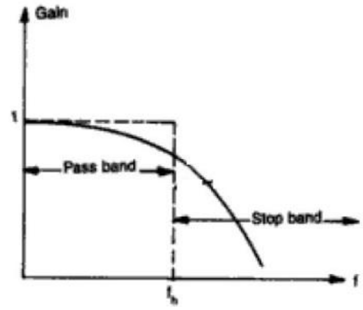


Figure 5.31b

- OP-AMP used here is in non inverting mode so gain of the amplifier is

$$A_{VF} = \frac{V_O}{V_S} = 1 + \frac{R_F}{R_1}$$

- Frequency response curve shows frequency signal below cut-off frequency is passed and it is consider as passband.
- Frequency signal above cut-off frequency is attenuated and it is consider as stopband.

2. ACTIVE HIGH PASS FILTERS

- Circuit diagram and frequency response of active high pass filter given below in figure.
- This circuits use R-C network for filtering and OP-AMP in non inverting mode as amplifier.
- R-C value decides the cut-off frequency of the filter.
- Resistor R1 and RF decide gain of the amplifier.

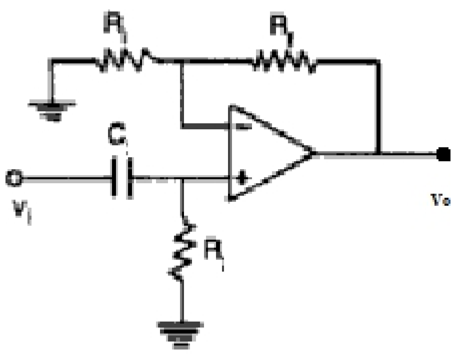


Figure 5.32

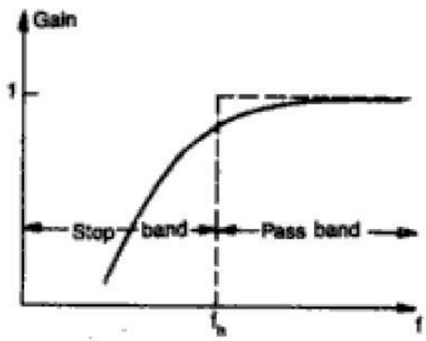


Figure 5.32b

➢ OP-AMP used here is in non inverting mode so gain of the amplifier is

$$A_{VF} = \frac{V_O}{V_S} = 1 + \frac{R_F}{R_1}$$

➢ Frequency response curve shows frequency signal below cut-off frequency is attenuated and it is consider as stopband.
➢ Frequency signal above cut-off frequency is passed and it is consider as passband.

PRACTICAL APPLICATION: A CASE STUDY

Automotive Power-Assisted Steering System

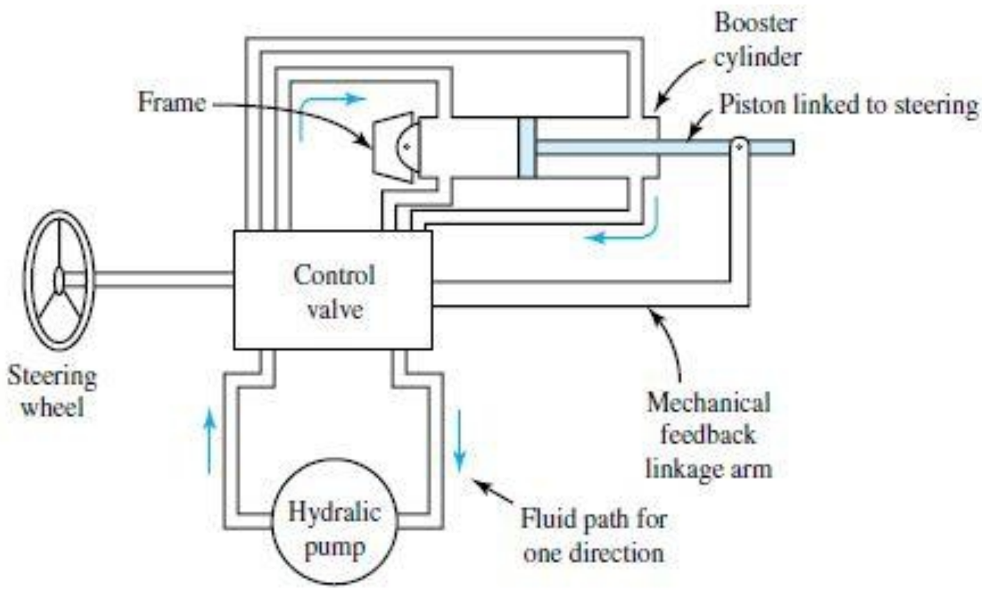


Figure 5.33

➢ In terms of negative feedback, there exists an analogy between the operational amplifier and the power-steering mechanism of an automobile.
➢ The hydraulic pump is analogous to the power supply in an op-amp circuit.
➢ The position of the booster-cylinder piston that is linked to the steering is analogous to the op-amp output signal.
➢ The mechanical linkage between the control valve and the booster-cylinder piston is analogous to the feedback circuit.
➢ The control-valve response to the difference between the input from the steering wheel and the position of the steering linkage is analogous to the op-amp response to its differential input signal.
➢ Thus the automotive power assisted steering system is but an example of negative feedback in a mechanical sense.
➢ Figure illustrates in a simplified manner how a hydraulic pump driven by the engine continuously supplies pressure to a control valve which in turn supplies the fluid to the two sides of the booster cylinder.

- A negative feedback path is established from the booster cylinder through the mechanical linkage back to the control valve. For straight steering, the pressure applied is equal on both sides of the cylinder and, as such, no turning force results.
- When the steering wheel is moved by the driver to turn the wheels in the desired direction, more pressure is applied to one side of the cylinder or the other. A mechanical feedback arm from the steering linkage causes the valve to return to its neutral position as the wheels turn, thereby allowing the driver to make a gradual turn. As and when the steering wheel is turned, the wheels move a proportional amount rather than moving all the way to the extreme position.

Question

1. Explain in detail: Ideal OP-AMP.
2. List the characteristics of Practical OP-AMP.
3. Describe the appearance of a typical operational amplifier packaged as an 8-pin integrated circuit.
4. List the terminals present on all OP-AMPs and state their functions.
5. Derive the expression of voltage gain for inverting for OP-AMP.
6. Derive the expression of voltage gain for non-inverting mode for OP-AMP.
7. Describe how an OP-AMP voltage follower works and give an example of its applications.
8. Explain OP-AMP parameter values:
   a) Input offset voltage
   b) Input bias current
   c) Common-mode rejection ratio
   d) Slew rate
   e) Open loop gain
   f) Output offset voltage
9. Define filter. How are filter classified?
10. What are a passband and a stopband for filter?
11. Explain active high pass filter.
12. Explain active low pass filter.

Example

1) Design a circuit for an OP-AMP inverting amplifier with a closed-loop gain of -200, assuming an ideal OP-AMP.

2) Design a circuit for an OP-AMP non-inverting amplifier with a closed-loop gain of 101, assuming an ideal OP-AMP.

3) An inverting OP-AMP amplifier with an input resistance of 2 kΩ an output resistance of 100Ω and an open-circuit voltage gain of −30 (an inverting decibel gain of 29.5 dB).find out output resistance.

4) Design an inverting summer which has an output voltage given by $V_O = 3 - 2V_I$ .Assume that +15V and −15V supply voltages are available.

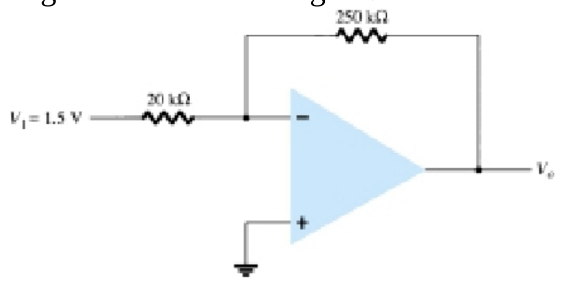
5) Calculate the output voltage of an op-amp summing amplifier for the following sets of voltages and resistors. Use R_f = 1 MΩ in all cases.

(a) $V_1 = +1$ V, $V_2 = +2$ V, $V_3 = +3$ V, $R_1 = 500$ kΩ, $R_2 = 1$ MΩ, $R_3 = 1$ MΩ.
(b) $V_1 = -2$ V, $V_2 = +3$ V, $V_3 = +1$ V, $R_1 = 200$ kΩ, $R_2 = 500$ kΩ, $R_3 = 1$ MΩ.

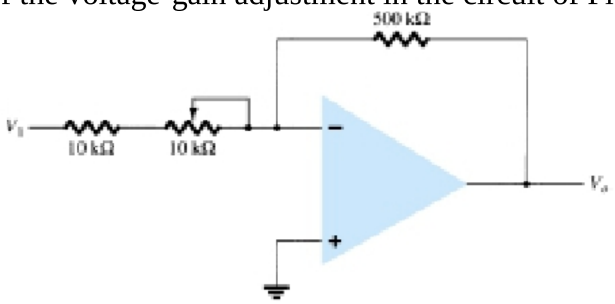
6) Calculate the CMRR (in dB) for the circuit measurements of V_d=1 mV, Vo=120 mV, and V_C =1 mV, Vo=20μV.

7) Determine the output voltage of an op-amp for input voltages of V_{i1}=200 μV and Vi2=140
μV. The amplifier has a differential gain of Ad=6000 and the value of CMRR is:
(a) 200.
(b) 105.

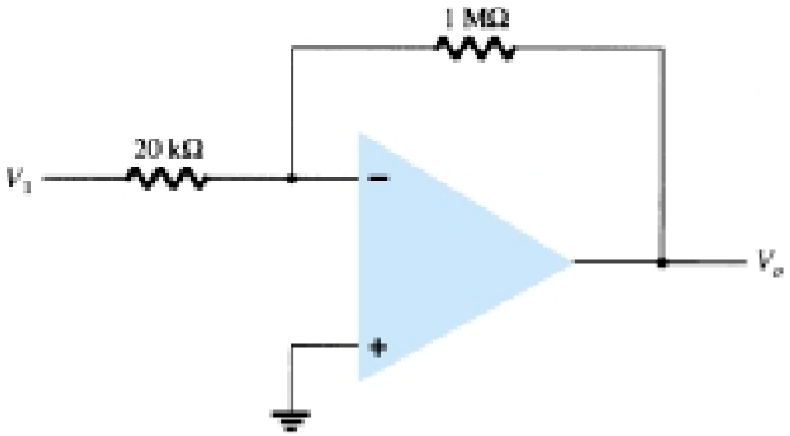
8) What is the output voltage in the circuit of Figure?



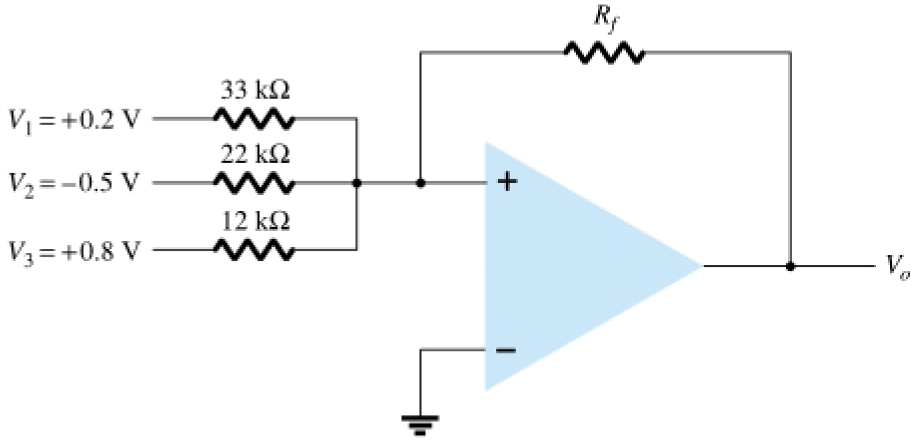
9) What is the range of the voltage-gain adjustment in the circuit of Figure?



10) What input voltage results in an output of 2 V in the circuit of Figure



11) Calculate the output voltage developed by the circuit of Figure for R_f =330 KΩ.


R_f
33 kΩ
$V_1 = +0.2$ V
22 kΩ
$V_2 = -0.5$ V
12 kΩ
$V_3 = +0.8$ V
+
−
V_o

CHAPTER 4
DIGITAL BUILDING BLOCKS & COMPUTER SYSTEMS

TOPICS COVERED IN THIS CHAPTER

1. **Introduction**
2. **Introduction to Digital System**
3. **Number systems**
4. **Logic gates**
5. **Boolean algebra**
6. **Different forms of boolean algebra**
7. **Simplification of logical function using karnaugh map (k-map)**
8. **Binary adders**
9. **Encoder**
10. **Decoder**
11. **Multiplexer**
12. **Demultiplexer**
13. **Sequential blocks**
14. **Shift registers**
15. **Counters**
16. **Digital-to-analog & analog-to-digital converter**
17. **Memory**
18. **Display Devices**
19. **Computer Systems**
20. **Computer Networks**
21. **A Case Study – Microcomputer – Controlled Bread making Machine**
22. **Questions**

4.1 INTRODUCTION:

- Electronic systems usually deal with information. Representation of information is called a **signal.** Signal in electronics is generally in form of voltage or current. Value of a signal is proportional to some physical quantity and it gives information about it. For example, temperature represented in terms of voltage signal.

- There are two types of signals which are different in terms of their characteristics with respect to time and value.
  1. Analog Signals
  2. Digital Signals

- A signal whose value is defined at all instances of time is called **continuous time signal.** On the other hand signal whose values are defined only at discrete instances of time is called **discrete time signal.** Most of the signals that occur in nature are analog in form. A discrete time signal can be obtained from continuous time signal by process called **sampling.** This has been illustrated in Fig. 4.1.

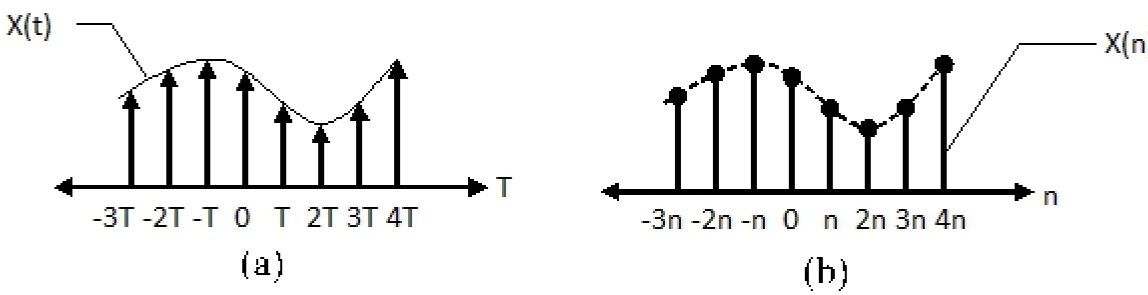


Fig. 4.1: (a) Continuous time signal x(t) sampled at every T interval, (b) Resulting discrete time signal x(n)

- Similarly if a signal can take any value in a given range between some minimum and maximum value then the signal is called **continuous value signal.** On the other hand if a signal takes only certain fixed values in a given range then it is called **discrete value signal**. The process of converting a continuous value signal to a discrete value signal is called **quantization.** This is illustrated in Fig. 4.2.

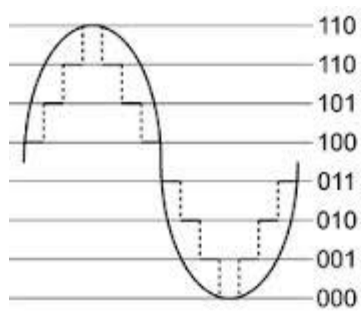


Fig. 4.2: Continuous value signal (solid line) and discrete value signal (dotted line)

Analog signal: Signals that are continuous in time and continuous in value are called analog signal.

Digital signal: Signals that are discrete in time and discrete in values are called digital signals. Digital signals are generally processed by digital systems like computers and hence their values are represented in terms of binary as shown in Fig. 4.2.

- Analog signal being continuous in time will have infinite values in any given period of time. Practically a digital system like computer cannot handle infinite values due to limited physical resources and processing power. This is the reason why a continuous time signal has to be sampled and converted to discrete time signal.

- Again analog signals are continuous in value and hence can take any value in a given range. Now ideally number of values in any given range will be infinite which cannot be represented by finite number of bits on a computer. For example, as shown in Fig. 4.2, with three bits used for representing values only eight different values can be represented. Thus a continuous value signal has to be quantized and converted to discrete value signal.

➢ Levels of Integration

- Digital electronic circuits have become increasingly popular and successful due to integrated circuit (IC) technology. Advancement in IC technology has made it possible to construct large number of devices (eg. transistor, diode, resistors, capacitors, etc) on a very small chip. Classification of IC technology based on number of components per chip is as follows.

  1. Small-scale integration (SSI), containing fewer than 100 components
  2. Medium-scale integration (MSI), containing 100 to 1000 components
  3. Large-scale integration (LSI), containing 1000 to 10,000 components
  4. Very large-scale integration (VLSI), containing more than 10,000 components

➢ Comparison of Analog and Digital Systems

	Analog Systems	Digital Systems
1	Analog systems operate on continuous time and continuous value signals.	Digital systems operate on discrete time and discrete value signals generally represented in binary.
2	Analog systems are difficult to design.	Digital systems are easy to design as most of the components are in form of Integrated circuits (IC).
3	Analog systems are mostly custom made and lack flexibility.	Digital systems have high degree of flexibility.
4	Less efficient in storage of information.	More efficient in storage of information.

5	Analog signal processed by these systems are affected by noise very easily.	Digital signal are more noise-immune compared to analog signals.
6	Relatively costly compared to digital system	Low cost due to mass production of components.
7	Analog systems are more sensitive to parameter variation.	Digital systems are less sensitive to parameter variation
8	No conversion of input signals are required before processing	Input signals are converted from analog to digital form before it is processed
9	As no conversion of input signal is required there is no loss of information.	Due to process of sampling and quantization there is loss of information.
10	Analog systems are more efficient for real time processing	Digital systems may offer limitations for real time processing

4.2 Introduction to Digital System:

- A digital system uses a building blocks approach. Many small operational units are interconnected to make up the overall system.
- The most basic logical unit system is gate circuit. There are several different types of gates with each perform differently from other logic gates.
- Digital signal consist of only two values, ‘0’ and ‘1’. These two values have logical meaning i.e. ‘1’ represents the existence of particular condition and ‘0’ represents the absence of condition.

- There are two types of tables are used in digital system:
  1. **Truth Table:**
     Truth table plots inputs and outputs in terms of 1s and 0s.

  2. **Function Table:**
     Function table plots inputs and outputs in term of HIGH and LOW voltage levels.

- The design of digital system may be roughly divided into three stages;

  1. **System Design:**
     It involves breaking the overall system into subsystem and specifying the characteristics of each subsystem. For example, the system design of a digital computers involves specifying the number and type of memory, ALU and i/p – o/p devices.

  2. **Logic Design:**
     It involves how to interconnect basic logic building blocks to perform specific function. For example, to make a flip flop different logic gates are needs to be connected in specific manner.

3. **Circuit Design:**
   It involves specifying the interconnection of specific components like resistors, transistors, diodes, CMOS etc. to create a logic gates.

➢ Advantages of Digital Systems

1. Digital systems are easier to design
2. Information storage is easy
3. Accuracy and precision are greater
4. Digital systems are more versatile
5. Digital circuits are less affected by noise
6. More digital circuitry can be fabricated on IC chips

➢ Logic Levels and Different types of Logics

- Digital system use the binary number system. Therefore, two-state devices are used to represent the two binary digits 1s & 0s by two different voltage levels, called HIGH and LOW.
- Normally, the binary 0 and 1 are represented by the logic voltage levels 0 V and +5 V.
- Usually any voltage between 0 V to 0.8 V represents the logic 0 and any voltage between 2 V to 5 V represents the logic 1. This voltage levels can be varies according to the different logical systems.
- There are three types of logics available in digital systems.
  1. Positive Logic
  2. Negative Logic
  3. Mixed Logic

1. Positive Logic:
In positive logic high voltage level is represent as logic 1 and low voltage level is represent as logic 0.

Fig. 4.3: Illustration of positive logic

2. Negative Logic:
In positive logic high voltage level is represent as logic 0 and low voltage level is represent as logic 1.

Fig. 4.4: Illustration of negative logic

3. **Mixed Logic:**
   This scheme uses positive logic in some portions (e.g inputs) of the system while applying negative logic (e.g. outputs) in other portion of the system.

- Suppose some function X = AB' + A'B for this function the representation of all the logics are as follow;

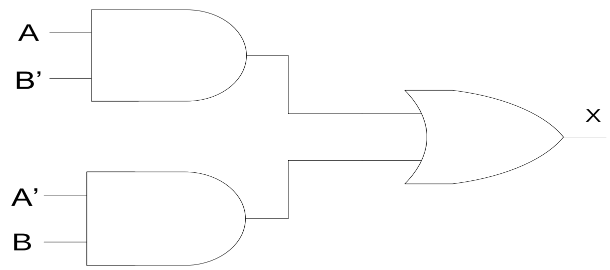


Fig. 4.5: Representation of function X = AB' + A'B

- Truth table of the given function for all the logics is shown as follow;

Table 4.1: Truth table of Positive logic, Negative logic, Mixed logic for X = AB' + A'B

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

Positive Logic

A	B	X
1	1	1
1	0	0
0	1	0
0	0	1

Negative Logic

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

Mixed Logic

4.3 NUMBER SYSTEMS:

4.3.1 Introduction to Number System:

➢ **Definition & Importance**

- Number system is the basis for counting various items. On hearing the word 'number', we immediately think of the familiar decimal number system with 10 digits 0 to 9. But modern computers communicate and operate with binary numbers which use only 2 digits 0 & 1. Also different types of number systems like octal and hexadecimal are also used widely. Depending upon the type of number system, we use different digits to represent various numbers.

➢ **Few Common Aspects to All Numbering Systems**

(i) Base or Radix
The number of symbols used for the representation of numbers in a number system is known as its Base or Radix and is generally denoted by r.

(ii) Digit

Each symbol in the number system is called a Digit.

(iii) The largest value of a digit is always one less than the base

For ex, in decimal system, the largest digit is 9 (since base is 10)

(iv) Each digit position (i.e. place) represents a different multiple of base

This means that the numbers have positional importance. Hence the number systems are known as **Positional Weighted Number System**. It means that the value attached to a symbol depends on its location with respect to the decimal point.

- For example decimal number 123.4 (base 10) can actually be represented as;

$(123.4)_{10} = 1x10^2 + 2x10^1 + 3x10^0 + 4x10^{-1}$

- In general, a number of any radix can be expressed as,

$Nr = \ldots + D3 \text{ x } r^3 + D2 \text{ x } r^2 + D1 \text{ x } r^1 + D0 \text{ x } r^0 + D\text{-}1 \text{ x } r^{-1} + D\text{-}2 \text{ x } r^{-2} + D\text{-}3 \text{ x } r^{-3} + \ldots$

Where;

r is the base and D_i is any valid digit in the number system of base r.

- The digits on the left side of the decimal point form the integer part of a number and those on the right side form the fractional part.
- The left most digit in any number representation, which has the greatest positional weight out of all the digits present in that number is called the most significant digit (MSD).
- The right most digit in any number representation, which has the least positional weight out of all the digits present in that number is called the least significant digit (LSD).

➢ **Various Numbering Systems**

- Different number systems are used in various applications. The commonly used number systems along with their base, 1st digit, last digit and available digits are as shown below:

Table 4.2: Illustration of various number system

Sr. No	Number System	Base	First digit	Last digit	All digits
1	Binary	2	0	1	0,1
2	Octal	8	0	7	0,1,2,3,4,5,6,7
3	Decimal	10	0	9	0,1,2,3,4,5,6,7,8,9
4	Hexadecimal	16	0	F	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Note: In hexadecimal number system, meaning of A≈10, B≈11, C≈12, D≈13, E≈14 & F≈15.

1. Decimal number system

- Decimal number system is the most familiar no. system used in day-to-day life. The decimal system consists of 10 unique symbols. Hence the **base or radix is 10**. It is a positional weighted system. In this system, any number (integer, fraction or mixed) of any magnitude can be represented by the use of these ten symbols only.

- The digits on the left side of the decimal point form the integer part of a decimal number while those on right side from the fractional part. The digits on the right of the decimal point have weights which are negative powers of 10 and the digits to the left of the decimal point have weights which are positive powers of 10. The sum of all the digits multiplied by their weights gives the total number being represented.

- In general, the value of any mixed decimal number

$$d_n\, d_{n-1}\, d_{n-2} \ldots d_1\, d_0 \,.\, d_{-1}\, d_{-2}\, d_{-3} \ldots d_{-k}$$

is given by

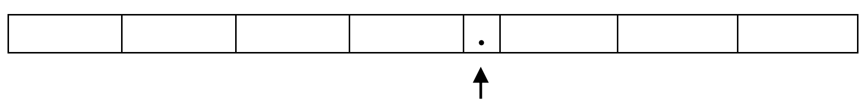
$(d_n \text{ x } 10^n) + (d_{n-1} \text{ x } 10^{n-1}) + \ldots + (d_1 \text{ x } 10^1) + (d_0 \text{ x } 10^0) + (d_{-1} \text{ x } 10^{-1}) + \ldots + (d_{-k} \text{ x } 10^{-k})$

- Consider a decimal no. 9256.26. We represent it as:

9256.26 = 9 x 1000 + 2 x 100 + 5 x 10 + 6 x 1 + 2 x (1/10) + 6 x (1/100)

$= 9 \text{ x } 10^3 + 2 \text{ x } 10^2 + 5 \text{ x } 10^1 + 6 \text{ x } 10^0 + 2 \text{ x } 10^{-1} + 6 \text{ x } 10^{-2}$

MSD . . . . 10^3 10^2 10^1 10^0 10^{-1} 10^{-2} 10^{-3} . . . . LSD



RADIX POINT

Fig. 4.6: Decimal position values as power of 10

2. BINARY NUMBER SYSTEM

- The binary number system is a positional weighted system. The **base or radix** of this number system is **2**. Hence, it has two independent symbols. The base itself cannot be a symbol. The symbols used are 0 & 1. A binary digit is called a *bit*. A binary number consists of a sequence of bits, each of which is either a 0 or a 1. The binary point separates the integer and fraction part. The weight of each bit position is one power of 2 greater than the weight of the position to its immediate right. The place values left on the binary point in binary are 64, 32, 16, 8, 4, 2 and 1.

- In general, the value of any mixed binary number

$$b_n\ b_{n-1}\ b_{n-2}\ .\ .\ .\ b_1\ b_0\ .\ b_{-1}\ b_{-2}\ b_{-3}\ .\ .\ .\ b_{-k}$$

is given by

$$(b_n \times 2^n) + (b_{n-1} \times 2^{n-1}) + \ldots + (b_1 \times 2^1) + (b_0 \times 2^0) + (b_{-1} \times 2^{-1}) + \ldots + (b_{-k} \times 2^{-k})$$

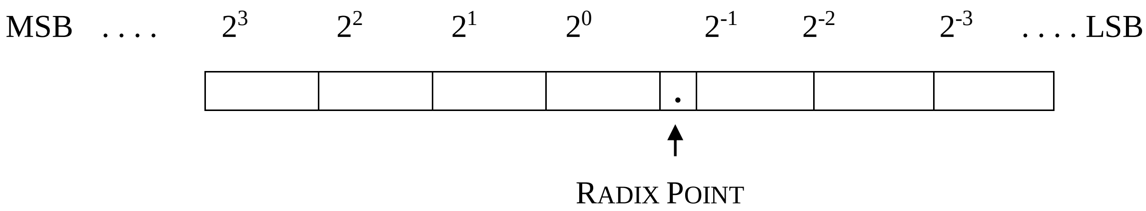


Fig. 4.7: Binary position values as power of 2

➢ Counting in Binary

- Counting in binary is very similar to decimal counting. Start counting with 0, the next count is 1. Moving ahead, we put 1 in the column to the left and continue the counting. Thus, 11 is the maximum we can count using two bits. Similarly, we can continue counting with 5, 6, ... bits.

Table 4.3: Counting in Binary

Decimal Number	Binary Number	Decimal Number	Binary Number
0	00000	11	01011
1	00001	12	01100
2	00010	13	01101
3	00011	14	01110
4	00100	15	01111
5	00101	16	10000
6	00110	17	10001
7	00111	18	10010
8	01000	19	10011
9	01001	20	10100
10	01010	21	10101

➢ Applications

- The binary number system is used in digital computers because the switching circuits used in these computers use two-state devices such as transistors, diodes, etc. These devices have to exist in one of the two possible states: ON of OFF, OPEN or CLOSED. So, these two states can be represented by the symbols 0 and 1, respectively.

3. OCTAL NUMBER SYSTEM

- The octal number system was extensively used by early minicomputers. It is also a positional weighted system. Its **base or radix is 8**. It has 8 independent symbols **0 to 7**.
- Since its base $8 = 2^3$, every 3-bit group of binary can be represented by an octal digit. An octal number is, thus 1/3 rd. the length of the corresponding binary number.

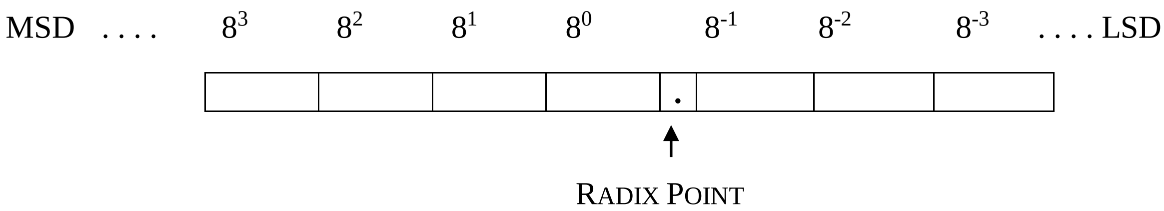


Fig. 4.8: Octal position values as power of 8

Table 4.4: Counting in Octal

Decimal Number	Octal Number	Decimal Number	Octal Number
0	0	11	13
1	1	12	14
2	2	13	15
3	3	14	16
4	4	15	17
5	5	16	20
6	6	17	21
7	7	18	22
8	10	19	23
9	11	20	24
10	12	21	25

➢ Usefulness of the Octal System

- In computer work, binary numbers up to 64 bits are not uncommon. These binary numbers do not always represent a numerical quantity; they often represent some type of code. While dealing with large binary numbers, it is convenient and more efficient for us to write the numbers in octal rather than binary. The ease with which conversions can be made between octal and binary makes the octal system more attractive as a shorthand means of expressing large binary numbers.

4. HEXADECIMAL NUMBER SYSTEM

- Binary numbers are too long. These numbers are fine for machines butt are too lengthy to be handled by human beings. So, there is a need to represent the binary numbers concisely. One number system developed with this objective is the hexadecimal number system (or Hex). Although it is somewhat difficult to interpret than the octal number system, it has become the most popular means of direct data entry and retrieval in digital systems.

- The hexadecimal number system is positional weighted system. The **base or radix is 16** that means, it has 16 independent symbols. The symbols used are **0 to 9 and A to F**. since its base is $16 = 2^4$, every 4 bit binary digit combination can be represented by one hexadecimal digit. So, a hexadecimal number is ¼ th the length of the corresponding binary number.

- A 4-bit group is called a *nibble*. Since computer words come in 8, 16, 32 bits and so on, they can be easily represented in hexadecimal. The hexadecimal number system is particularly used for human communications with computers. It is used in both large and small computers.

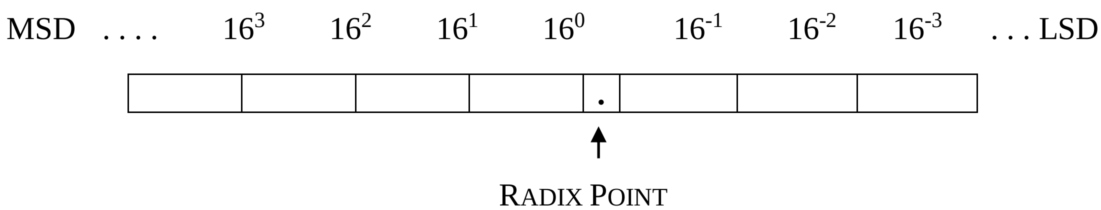


Fig. 4.9: Hexadecimal position values as power of 16

Table 4.5: Counting in Hexadecimal

Decimal Number	Hexadecimal Number	Decimal Number	Hexadecimal Number
0	0	21	15
1	1	22	16
2	2	23	17
3	3	24	18
4	4	25	19
5	5	26	1A
6	6	27	1B
7	7	28	1C
8	8	29	1D
9	9	30	1E
10	A	31	1F
11	B	32	20
12	C	33	21
13	D	34	22
14	E	35	23

15	F	36	24
16	10	37	25
17	11	38	26
18	12	39	27
19	13	40	28
20	14	41	29

4.3.2 Number Base Conversions:

➢ **Definition and Importance**

- The human beings use decimal number system while computer uses binary number system. Therefore, it is essential to convert decimal number into its equivalent binary while feeding number into computer and to convert binary number into its decimal equivalent while displaying result of operation to the human beings.

- However, dealing with a large quantity of binary numbers of many bits is inconvenient for human beings. Therefore, octal and hexadecimal numbers are used as a shorthand means of expressing large binary numbers. Hence inter conversion among different number systems is required.

➢ **Conversions between Decimal, Binary, Octal and Hexadecimal**

- The below table shows the decimal, binary, octal and hexadecimal numbers.

Table 4.6: Conversions between Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C

13	1101	15	D
14	1110	16	E
15	1111	17	F

1. Binary to Decimal Conversion

- Binary numbers can be converted to their decimal equivalents by the positional weights method. In this method, each binary digit of the number is multiplied by its position weight and the product terms are added to obtain the decimal number.

Ex 1: Convert $(10101)_2$ to decimal

Solution:
Positional weights are: 2^4 2^3 2^2 2^1 2^0
$1\ 0\ 1\ 0\ 1 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
$= 16 + 0 + 4 + 0 + 1$
$= 21$
Hence, $(10101)_2 = (21)_{10}$

Ex 2: Convert $(11011.101)_2$ to decimal

Solution:
Positional weights are: 2^4 2^3 2^2 2^1 2^0 . 2^{-1} 2^{-2} 2^{-3}
$1\ 1\ 0\ 1.1\ 0\ 1 = (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (0 \times 2^{-2}) +$
(1×2^{-3})
$= 16 + 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125$
$= 27.625$
Hence, $(11011.101)_2 = (27.625)_{10}$

2. Octal to Decimal Conversion

- To convert an octal number to a decimal number, multiply each digit in the octal number by the weight of its position and add all the product terms.

Ex 3: Convert $(4057.06)_8$ to decimal

Solution:
Positional weights are: 8^3 8^2 8^1 8^0 . 8^{-1} 8^{-2}
$4057.06_8 = (4 \times 8^3) + (0 \times 8^2) + (5 \times 8^1) + (7 \times 8^0) + (0 \times 8^{-1}) + (6 \times 8^{-2})$
$= 2048 + 0 + 40 + 7 + 0 + 0.0937$

= 2095.0937

Hence, $(4057.06)_8 = (2095.0937)_{10}$

3. Hexadecimal to Decimal Conversion

- Multiply each digit in the hex number by its position weight and add all those product terms. In this way, we get the decimal equivalent of the hexadecimal number

Ex 4: Convert $(5C7)_{16}$ to decimal

Solution:

Positional weights are: 16^2 16^1 16^0

$5C7_{16} = (5 \times 16^2) + (12 \times 16^1) + (7 \times 16^0)$

$= 1280 + 192 + 7$

$= 1479_{10}$

Hence, $(5C7)_{16} = (1479)_{10}$

Ex 5: Convert $A0F9.0EB_{16}$ to decimal

Solution:

Positional weights are: 16^3 16^2 16^1 16^0.16^{-1} 16^{-2} 16^{-3}

$A0F9.0EB_{16} = (10 \times 16^3) + (0 \times 16^2) + (15 \times 16^1) + (9 \times 16^0) + (0 \times 16^{-1}) + (14 \times 16^{-2}) + (11 \times 16^{-3})$

$= 40960 + 0 + 240 + 9 + 0 + 0.0546 + 0.0026$

$= 41209.0572_{10}$

Hence, $(A0F9.0EB)_{16} = (41209.0572)_{10}$

4. Decimal to Binary Conversion

- The conversion of decimal number to binary is carried out in 2 steps. In step 1, we have to convert integer part and in step 2, we have to convert fractional part.

- For **integer part** conversion, we use successive division-by-2 method. In this method we repeatedly divide the integer part of the decimal number by 2 until the quotient is zero. The remainder of each division becomes the numeral in the new radix. The remainders are taken in the reverse order to form a new radix number. This means that the first remainder is the LSD and the last remainder is the MSD in the new radix number. Thus the integers read from **bottom to top** give the equivalent binary fraction.

- Similarly, for **fractional part**, we use successive multiplication-by-2 method. In this method, the number to be converted is multiplied by the radix of new number, producing a product that has an integer part and a fractional part. The integer part (carry) of the product becomes a numeral in the new radix number. The fractional part is again multiplied by the radix and this process is repeated until fractional part reaches 0 or until the new radix number is carried out to significant digits. The integer part (carry) of each product is read from **top to bottom** to represent the new radix number.

Ex 6: Convert 52_{10} to binary

- Here the number is integer number so we need to divide the given decimal number by 2 and read the remainders from bottom to top to get the equivalent binary number.

2	52	Remainder
2	26	0
2	13	0
2	6	1
2	3	0
2	1	1
	0	1

Hence, $(52)_{10} = (110100)_2$

Ex 7: Convert $(163.875)_{10}$ to binary

Solution:

Step 1: Separate the integer and fractional parts of the decimal number. Now for integer part, we carry successive division-by-2 method as follows:

2	163	Remainder
2	81	1
2	40	1
2	20	0
2	10	0
2	5	0
2	2	1
2	1	0
	0	1

So, $163_{10} = (10100011)_2$

Step 2: Now the fraction part is 0.875_{10}. Carrying out successive multiplication-by-2 as follows:

0.875 x 2 = 1.75	1	↓
0.75 x 2 = 1.5	1	
0.5 x 2 = 1.0	1	

So, $0.875_{10} = 0.111_2$

Hence. $(163.875)_{10} = (10100011.111)_2$

5. Decimal to Octal Conversion

- Decimal to Octal conversion can be done in similar way as decimal to binary conversion. The integer and fractional parts are to be separated and the same procedure is carried out. But the division and multiplication are carried out by 8 as the base of octal number is 8. Following the same steps, we can get the equivalent octal number of the given decimal number.

Ex 8: Convert 378.93_{10} to octal

Solution:

Step1: Conversion of integer part by successive division-by-8 method.

		Remainder	
8	378		↑
8	47	2	
8	5	7	
	0	5	

So, $(378)_{10} = (572)_8$

Step 2: Conversion of fractional part by successive multiplication-by-8 method.

0.93 x 8 = 7.44	7	↓
0.44 x 8 = 3.52	3	
0.52 x 8 = 4.16	4	
0.16 x 8 = 1.28	1	

So, $0.93_{10} = 0.7341_8$

Hence, $(378.93)_{10} = (572.7341)_8$

6. Decimal to Hexadecimal Conversion

- Decimal to hexadecimal conversion is carried out by 2 steps. In the first step, the integer part of the decimal number is divided by 16 successively and the remainder is noted. The remainders read from bottom to top gives the equivalent hexadecimal integer. In the second step, the successive multiplication of fractional part by 16 is done and the integers are noted down. Reading the integers from bottom to top gives the hexadecimal fraction.

Ex 9: Convert $(3509.75)_{10}$ to hexadecimal

Solution:

Step1: Conversion of integer part by successive division-by-16 method

		Remainder
16	3509	
16	219	5
16	13	11 = B
	0	13 = D

So, $(3509)_{10} = (DB5)_{16}$

Step 2: Conversion of fractional part by successive multiplication-by-16 method.

0.75 x 16 = 12.0 12 = C

So, $(0.75)_{10} = (0.C)_{16}$

Hence, $(3509.75)_{10} = (DB5.C)_{16}$

7. Octal to Binary Conversion

- To convert a given octal number to binary, just replace each octal digit by its 3-bit binary equivalent.

Ex 9: Convert 367.52_8 to binary.

Solution:

Given octal number is	3	6	7	.	5	2
Convert each octal digit to binary	011	110	111	.	101	010

Hence, $(367.52)_8 = (11110111.101010)_2$

8. Hexadecimal to Binary Conversion

- To convert a given hexadecimal number to binary, just replace each hexadecimal digit by its 4-bit binary equivalent.

Ex 10: Convert $4BAC_{16}$ to binary.

Solution:

Given hexadecimal number is	4	B	A	C
Convert each digit to 4-bit binary	0100	1011	1010	1100

Hence, $(4BAC)_{16} = (100101110101100)_2$

Ex 11: Convert $3A9.B0D_{16}$ to binary.

Solution:

Given hexadecimal number is	3	A	9	.	B	0	D
Convert each digit to 4-bit binary	0011	1010	1001	.	1011	0000	110

Hence, $(3A9.B0D)_{16} = (1110101001.101100001101)_2$

9. Binary to Octal Conversion

- To convert a binary number to an octal number, starting from the binary point make groups of 3 bits each, on either side of the binary point and replace each 3-bit binary group by the equivalent octal digit.

Ex 12: Convert 110101.101010_2 to octal.

Solution:

Group of 3 bits are	110	101	.	101	010
Convert each group to octal	6	5	.	5	2

Hence, $(110101.101010)_2 = (65.52)_8$

Ex 13: Convert 10101111001.0111_2 to octal.

Solution:

Group of 3 bits are	10	101	111	001	.	011	1
=	010	101	111	001	.	011	100
Convert each group to octal	2	5	7	1	.	3	4

Hence, $(10101111001.0111)_2 = (2571.34)_8$

10. Binary to Hexadecimal Conversion

- To convert a binary number to an octal number, starting from the binary point make groups of 4 bits each, on either side of the binary point and replace each 4-bit binary group by the equivalent hexadecimal digit.

Ex 14: Convert 1011111011.011111_2 to hexadecimal.

Solution:

Group of 4 bits are	10	1111	1011	.	0111	11
	0010	1111	1011	.	0111	1100
Convert each group to hex	2	F	B	.	7	C

Hence, $(1011111011.011111)_2 = (2FB.7C)_{16}$

11. Octal to Hexadecimal Conversion

- To convert an octal number to hexadecimal, the simplest way is to first convert the given octal number to binary and then the binary number to hexadecimal.

Ex 15: Convert 1245 to hex.

Solution:

Given octal number is	1	2	4	5
Convert each octal digit to binary	001	010	100	110
Group of 4 bits are	0010	1010	0110	
Convert each 4–bit group to hex	2	A	6	

Hence, $(1245)_8 = (2A6)_{16}$

Ex 16: Convert 756.603_8 to hex.

Solution:

Given octal number is	7	5	6	.	6	0	3
Convert each octal digit to binary	111	101	110	.	110	000	011

Group of 4 bits are	0001	1110	1110	.	1100	0001	1000
Convert each 4–bit group to hex	1	E	E	.	C	1	8

Hence, $(756.603)_8 = (1EE.C18)_{16}$

12. Hexadecimal to Octal Conversion

- To convert hexadecimal number to octal, the simplest way is to first convert the given hexadecimal number to binary and then the binary number to octal.

Ex 17: Convert B9F.AE$_{16}$ to octal.

Solution:

Given hex number is	B	9	F	.	A	E		
Convert each hex digit to binary	1011	1001	1111	.	1010	1110		
Group of 3 bits are	101	110	011	111	.	101	011	100
Convert each 3–bit group to octal	5	6	3	7	.	5	3	4

Hence, $(B9F.AE)_{16} = (5637.534)_8$

13. Any radix r number to Decimal Conversion

- We can convert a given number in radix r to decimal by multiplying each digit by its positional weights and taking sum of all the products.

Ex 18: Convert 1221$_3$ to decimal.

Solution:

Here, the given number is in base 3. Its positional weights are: $3^3\ 3^2\ 3^1\ 3^0$

$$\begin{aligned} 1\ 2\ 2\ 1 &= (1 \text{ x } 3^3) + (2 \text{ x } 3^2) + (2 \text{ x } 3^1) + (1 \text{ x } 3^0) \\ &= 27 + 18 + 6 + 1 \\ &= 52 \end{aligned}$$

Hence, $(1221)_3 = (52)_{10}$

Ex 19: Convert 234.02$_5$ to decimal.

Solution:

Here, the given number is in base 5. Its positional weights are: $5^2\ 5^1\ 5^0\ .\ 5^{-1}\ 5^{-2}$

$2\,3\,4\,.\,0\,2 = (2 \times 5^{2}) + (3 \times 5^{1}) + (4 \times 5^{0}) + (0 \times 5^{-1}) + (2 \times 5^{-2})$
$= 50 + 15 + 4 + 0 + 0.08$
$= 69.08$

Hence, $(234.02)_5 = (69.08)_{10}$

14. Decimal to any radix r Conversion

- Decimal number can be converted in any radix by 2 steps. In step1, the integer part of the decimal number is divided successively by the radix r and the remainders are noted down. Taking the remainders from bottom to top gives the radix r equivalent of the integer part. Similarly, the fractional part is successively multiplied by the radix r and the integer part of the result is noted down. Noting the carry from top to bottom gives the fractional part equivalent in radix r.

Ex 20: Convert 1989.35_{10} to base 12.

Solution:

Step 1: Separate the integer and fractional parts of the decimal number. Now for integer part, we carry successive division-by-12 method as follows:

12	1989	Remainder
12	165	9
12	13	9
12	1	1
	0	1

So, $(1989)_{10} = (1199)_{12}$

Step 2: Now the fraction part is 0.35_{10}. Carrying out successive multiplication-by-12 as follows:

$0.35 \times 12 = 4.2$	4
$0.2 \times 12 = 2.4$	2
$0.4 \times 12 = 4.8$	4

So, $0.35_{10} = 0.424_{12}$

Hence. $(1989.35)_{10} = (1199.424)_{12}$

15. Find the value of unknown base

Ex 21: Determine b if $(33)_{10} = (201)_b$

Solution:

We have, $(33)_{10} = (201)_b$

$$33 = 2 \times b^2 + 0 \times b^1 + 1 \times b^0$$
$$= 2b^2 + 1$$
$$2b^2 = 32$$
$$b^2 = 16$$
$$b = \pm 4$$

But base of any number cannot be negative. Hence value of b = 4.

Ex 22: Determine b if $(193)_b = (623)_8$

Solution:

We have, $(193)_b = (623)_8$

$1 \times b^2 + 9 \times b^1 + 3 \times b^0 = 6 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$

$b^2 + 9b + 3 = 384 + 16 + 3$

$b^2 + 9b + 3 = 403$

$b^2 + 9b - 400 = 0$

$b = 16, b = -25$

As base is always positive, value of b=16.

16. Binary Coded Decimal (BCD)

- It is a numeric code that is used to represent decimal using binary bits i.e. 1's and 0's. It is different from representation of a decimal number in binary system i.e. base 2 system.
- In BCD representation each digit of a decimal number is represented by a group of four bits. These bits are given with weights of 8-4-2-1 and hence many a times BCD code is also called 8421 code. Code for each digit of decimal is as follows.

Table 4.7: Conversions between Decimal to BCD code

Decimal Digit	BCD code 8421
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101

6	0110
7	0111
8	1000
9	1001

Ex 23: $(58)_{10}$ = ($)_{BCD}$

Solution:

Decimal: 5 → 0101, 8 → 1000

0101 1000

Thus $(58)_{10} = (01011000)_{BCD}$.

- It should be noted that binary representation of $(58)_{10}$ will be $(111010)_2$ which is quite different from the BCD representation.

Ex 23: $(001001011001)_{BCD} = ()_{10}$

Solution:

BCD: 0010 0101 1001

0010 → 2, 0101 → 5, 1001 → 9

2 5 9

$(001001011001)_{BCD} = (259)_{10}$

- It can be observed that BCD codes are less efficient for representation compared to binary as it requires more number of bits then required in binary representation. However, it is popular because of its ease of conversion to and from decimal.

4.4 LOGIC GATES:

- Logic gates are the fundamental building blocks of digital systems. They are the physical devices that performs the basic Boolean operations of AND, OR and NOT.
- Input and outputs of logic gates (that is basically a voltage signal) can occur only in two levels. These two levels are termed as High and Low or True and False or ON and OFF or simply 1 and 0. In representation of higher of the two voltage levels is symbolized as 1 and lower symbolized as 0 the gate is said to be positive logic gate. However, if higher of the two voltage levels is symbolized as 0 and lower as 1 then it is said to be negative logic gate.
- Input output behavior of a gate is generally represented using truth table. It is a table that lists output for all possible combinations of inputs.
- There are total seven logic gates in which three are **basic logic gates** (AND, OR, NOT) and two are **universal logic gates** (NAND, NOR).

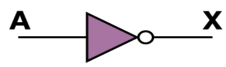
➢ Various basic gates are discussed as follows;

1. ***NOT Gate:***

- NOT gate has one inputs and one output. The output becomes logic 1 when input is at logic 0 and output becomes logic 0 when the input is at logic 1. Thus it inverts or complements the logic available at input and hence called and **inverter or complement.** It is represented by a bar over the variable “ ¯ ”or with a symbol “ ’ ”. Thus, for example, X = A′ or X = $\overline{A}$ read as “X is equal to Not A or A bar or A complement”. NOT gate and its truth table are shown in fig. 4.10.

Boolean Expression **Logic Diagram Symbol** **Truth Table**

X = A'



A	X
0	1
1	0

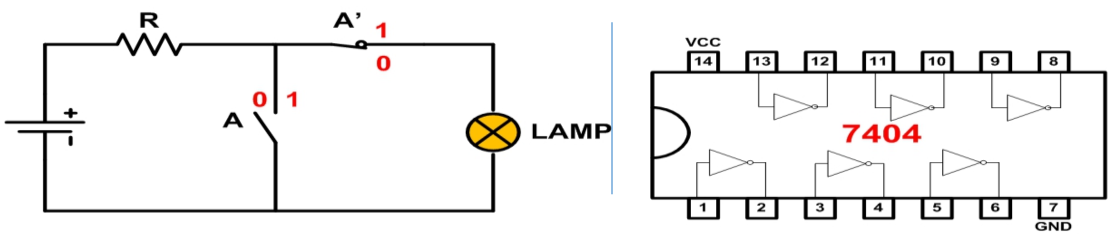


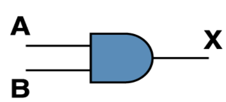
Fig. 4.10: Illustration of NOT gate

2. ***AND Gate:***

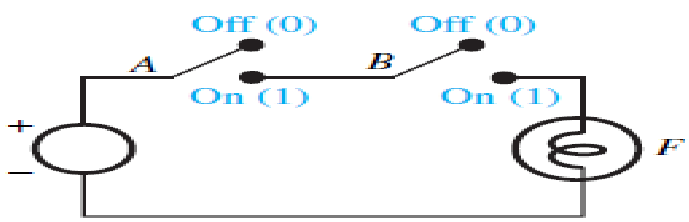
- AND gate means all or nothing logic
- AND gate has two or more inputs and one output. The output becomes logic 1 only when each one of its input is at logic 1. For all other input combinations it gives output logic 0. It is represented by a symbol •. Thus, for example, X = A · B (also written simply as X = AB) is read as “X is equal to A AND B”. Two input AND gate and its truth table is shown in fig. 4.11.

Boolean Expression **Logic Diagram Symbol** **Truth Table**

X = A · B



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1



$F = 0$ (light is off)
$F = 1$ (light is on) if and only if $A = B = 1$

Fig. 4.11: Illustration of AND gate

- Operation of AND gate can be understood through the example of two switches connected in series as shown in fig. 4.11. Here we assume switches A and B to present logic 1 when in ON condition and logic 0 in OFF condition. Similarly, if lamp is ON we assume logic 1 and in OFF condition we assume it as logic 0. Then it can be determined that the lamp will be ON (at logic 1) only when both the switches A and B are ON (at logic 1).

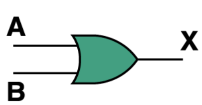
3. OR Gate:

- OR gate means any or all logic
- OR gate has two or more inputs and one output. The output becomes logic 1 when at least (minimum) one of the inputs is at logic 1. It is represented by a symbol +. Thus, for example, X = A + B is read as "X is equal to A OR B". Two input AND gate and its truth table is shown in fig. 4.12.

Boolean Expression

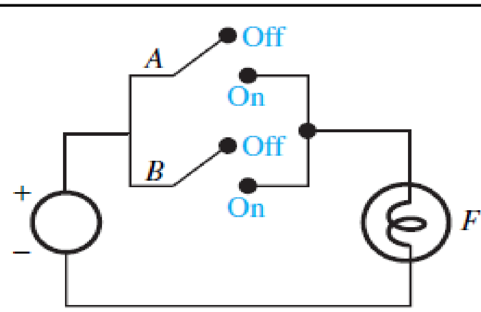
$X = A + B$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1



$F = 1$ (light is on) if either A or B (or both) is equal to 1 (switch on)

Fig. 4.12: Illustration of AND gate

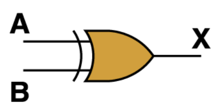
4. *Exclusive OR Gate (EX-OR):*

- It also means Inequality detector because it gives output high when both inputs are different.
- Exclusive OR gate give output equal to 1 when the two inputs are exclusively different. This is the reason why it is also known as inequality gate. The schematic symbol and truth table of the gate is shown in fig. 4.13. It is represented by a symbol $\oplus$. Thus, for example, $X = A \oplus B$ is read as "X is equal to A XOR B." The logic expression this gate in terms of AND, OR and NOT operation is $X = A \oplus B = \overline{A}B + A\overline{B}$.

Boolean Expression

$X = A \oplus B$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

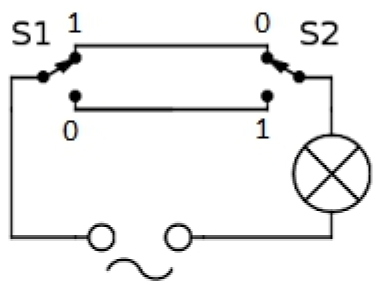


Fig. 4.13: Illustration of EX-OR gate

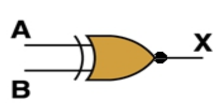
5. *Exclusive NOR Gate (EX-NOR):*

- It also means equality detector because it gives output high when both inputs are same.
- Exclusive NOR gate is XOR gate followed by inverter. Thus it is complement of XOR gate. This is the reason why it is also known as equality gate. The logic symbol, logic expression, chematic symbol, truth table of the gate is shown in fig. 4.14.
- **X = AB + A'B'**

Boolean Expression

$X = A \odot B$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

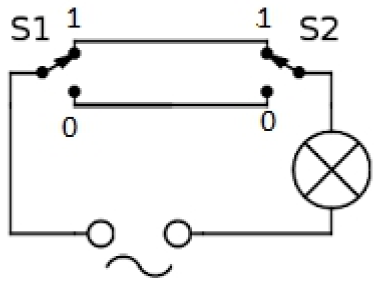


Fig. 4.14: Illustration of EX-NOR gate

➢ ***Universal Gates:***

- NAND and NOR gates are known as a universal gates because from this two gates all other gates can be constructed.

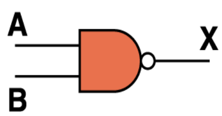
6. NAND Gate:

- NAND gate represents combination of AND gate followed by NOT gate. It represents complement of AND operation. Schematic symbol of NAND gate and its truth table are shown in fig. 4.15. The logic expression is given as $X = \overline{(A \cdot B)}$ or X = (A·B)’.

Boolean Expression

X = (A · B)'

Logic Diagram Symbol



Truth Table

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

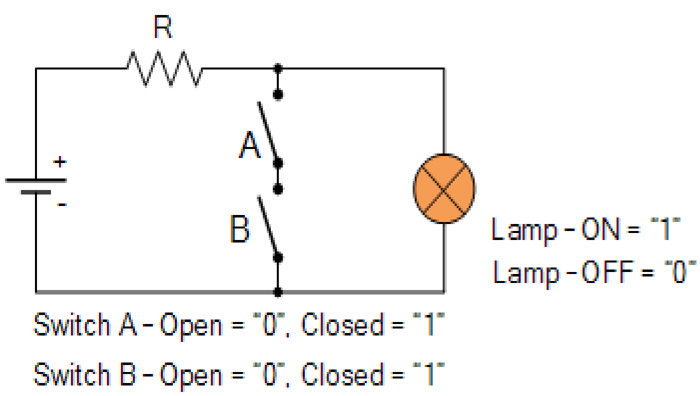
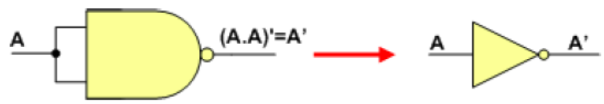


Fig. 4.15: Illustration of NAND gate

- **NAND gate as Universal gate**

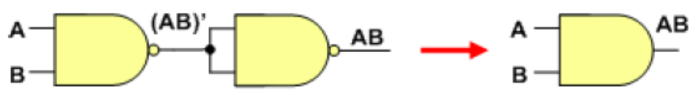
1. Implementing NOT gate

- All NAND input pins connect to the input signal A gives an output A'.



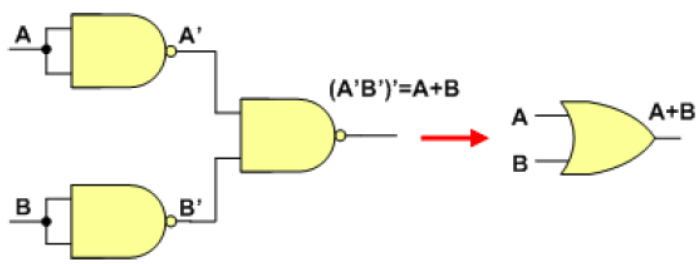
2. Implementing AND gate

- The AND is replaced by a NAND gate with its output complemented by a NAND gate inverter.



3. Implementing OR gate

- The OR gate is replaced by a NAND gate with all its inputs complemented by NAND gate inverters.

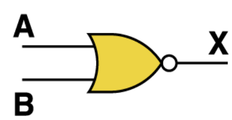


7. NOR Gate:

- NOR gate represents combination of OR gate followed by NOT gate. It represents complement of OR operation. Schematic symbol of NOR gate and its truth table are shown in fig. 4.16. The logic expression is given as $X = \overline{(A + B)}$ or $X = (A+B)'$.

Boolean Expression **Logic Diagram Symbol** **Truth Table**

X = (A + B)'



A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

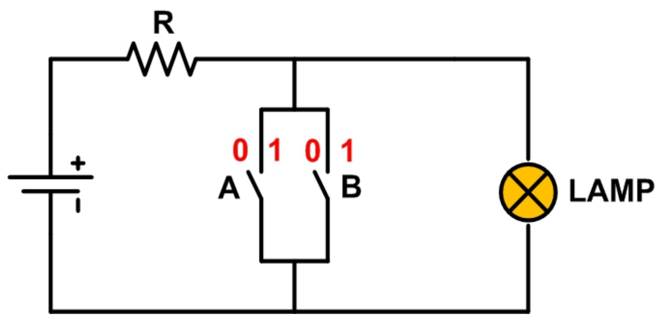
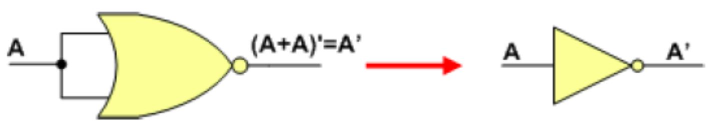


Fig. 4.16: Illustration of NOR gate

- **NOR gate as Universal gate.**

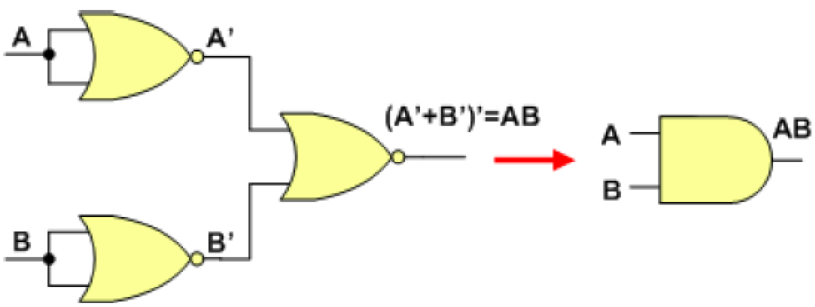
1. Implementing NOT gate

- All NOR input pins connect to the input signal A gives an output A`.



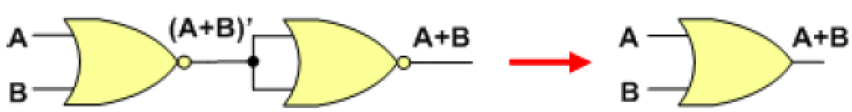
2. Implementing AND gate

- The AND gate is replaced by a NOR gate with all its inputs complemented by NOR gate inverters.



3. Implementing OR gate

- The OR is replaced by a NOR gate with its output complemented by a NOR gate inverter.



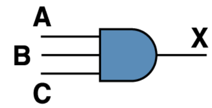
- **Gates with More Inputs**

- Any logic gates can be designed to accept three or more input values.
- As an example, three-input AND gate produces an output of 1 only if all input values are 1.

Boolean Expression

$X = A \cdot B \cdot C$

Logic Diagram Symbol



Truth Table

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Fig. 4.17: Illustration of gates with more inputs

➢ Constructing Gates

- A transistor is a device that acts, depending on the voltage level of an input signal, either as a wire that conducts electricity or as a resistor that blocks the flow of electricity
- A transistor has no moving parts, yet acts like a switch.
- It is made of a semiconductor material, which is neither a particularly good conductor of electricity, such as copper, nor a particularly good insulator, such as rubber.
- A transistor is shown in fig. 4.18.

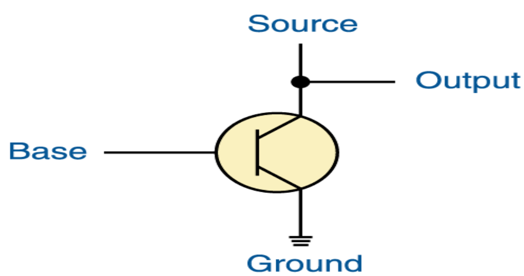


Fig. 4.18: Transistor

- A transistor has three terminals
  1. A source
  2. A base
  3. An emitter, typically connected to a ground wire
- If the electrical signal is grounded, it is allowed to flow through an alternative route to the ground (literally) where it can do no harm.
- It turns out that, because the way a transistor works, the easiest gates to create are the NOT, NAND, and NOR gates shown in fig. 4.19.

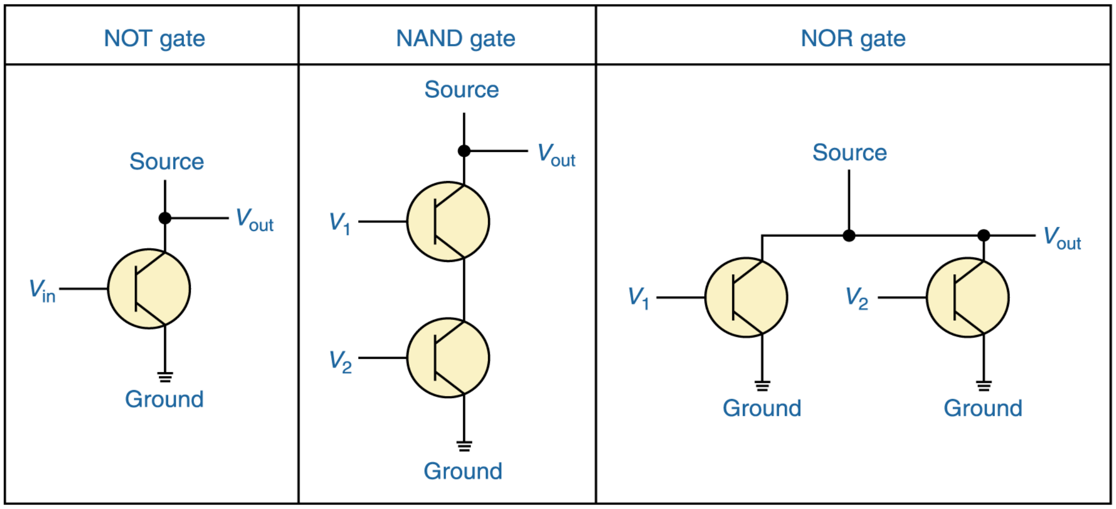


Fig. 4.19: Logic gates using transistor

4.5 BOOLEAN ALGEBRA:

- Inventor of Boolean algebra was George Boole (1815 - 1864).

- Designing of any digital system there are three main objectives;
  1. Build a system which operates within given specifications
  2. Build a reliable system
  3. Minimize resources

- Boolean algebra is a system of mathematical logic.
- Any complex logic can be expressed by Boolean function.
- Boolean algebra is governed by certain rules and laws.
- Boolean algebra is different from ordinary algebra & binary number system.
  In ordinary algebra; A + A = 2A and AA = A^2, here A is numeric value.
- In Boolean algebra;
  A + A = A and AA = A, here A has logical significance, but no numeric significance.

Binary no. system	Ordinary no. system	Boolean algebra
1 + 1 = 10	1 + 1 = 2	1 + 1 = 1

- In Boolean algebra there is nothing like subtracting or division, no negative or fractional numbers.
- Boolean algebra represent logical operation only. Logical multiplication is same as AND operation and logical addition is same as OR operation.
- Boolean algebra has only two values 0 & 1.
- In Boolean algebra;

If A = 0 then A ≠ 1.
If A = 1 then A ≠ 0.

➢ **Let's introduce Boolean algebra by considering a practical problem**

- Suppose a system which transmitting 2 bit binary information over 2 line to another system.
- So with 2 line, 4 unique code could be represented.
- The receiving system may need to identify the presence of certain transmitted codes.
- As an example suppose system is to identify the occurrence of the code representing the decimal number 1 and 2. Each one of these code appear on the 2 lines, a circuit is to generate an output 1.
- When any other code present the output should be 0.
- The method of implementing this system is shown in table 4.8.

Table 4.8: Binary to Decimal conversion

Binary A B	**Decimal**
0 0	0
0 1	1
1 0	2
1 1	3

- We want output 1 if decimal 1 & 2 otherwise 0 as defined above.
- Below circuit generate output 1 when A = 0 & B = 1 or A = 1 & B = 0.

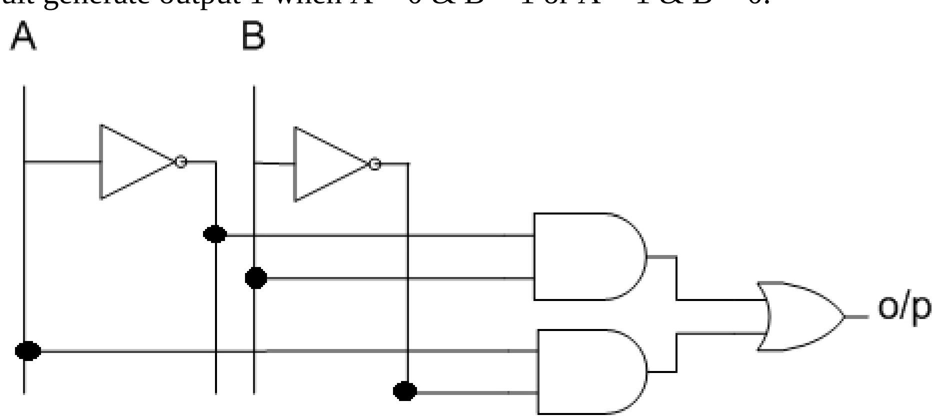
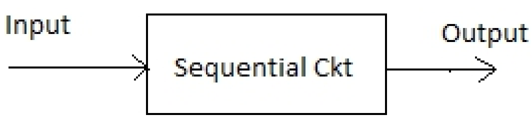
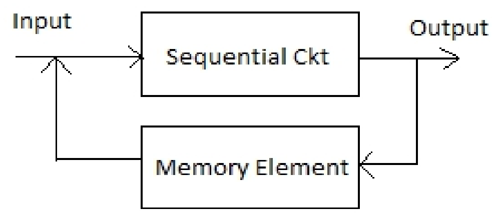


Fig. 4.20

➢ **Types of logic circuit**

- There are two types of logic circuit;
  1. Sequential circuit
  2. Combinational circuit

- ***Difference between combinational & sequential circuit:***

Combinational Circuit	Sequential Circuit
Output is dependent only on the input at the same instant of time	Output is dependent on present input and past output.
It does not contains memory elements.  	It contains memory element. 
Its behavior is described by the set of output function.	Its behavior is described by the set of next state function and the set of output function.
No feedback is available.	Feedback is available.
It does not contains periodic clock signal.	It contains clock signals.
Faster than sequential circuit.	Slower than combinational circuit.
e.g. half adder, full adder, etc.	e.g. Flip flop, counter, etc.

- **Advantages of Boolean Algebra**

1. Minimize the no. of gates used in circuit.
2. Decrease the cost of circuit.
3. Minimize the resources.
4. Less fabrication area is required to design a circuit.
5. Minimize the designer's time.
6. Reducing to a simple form. Simpler the expression more simple will be hardware.
7. Reduce the complexity.

- **Axioms of Boolean Algebra**

- Axioms or postulate of Boolean algebra are a set of logical expression that we accept without proof & upon which we can build a set of useful theorems.

Axioms 1: $0 \cdot 0 = 0$
Axioms 2: $0 \cdot 1 = 0$
Axioms 3: $1 \cdot 0 = 0$
Axioms 4: $1 \cdot 1 = 1$
Axioms 5: $0 + 0 = 0$
Axioms 6: $0 + 1 = 1$
Axioms 7: $1 + 0 = 1$

Axioms 8: $1 \cdot 1 = 1$
Axioms 9: $1' = 0$
Axioms 10: $0' = 1$

4.5.1 Laws of Boolean Algebra

1. Complementation Laws:

- The term complement simply means to invert, i.e. to change 0’s to 1’s and 1’s to 0’s.
  Law 1: $0' = 1$
  Law 2: $1' = 0$
  Law 3: If $A = 0$ then $A' = 1$
  Law 4: If $A = 1$ then $A' = 0$
  Law 5: $A'' = A$

2. AND Laws:

Law 1: $A \cdot 0 = 0$
Law 2: $A \cdot 1 = A$
Law 3: $A \cdot A = A$
Law 4: $A \cdot A' = 0$

3. OR Laws:

Law 1: $A + 0 = A$
Law 2: $A + 1 = 1$
Law 3: $A + A = A$
Law 4: $A + A' = 1$

4. Commutative Laws:

- Commutative laws allow change in position of AND or OR variables.

Law 1: $A + B = B + A$
Proof:

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	1

=

B	A	B + A
0	0	0
0	1	1
1	0	1
1	1	1

Fig. 4.21

- This law can be extended to any numbers of variables for e.g.
  A + B + C = B + A + C = C + B + A = C + A + B

  Law 2: A · B = B · A
  Proof:

A	B	A · B
0	0	0
0	1	0
1	0	0
1	1	1

=

B	A	B · A
0	0	0
0	1	0
1	0	0
1	1	1

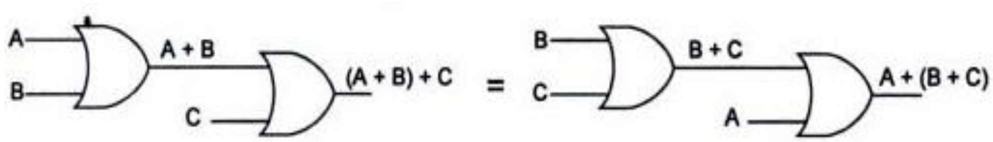
Fig. 4.22

- This law can be extended to any numbers of variables for e.g.
  A · B · C = B · A · C = C · B · A = C · A · B

5. Associative Laws:

- The associative laws allow grouping of variables.

  Law 1: (A + B) + C = A + (B + C)
  Proof:



A	B	C	A + B	(A + B) + C
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

=

A	B	C	B + C	A + (B + C)
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	1
1	0	0	0	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Fig. 4.23

- This law can be extended to any no. of variables for e.g.
  A + (B + C + D) = (A + B + C) + D = (A + B) + (C + D)

Law 2: (A · B) · C = A · (B · C)
Proof:

A	B	C	AB	(AB)C
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

=

A	B	C	BC	A(BC)
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

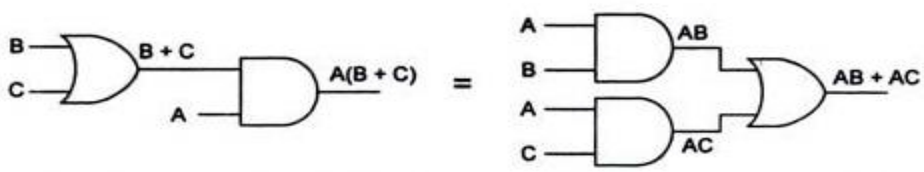
Fig. 4.24

- This law can be extended to any no. of variables for e.g.
  A · (B · C · D) = (A · B · C) · D = (A · B) · (C · D)

6. Distributive Laws:

- The distributive laws allow factoring or multiplying out of expressions.

Law 1: A (B + C) = AB + AC
Proof:



A	B	C	B + C	A(B + C)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

=

A	B	C	AB	AC	AB + AC
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

Fig. 4.25

Law 2: A + BC = (A + B) (A + C)
Proof: R.H.S. = (A + B) (A + C)
= AA + AC + BA + BC
= A + AC + BA + BC
= A + BC   (B'cz 1 + C + B = 1 + B = 1)
= L.H.S.

Law 3: $A + A'B = A + B$
Proof: L.H.S. $= A + A'B$
$= (A + A')(A + B)$
$= A + B$
$=$ R.H.S.

7. Idempotence Laws:

- Idempotence means the same value.

Law 1: $A \cdot A = A$
Proof:
Case 1: If $A = 0 \rightarrow A \cdot A = 0 \cdot 0 = 0 = A$
Case 2: If $A = 1 \rightarrow A \cdot A = 1 \cdot 1 = 1 = A$

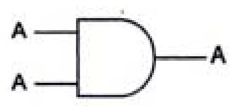


Fig. 4.26

Law 2: $A + A = A$
Proof:
Case 1: If $A = 0 \rightarrow A + A = 0 + 0 = 0 = A$
Case 2: If $A = 1 \rightarrow A + A = 1 + 1 = 1 = A$

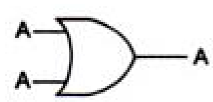


Fig. 4.27

8. Complementation Law / Negation Law:

Law 1: $A \cdot A' = 0$
Proof:
Case 1: If $A = 0 \rightarrow A' = 1$ So, $A \cdot A' = 0 \cdot 1 = 0$
Case 2: If $A = 1 \rightarrow A' = 0$ So, $A \cdot A' = 1 \cdot 0 = 0$

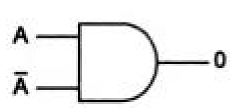


Fig. 4.28

Law 2: $A + A' = 1$
Proof:
Case 1: If $A = 0 \rightarrow A' = 1$ So, $A + A' = 0 + 1 = 1$
Case 2: If $A = 1 \rightarrow A' = 0$ So, $A + A' = 1 + 0 = 1$

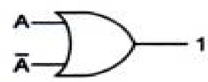


Fig. 4.29

9. Double Negation Law:

- This law states that double negation of a variables is equal to the variable itself.
  Law 1: A'' = A
  Proof:
  Case 1: If A = 0 → A'' = 0'' = 1' = A
  Case 2: If A = 1 → A'' = 1'' = 0' = A

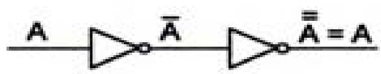


Fig. 4.30

- Any odd no. of inversion is equivalent to single inversion.
- Any even no. of inversion is equivalent to no inversion at all.

10. Identity Law:

Law 1: A · 1 = A
Proof:
Case 1: If A= 1 → A · 1 = 1 · 1 = 1 = A
Case 2: If A= 0 → A · 0 = 0 · 0 = 0 = A

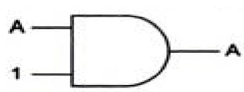


Fig. 4.31

Law 2: A + 1 = 1
Proof:
Case 1: If A= 1 → A + 1 = 1 + 1 = 1 = A
Case 2: If A= 0 → A + 0 = 0 + 0 = 0 = A

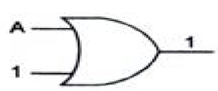


Fig. 4.32

11. Null Law:

Law 1: A · 0 = 0
Proof:
Case 1: If A= 1 → A · 0 = 1 · 0 = 0 = 0
Case 2: If A= 0 → A · 0 = 0 · 0 = 0 = 0

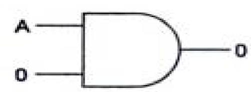


Fig. 4.33

Law 2: A + 0 = A
Proof:
Case 1: If A= 1 → A + 0 = 1 + 0 = 1 = A
Case 2: If A= 0 → A + 0 = 0 + 0 = 0 = A

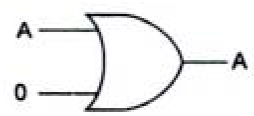


Fig. 4.34

12. Absorption Law:

Law 1: A + AB = A
Proof: L.H.S. = A + AB
= A (1 + B)
= A (1)
= A
R.H.S.

Law 2: A (A + B) = A
Proof: L.H.S. = A (A + B)
= A · A + AB
= A + AB
= A (1 + B)
= A (1)
= A
= L.H.S.

13. Consensus Theorem:

Theorem 1: A · B + A’C + BC = AB + A’C
Proof: L.H.S. = AB + A’C + BC
= AB + A’C + BC (A +A’)
= AB + A’C + BCA + BCA’
= AB (1 + C) + A’C (1 + B)
= AB + A’C
= R.H.S.

- This theorem can be extended as,
AB + A’C + BCD = AB + A’C

Theorem 2: (A + B) (A’ + C) (B + C) = (A + B) (A’ + C)
Proof: L.H.S. = (A + B) (A’ + C) (B + C)
= (AA’ + AC + A’B + BC) (B + C)
= (0 + AC + A’B + BC) (B + C)
= ACB +ACC +A’BB + A’BC + BCB + BCC
= ABC + AC + A’B + A’BC + BC + BC
= ABC + AC + A’B + A’BC + BC
= AC (1 + B) + A’B (1 + C) + BC
= AC + A’B + BC …………………………(1)

R.H.S. = (A + B) (A’ + C)
= AA’ + AC + BA’ + BC
= 0 + AC + BA’ + BC
= AC + A’B + BC …………………………(2)
Equation (1) = Equation (2)
So. L.H.S = R.H.S.

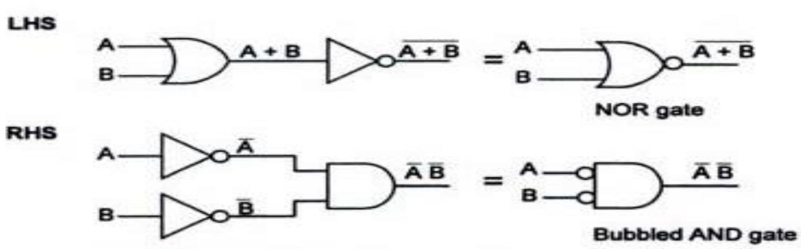
- This theorem can be extended to any no. of variables.
  (A + B) (A’ + C) (B + C + D) = (A + B) (A’ + C)

14. Transposition theorem:

Theorem: AB + A’C = (A + C) (A’ +B)
Proof: R.H.S. = (A + C) (A’ +B)
= AA’ + AB + CA’ + CB
= 0 + AB + CA’ + CB
= AB + CA’ + CB
= AB + A’C (B’cz of AB + A’C + BC = AB + A’C)
=L.H.S.

15. De Morgan’s Theorem:

Law 1: (A + B)’ = A’ · B’
Proof:



A	B	A + B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

=

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}\cdot\bar{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Fig. 4.35

Law 2: (A· B)’ = A’ + B’
Proof:

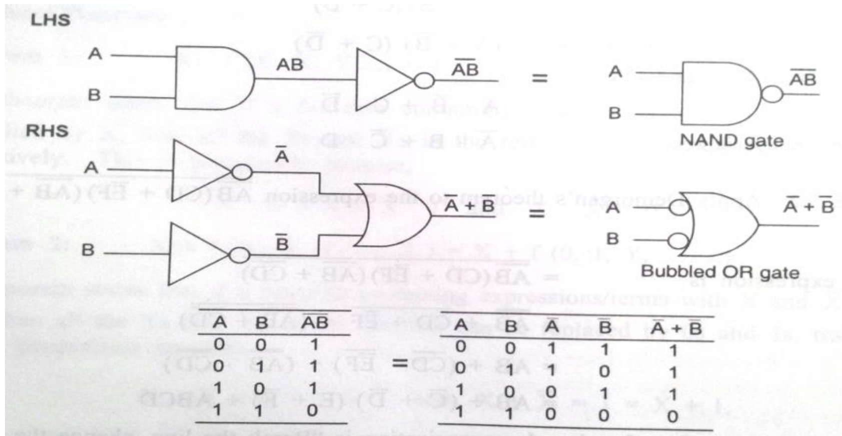


Fig. 4.36

16. Duality Theorem:

- Duality theorem arises as a result of presence of two logic system i.e. positive & negative logic system.
- This theorem helps to convert from one logic system to another.
- From changing one logic system to another following steps are taken:
  1. 0 becomes 1, 1 becomes 0.
  2. AND becomes OR, OR becomes AND.
  3. ‘+’ becomes ‘·’, ‘·’ becomes ‘+’.
  4. Variables are not complemented in the process.

Given Expression	*Dual*
1. $\overline{0} = 1$	$\overline{1} = 0$
2. $0 \cdot 1 = 0$	$1 + 0 = 1$
3. $0 \cdot 0 = 0$	$1 + 1 = 1$
4. $1 \cdot 1 = 1$	$0 + 0 = 0$
5. $A \cdot 0 = 0$	$A + 1 = 1$
6. $A \cdot 1 = A$	$A + 0 = A$
7. $A \cdot A = A$	$A + A = A$
8. $A \cdot \overline{A} = 0$	$A + \overline{A} = 1$
9. $A \cdot B = B \cdot A$	$A + B = B + A$
10. $A \cdot (B \cdot C) = (A \cdot B) \cdot C$	$A + (B + C) = (A + B) + C$
11. $A \cdot (B + C) = AB + AC$	$A + BC = (A + B)(A + C)$
12. $A(A + B) = A$	$A + AB = A$
13. $A \cdot (A \cdot B) = A \cdot B$	$A + A + B = A + B$
14. $\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\ \overline{B}$

4.5.2 Reduction of Boolean Expressions

- ***Demorganize the following functions:***

Ex 24: [(A + B’) (C + D’)]’

Solution:

[(A + B’) (C + D’)]’
= (A + B’)’ + (C + D’)’
= A’ B’’ + C’D’’
= A’B + C’D

Ex 25: [(AB)’ (CD + E’F) ((AB)’ + (CD)’)]’

Solution:

[(AB)’ (CD + E’F) ((AB)’ + (CD)’)]’
= (AB)’’ + (CD + E’F)’ + ((AB)’ + (CD)’)’
= AB + [(CD)’ (E’F)’] + [(AB)’’ (CD)’’]
= AB + (C’ + D’) (E + F’) + ABCD

Ex 26: [(AB)’ + A’ + AB]’

Solution:

[(AB)’ + A’ + AB]’
= AB’’ · A’’ · AB’
= ABA (A’ + B’)
= AB (A’ + B’)
= ABA’ + ABB’
= 0

Ex 27: [P (Q + R)]’

Solution:

[P (Q + R)]’
= P’ + (Q + R)’
= P’ + Q’ R’

Ex 28: [(P + Q’) (R’ + S)]’

Solution:

[(P + Q’) (R’ + S)]’
= (P + Q’)’ + (R’ + S)’
= P’ Q’’ + R’’ S’
= P’Q + RS’

Ex 29: [[(A + B)’ (C + D)’]’ [(E + F)’ (G + H)’]’]’

Solution:

$$
\begin{aligned}
&[[(A + B)' (C + D)']' [(E + F)' (G + H)']']' \\
&= [(A + B)' (C + D)']'' + [(E + F)' (G + H)']'' \\
&= [(A + B)' (C + D)'] + [(E + F)' (G + H)'] \\
&= A'B'C'D' + E'F'G'H'
\end{aligned}
$$

- ***Reducing the following functions:***

Ex 30: A [B + C’ (AB + AC’)’]

Solution:

$$
\begin{aligned}
&A [B + C' (AB + AC')'] \\
&= A [B + C' (AB)' (AC')'] \\
&= A [B + C' (A' + B') (A' + C)] \\
&= A [B + (A' C' + B' C') (A' + C)] \\
&= A [B + (A' C'A' + B' C'A') (A' C' C + B' C' C)] \\
&= A [B + (A'C' + B' C'A') (0 + 0)] \\
&= A [B + A'C' (1 + B')] \\
&= A [B + A'C'] \\
&= AB + A'AC' \\
&= AB + 0 \\
&= AB
\end{aligned}
$$

Ex 31: A + B [AC + (B + C’)D]

Solution:

$$
\begin{aligned}
&A + B [AC + (B + C')D] \\
&= A + B [AC + (BD + C'D)] \\
&= A + ABC + BBD + BC'D \\
&= A + ABC + BD + BC'D \\
&= A (1 + BC) + BD (1 + C') \\
&= A (1) + BD (1) \\
&= A + BD
\end{aligned}
$$

Ex 32: (A + (BC)’)’ (AB’ + ABC)

Solution:

$$
\begin{aligned}
&(A + (BC)')' (AB' + ABC) \\
&= (A' (BC)'') (AB' + ABC) \\
&= (A'BC) (AB' + ABC) \\
&= A'BCAB' + A'BCABC \\
&= 0 + 0 \\
&= 0
\end{aligned}
$$

Ex 33: (B + BC) (B + B'C) (B + D)

Solution:

(B + BC) (B + B'C) (B + D)
= (BB + BB'C + BBC + BCB'C) (B + D)
= (B + 0 + BC + 0) (B + D)
= B (1 + C) (B + D)
= B (B + D)
= BB + BD
= B + BD
= B (1 + D)
= B

Ex 34: AB + AB'C +BC'

Solution:

AB + AB'C +BC'
= A (B + B'C) + BC'
= A (B + B') (B + C) + BC'
= A (1) (B + C) + BC'
= AB + AC + BC'
= CA + C'B + AB
= CA + C'B (B'cz of Consensus theorem 1)

Ex 35: AB'C + B + BD' + ABD' + A'C

Solution:

AB'C + B + BD' + ABD' + A'C
= AB'C + B (1 + D' + AD') + A'C
= AB'C + B + A'C
= C (A' + AB') + B
= C (A' + A) (A' + B') + B
= C (1) (A' + B') + B
= C (A' + B') + B
= A'C + CB' + B
= A'C + (C + B) (B' + B)
= A'C + (B + C) (1)
= A'C + B + C
= C (1 + A') + B
= B + C

Ex 36: A'B' + A'B

Solution:

A'B' + A'B = A' (B' + B) = A'

Ex 37: A'B' + AB'

Solution:

A'B' + AB'
= B' (A' + A)
= B'

Ex 38: A'B + AB

Solution:

A'B + AB
= B (A' + A)
= B (1)
= B

Ex 39: A'B' + A'B + AB' + AB

Solution:

A'B' + A'B + AB' + AB
= A' (B' + B) + A (B' + B)
= A' + A
= 1

Ex 40: [(A + B) (A' + B)] + [(A + B) (A + B')]

Solution:

[(A + B) (A' + B)] + [(A + B) (A + B')]
= [AA' + AB + BA' + BB] + [AA + AB' + BA + BB"]
= [0 + AB + A'B + B] + [A + AB' + AB + 0]
= [B (A + A' + 1)] + [A (1 + B' + B)]
= B + A
= A + B

Ex 41: [(A + B') (A' + B')] + [(A' + B') (A' + B')]

Solution:

[(A + B') (A' + B')] + [(A' + B') (A' + B')]
= [AA' + AB' + B'A' + B'B'] + [A'A' + A'B' + B'A' + B'B']
= [0 + AB' + A'B' + B'] + [A' + A'B' + B']
= [B' (A + A' + 1)] + [A' + B'(1)]
= B' + A' + B'
= A' + B'

Ex 42: (A + B) (A + B’) (A’ + B) (A’ + B’)

Solution:

(A + B) (A + B’) (A’ + B) (A’ + B’)
= (AA + AB’ + BA + BB’) (A’A’ + A’B’ + BA’ + BB’)
= [A (1+ B’ + B)] [A’ (1 + B’ + B)]
= AA’
= 0

- ***Examples of Logic Circuits and Boolean Expressions***

Ex 43: For the logic circuit shown fig. 4.37, find the Boolean expression and the truth table. Identify the gate that given circuit realizes.

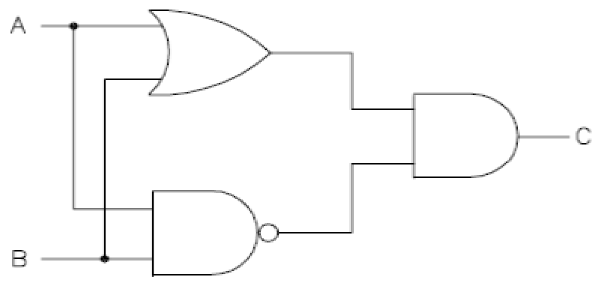


Fig. 4.37

Solution:
Here,
Output of OR gate will be (A+B)
Output of NAND gate will be (AB)’
Hence the final out C will be AND of these two outputs
Hence

C = (A+B) · (A·B)’

Truth table for the same can be given in table 4.9.

Table 4.9

Input					Output
A	B	A+B	A·B	(A·B)’	(A+B) · (A·B)’
0	0	0	0	1	0
0	1	1	0	1	1
1	0	1	0	1	1
1	1	1	1	0	0

From the truth table it is clear that the circuit realizes Ex-OR gate.

Ex 44: For the logic circuit shown fig. 4.38, find the Boolean expression.

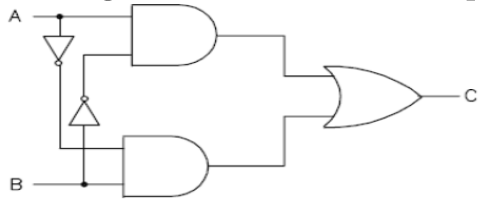


Fig. 4.38

Solution:

Here,
Output of top AND gate will be AB'
Output of bottom AND gate will be A'B
Hence the final out C will be OR of these two outputs
Hence

C = AB'+A'B

Ex 45: For the logic circuit shown fig. 4.39, find the Boolean expression and the truth table. Identify the gate that given circuit realizes.

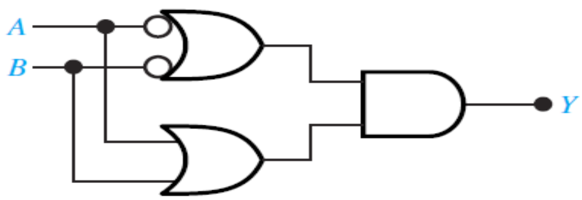


Fig. 4.39

Solution:

Here, bubble indicates inversion.
Hence input of top OR gate is A' and B' and hence its output will be A'+B'
Output of bottom OR gate will be A+B
Hence the final output Y will be AND of these two outputs.
Hence,

Y = (A'+B')(A+B).

Truth table for the same can be given in table 4.10.

Table 4.10

Input						Output
A	B	A'	B'	A'+B'	A+B	(A'+B') (A+B)
0	0	1	1	1	0	0
0	1	1	0	1	1	1
1	0	0	1	1	1	1
1	1	0	0	0	1	0

Note: that circuit in example 44 is same as circuit in this example. This is because NAND gate is equivalent to Bubbled OR gate.

Ex 46: For the given Boolean expression draw the logic circuit.

F = X + (Y' + Z)

Solution:

The expression primarily involves three operators i.e NOT, AND and OR
To generate Y' a NOT gate is required.
To generate Y'Z an AND gate is required.
To generate final output OR gate is required.
The circuit can be drawn as shown in fig. 4.40.

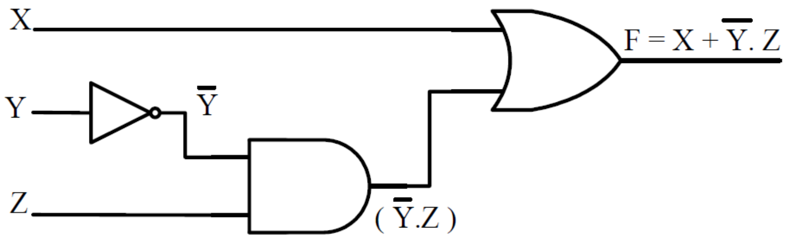


Fig. 4.40

4.6 DIFFERENT FORMS OF BOOLEAN ALGEBRA:

- There are two types of boolean form
  1. Standard form
  2. Canonical form

1. Standard Forms

- In this configuration, the terms that form the function may contain one, two, or any number of literals.
- There are two types of standard forms: (i) sum of product (SOP) (ii) product of sum (POS).

➢ **Sum of Product (SOP)**

- The SOP is a Boolean expression containing AND terms, called product terms, of one or more literals each. The sum denote the ORing of these terms.
- An example of a function expressed in sum of product is:

F = Y' + XY + X'YZ'

➢ **Product of Sum (POS)**

- The POS is a Boolean expression containing OR terms, called sum terms. Each terms may have any no. of literals. The product denotes ANDing of these terms.
- An example of a function expressed in product of sum is:

F = X (Y' + Z) (X' + Y + Z' + W)

- A Boolean expression function may be expressed in a nonstandard form. For example the function:

  $$F = (AB + CD)(A'B' + C'D')$$

  Above function is neither sum of product nor in product sums. It can be changed to a standard form by using distributive law as below;

  $$F = ABC'D' + A'B'CD$$

2. Canonical Forms

- Any boolean expression can be expressed in Sum of Product (SOP) form or Product of Sum (POS) form, they are called **canonical form**.

➢ SOP - Sum of Product

- A standard SOP form is one in which a no. of product terms, each one of which contains all the variables of the function either in complemented or non-complemented form, summed together
- Each of the product term is called **MINTERM.**
- For minterms,

  Each non-complemented variable → 1

  Each complemented variable → 0
- Decimal equivalent is expressed in terms of lower case 'm'.

- **For example,**

  1. $XYZ = 111 = m_7$
  2. $A'BC = 011 = m_3$
  3. $P'Q'R' = 000 = m_0$
  4. $T'S' = 00 = m_0$
  5. $B'C = 01 = m_1$

Ex 47: $F_1 = X'Y'Z + XY'Z' + XYZ$

Solution:

$F_1 = X'Y'Z + XY'Z' + XYZ$

$= 001 + 100 + 111$

$= m_1 + m_4 + m_7$

$= \Sigma m(1, 4, 7)$

Ex 48: $F_2 = P'Q' + PQ$

Solution:

$F_2 = P'Q' + PQ$

$= 00 + 11$

$= m_0 + m_3 = \Sigma m(0, 3)$

Ex 49: F_3 = XY’ZW + XYZ’W’ + X’Y’Z’W’

Solution:

F_3 = XY’ZW + XYZ’W’ + X’Y’Z’W’
= 1011 + 1100 + 0000
= $m_{11} + m_{12} + m_0$
= Σm (0, 11, 12)

- **POS - Product of Sum**

- A standard POS form is one in which a no. of sum terms, each one of which contains all the variables of the function either in complemented or non-complemented form, are multiplied together
- Each of the product term is called **MAXTERM**
- For maxterms,
  Each non-complemented variable → 0
  Each complemented variable → 1
- Decimal equivalent is expressed in terms of upper case ‘M’.

- **For example,**
  1. X+Y+Z = 000 = M_0
  2. P’+Q’+R’ = 111 = M_7
  3. A’+B+C’+D = 1010 = M_{10}

Ex 50: F_1 = (P’+Q) (P+Q’)

Solution:

F_1 = (P’+Q) (P+Q’)
= (10) (01) = $M_2 \cdot M_1$
= ΠM (1, 2)

Ex 51: F_2 = (X’+Y’+Z’+W) (X’+Y+Z+W’) (X+Y’+Z+W’)

Solution:

F_2 = (X’+Y’+Z’+W) (X’+Y+Z+W’) (X+Y’+Z+W’)
= (1110) (1001) (0101) = $M_{14} \cdot M_9 \cdot M_5$
= ΠM (5, 9, 14)

Ex 52: F_3 = (A’+B+C) (A+B’+C) (A+B+C’)

Solution:

F_3 = (A’+B+C) (A+B’+C) (A+B+C’)
= (100) (010) (001)
= M_4 M_2 M_1
= ΠM (1, 2, 4)

4.6.1 Minterms & Maxterms for 3 variables

Table 4.11: Representation of Minterms & Maxterms for 3 Variable

Row No.	A B C	Minterms	Maxterms
0	0 0 0	$A'B'C' = m_0$	$A + B + C = M_0$
1	0 0 1	$A'B'C = m_1$	$A + B + C' = M_1$
2	0 1 0	$A'BC' = m_2$	$A + B' + C = M_2$
3	0 1 1	$A'BC = m_3$	$A + B' + C' = M_3$
4	1 0 0	$AB'C' = m_4$	$A' + B + C = M_4$
5	1 0 1	$AB'C = m_5$	$A' + B + C' = M_5$
6	1 1 0	$ABC' = m_6$	$A' + B' + C = M_6$
7	1 1 1	$ABC = m_7$	$A' + B' + C' = M_7$

4.6.2 Conversion between Canonical forms

- The complement of a function expressed as the sum of minterms equals the sum of minterms missing from original function
- This is because the original function is expressed by those minterms that make the function equal to 1, while its complement is 1 for those minterms that the function is 0.

Ex 53: F (A, B, C) = Σ (1, 4, 5, 6, 7)

Solution:

STEP 1:

Take complement of the given function;

$F'(A, B, C) = \Sigma(0, 2, 3) = (m_0 + m_2 + m_3)'$

STEP 2:

Put value of MINTERM in form of variables;

$F' = (A'B'C' + A'BC' + A'BC)'$

$= (A+B+C)(A+B'+C)(A+B'+C')$

$= M_0 \cdot M_2 \cdot M_3$

$= \Pi M(0, 2, 3)$

In general, $m_j' = M_j$

Ex 54: F (A, B, C, D) = ΠM (0, 3, 7, 10, 14, 15)

Solution:

STEP 1:

Take complement of the given function;

$F'(A,B,C,D) = \Pi M(1,2,4,5,6,8,9,11,12,13) = (M_1 M_2 M_4 M_5 M_6 M_8 M_9 M_{11} M_{12} M_{13})'$

STEP 2:
Put value of MAXTERM in form of variables;

$$F' = [(A+B+C+D')\,(A+B+C'+D)\,(A+B'+C+D)\,(A+B'+C+D')\,(A+B'+C'+D)\,(A'+B+C+D)\,(A'+B+C+D')\,(A'+B+C'+D')\,(A'+B'+C+D)\,(A'+B'+C+D')]'$$
$$= (A'B'C'D) + (A'B'CD') + (A'BC'D') + (A'BC'D) + (A'BCD')\ (AB'C'D') + (AB'C'D) + (AB'CD) + (ABC'D') + (ABC'D)$$
$$= m_1 + m_2 + m_4 + m_5 + m_6 + m_8 + m_9 + m_{11} + m_{12} + m_{13}$$
$$= \Sigma m\,(1, 2, 4, 5, 6, 8, 9, 11, 12, 13)$$

4.6.3 Convert to Minterms

Ex 55: F = A + B'C

Solution:

A → B & C is missing. So multiply with (B + B') & (C + C').
B'C → A is missing. So multiply with (A + A').

$$A = A\,(B + B')\,(C + C')$$
$$= (AB + AB')\,(C + C')$$
$$= ABC + AB'C + ABC' + AB'C'$$

$$B'C = B'C\,(A + A')$$
$$= AB'C + A'B'C$$

$$\text{So, } A + B'C = ABC + AB'C + ABC' + AB'C' + AB'C + A'B'C$$
$$= ABC + AB'C + ABC' + AB'C' + A'B'C$$
$$= 111 + 101 + 110 + 100 + 001$$
$$= m_7 + m_6 + m_5 + m_4 + m_1$$
$$= \Sigma m\,(1, 4, 5, 6, 7)$$

4.6.4 Convert to Maxterms

Ex 56: F = A (B + C')

Solution:

A → B & C is missing. So add BB' & CC'.
B + C' → A is missing. So add AA'.

$$A = A + BB' + CC'$$
$$= (A + B)\,(A + B') + CC$$
$$= (A + B + CC')\,(A + B' + CC')$$
$$= (A + B + C)\,(A + B + C')\,(A + B' + C)\,(A + B' + C')$$

$$B + C' = B + C' + AA' = (A + B + C')\,(A' + B + C')$$

So, A (B + C’) = (A + B + C) (A + B + C’) (A + B’ + C) (A + B’ + C’)
(A + B + C’) (A’ + B + C’)
= (A + B + C) (A + B + C’) (A + B’ + C) (A + B’ + C’)
(A’ + B + C’)
= (000) (001) (010) (011) (101)
= ΠM (0, 1, 2, 3, 5)

Ex 57: F = XY + X’Z

Solution:

F = XY + X’Z
= (XY + X’) (XY + Z)
= (X +X’) (Y + X’) (X + Z) (Y + Z)
= (Y + X’) (X + Z) (Y + Z)

X’ + Y = X’ + Y + ZZ’
= (X’ + Y + Z) (X’ + Y + Z’)
= (100) (101)

X + Z = X’ + Z + YY’
= (X + Y + Z) (X + Y’ + Z)
= (000) (010)

Y + Z = Y + Z + XX’
= (X + Y + Z) (X’ + Y + Z)
= (000) (100)

So, F = XY + X’Z
= (100) (101) (000) (010)
= $M_4\ M_5\ M_0\ M_2$
F = ΠM (0, 2, 4, 5)

4.7 SIMPLIFICATION OF LOGICAL FUNCTION USING KARNAUGH MAP (K-MAP):

4.7.1 Karnaugh Map (k-map) Introduction

- A Boolean expression may have many different forms.
- With the use of K-map, the complexity of reducing expression becomes easy and Boolean expression obtained is simplified.
- K-map also be said as **pictorial form** of truth table.

- K-map is alternative way of simplifying logic circuits.
- Instead of using Boolean algebra simplification techniques, you can transfer logic values from a Boolean statement or a truth table into a Karnaugh map (k-map)
- Tool for representing Boolean functions of up to six variables.
- K-maps are tables of rows and columns with entries represent 1's or 0's of SOP and POS representations.

- K-map cells are arranged such that adjacent cells correspond to truth rows that differ in only one bit position (*logical adjacency*)
- K-Map are often used to simplify logic problems with up to 6 variables
- ***No. of Cells = 2^n, where n is a number of variables.***

- The Karnaugh map is completed by entering a '1' (or '0') in each of the appropriate cells.
- Within the map, adjacent cells containing 1's (or 0's) are grouped together in twos, fours, or eights and so on.

4.7.2 2 variable k-map

- For 2 variable k-map, there are 2^2 = 4 input combinations.
- If A & B are two variables then;

SOP → Minterms → A'B' (m_0, 00) ; A'B (m_1, 01) ; AB' (m_2, 10) ; AB (m_3, 11)
POS → Maxterms → A + B (M_0, 00) ; A + B' (M_1, 01) ; A' + B (M_2, 10) ; A' + B' (M_3, 11)

➢ ***Mapping of SOP Expression:***

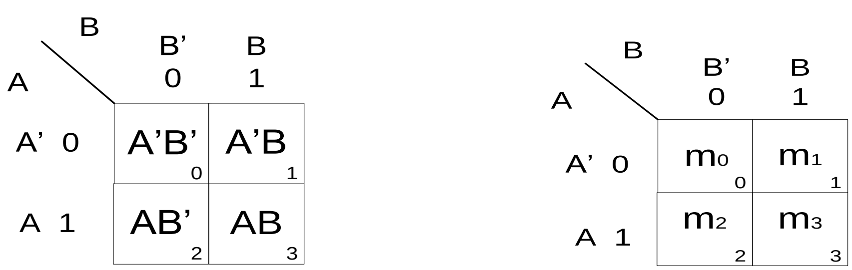


Fig. 4.41: Mapping of SOP form for two variable K-Map

- 1 in a cell indicates that the minterm is included in Boolean expression.
- For e.g. if F = ∑m (0, 2, 3), then 1 is put in cell no. 0, 2, 3 as shown below.

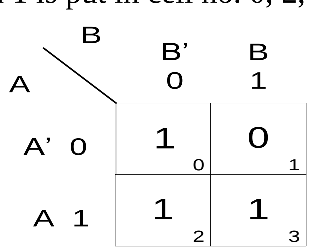
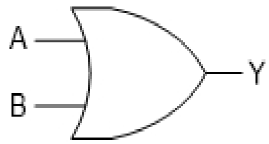


Fig. 4.42

Ex. 58: Map for a 2-input OR gate.



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

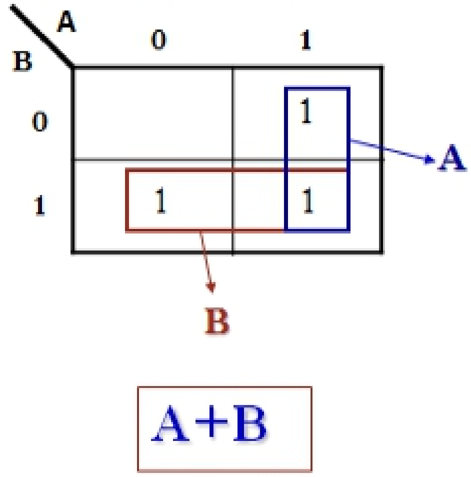


Fig. 4.43

Ex. 59: Map for a 2-input EX-OR gate.

A	B	X
0	0	1 → $\bar{A}\bar{B}$
0	1	0
1	0	0
1	1	1 → AB

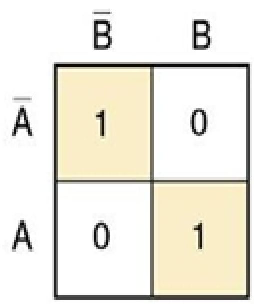


Fig. 4.44

F = A'B' + AB

➢ ***Map following SOP expressions:***

Ex. 60: F = AB

Solution:

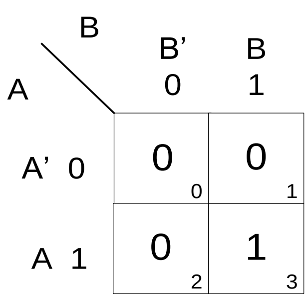


Fig. 4.45

Ex. 61: F = AB' + A'B +A'B'

Solution:

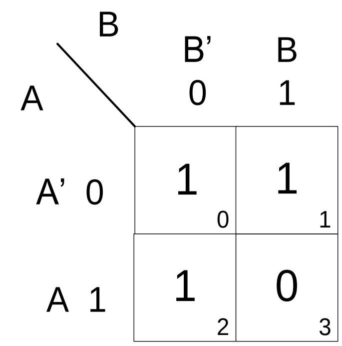


Fig. 4.46

Ex. 62: F (A, B) = ∑ (0, 2)

Solution:

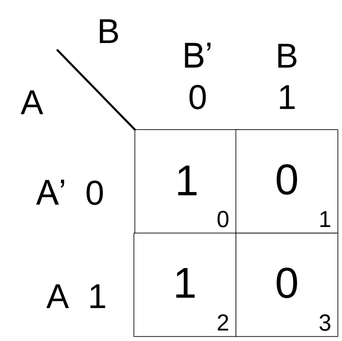


Fig. 4.47

Ex. 63: F = $m_0 + m_1$

Solution:

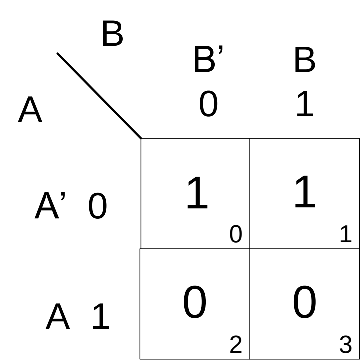


Fig. 4.48

➢ ***Map following POS expressions:***

A \ B	B 0	B’ 1
A 0	A+B [0]	A+B’ [1]
A’ 1	A’+B [2]	A’+B’ [3]

A \ B	B 0	B’ 1
A 0	M_0 [0]	M_1 [1]
A’ 1	M_2 [2]	M_3 [3]

Fig. 4.49: Mapping of POS form for two variable K-Map

- 0 in a cell indicates that the maxterm is included in Boolean expression.
- For e.g. if F = Π M (0, 2, 3), then 0 is put in cell no. 0, 2, 3 as shown below.

A \ B	B’ 0	B 1
A’ 0	0 [0]	1 [1]
A 1	0 [2]	0 [3]

Fig. 4.50

Ex 64: F (A, B) = (A+B) (A’+B)

Solution:

A \ B	B 0	B’ 1
A 0	0 [0]	1 [1]
A’ 1	0 [2]	1 [3]

Fig. 4.51

Ex 65: F = $M_0 \cdot M_1 \cdot M_2$

Solution:

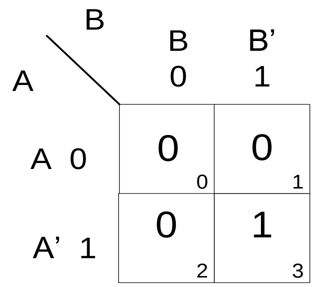


Fig. 4.52

Ex 66: F = ΠM (1, 3)

Solution:

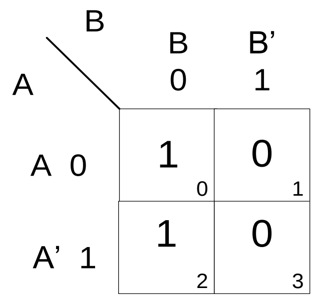


Fig. 4.53

- ***Reduce following SOP expressions:***

Ex 67: F = $m_0 + m_1$

Solution:

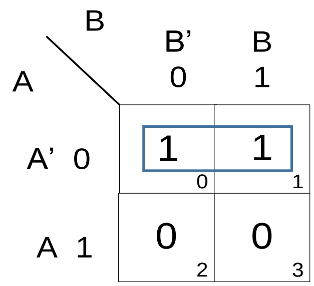


Fig. 4.54

F = A’

Ex 68: F = A'B' + AB'

Solution:

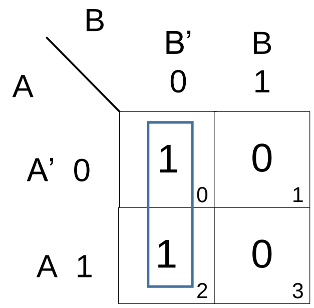


Fig. 4.55

F = B'

Ex 69: F = Σ (1, 3)

Solution:

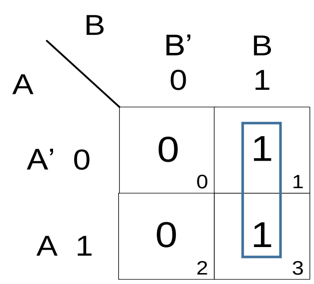


Fig. 4.56

F = B

Ex 70: F = $m_0 + m_3$

Solution:

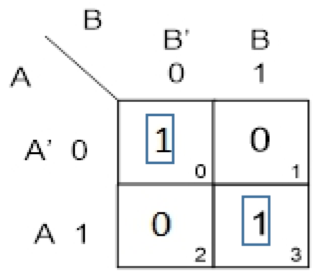


Fig. 4.57

F = A'B' + AB

Ex 71: F = ∑m (0, 1, 2, 3)

Solution:

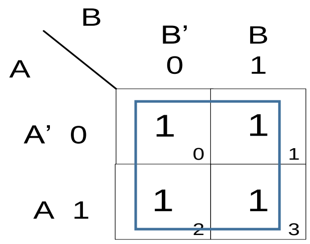


Fig. 4.58

F = 1

➢ ***Reduce following POS expressions:***

Ex 72: F = Π (0, 2, 3)

Solution:

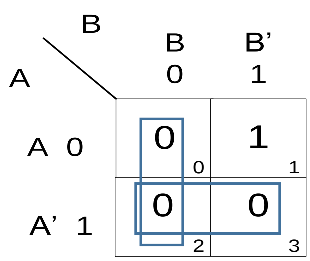


Fig. 4.59

F = A’B

Ex 73: F = (A+B) (A’+B) (A+B’)

Solution:

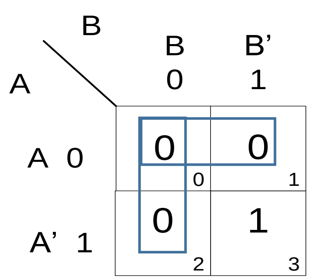


Fig. 4.60

F = AB

Ex 74: F = $M_3 \cdot M_1 \cdot M_2$

Solution:

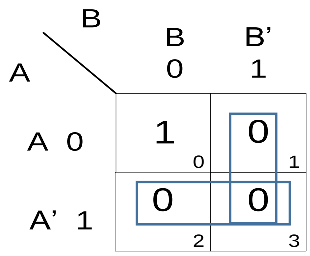


Fig. 4.61

F = A’B’

Ex 75: F = $m_2 + m_3$

Solution:

F = $m_2 + m_3 = \Pi$ (0, 1)

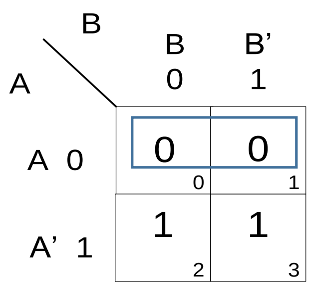


Fig. 4.62

F = A

Ex 76: F = ΠM (0, 1, 2, 3)

Solution:

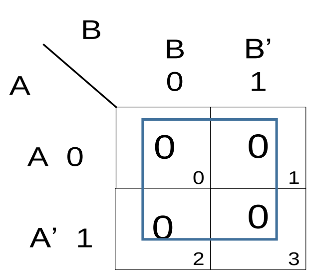


Fig. 4.63

F = 0

4.7.3 3 variable k-map

➢ ***Reduce following SOP expression:***

- For 3 variables, in SOP form there are 8 combinations as follow;

Table 4.12

A'B'C'	(m_0, 000)
A'B'C	(m_1, 001)
A'BC'	(m_2, 010)
A'BC	(m_3, 011)
AB'C'	(m_4, 100)
A'BC'	(m_5, 101)
A'B'C	(m_6, 110)
ABC	(m_7, 111)

- For the case of 3 variables, we form a map consisting of 2^3=8 cells as shown in Figure

A \ BC	00	01	11	10
0	000 (0)	001 (1)	011 (3)	010 (2)
1	100 (4)	101 (5)	111 (7)	110 (6)

A \ BC	$\overline{B}\,\overline{C}$ 00	$\overline{B}C$ 01	BC 11	$B\overline{C}$ 10
$\overline{A}$ 0	$\overline{A}\,\overline{B}\,\overline{C}$ (0)	$\overline{A}\,\overline{B}C$ (1)	$\overline{A}BC$ (3)	$\overline{A}B\overline{C}$ (2)
A 1	$A\overline{B}\,\overline{C}$ (4)	$A\overline{B}C$ (5)	ABC (7)	$AB\overline{C}$ (6)

Fig. 4.64: Mapping of SOP form for 3 variable K-Map

Ex 77: F = A'B'C' + ABC + A'BC'

Solution:

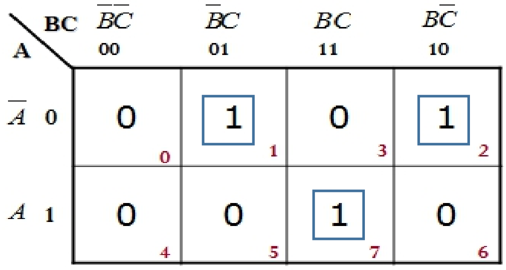


Fig. 4.65

F = A'B'C + ABC + A'BC'

Ex 78: F = Σ (1, 6, 7)

Solution:

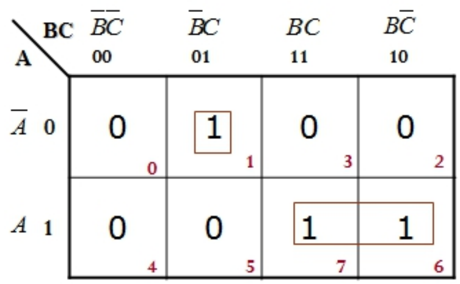


Fig. 4.66

F = A'B'C + AB

Ex 79: F = A'B'C' + ABC' + AB'C' + A'BC

Solution:

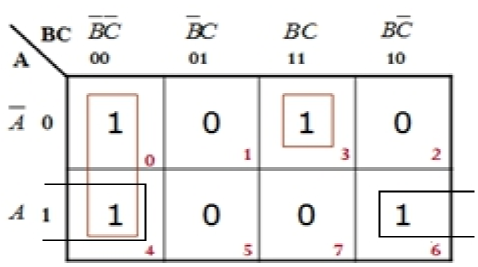


Fig. 4.67

F = B'C' + AC' + A'BC

Ex 80: F = Σm (0, 1, 2, 4, 5, 6)

Solution:

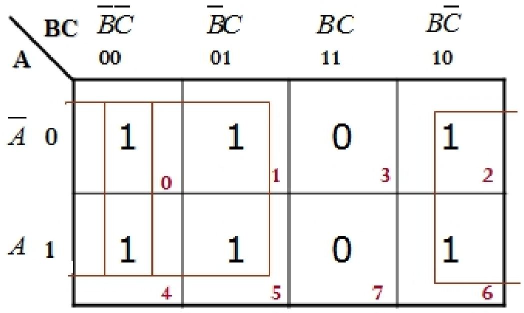


Fig. 4.68

F = B’ + C’

Ex 81: F = $m_3 + m_4 + m_6 + m_7$

Solution:

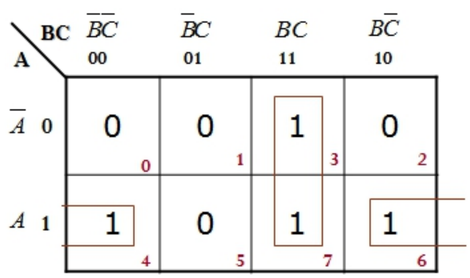


Fig. 4.69

F = BC + AC’

Ex 82: F = Σm (3, 7, 1, 6, 0, 2, 5, 4)

Solution:

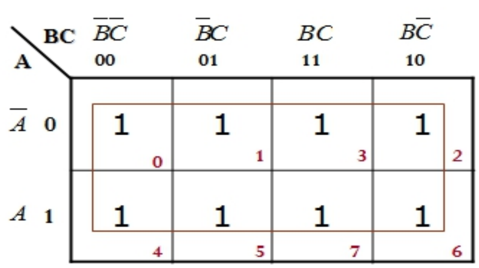


Fig. 4.70

F = 1

➢ ***Reduce following POS expression***

- For 3 variables, in POS form there are 8 combinations as follow;

Table 4.13

A + B + C	(M_0, 000)
A + B + C’	(M_1, 001)
A + B’ + C	(M_2, 010)
A + B’ + C’	(M_3, 011)
A’ + B + C	(M_4, 100)
A’ + B + C’	(M_5, 101)
A’ + B’ + C	(M_6, 110)
A’ + B’ + C’	(M_7, 111)

- For the case of 3 variables, we form a map consisting of 2^3=8 cells as shown in Figure

A \ BC	$B+C$ 00	$B+\overline{C}$ 01	$\overline{B}+\overline{C}$ 11	$\overline{B}+C$ 10
A 0	$A+B+C$ 0	$A+B+\overline{C}$ 1	$A+\overline{B}+\overline{C}$ 3	$A+\overline{B}+C$ 2
$\overline{A}$ 1	$\overline{A}+B+C$ 4	$\overline{A}+B+\overline{C}$ 5	$\overline{A}+\overline{B}+\overline{C}$ 7	$\overline{A}+\overline{B}+C$ 6

Fig. 4.71: Mapping of POS form for 3 variable K-Map

Ex 83: F = (A’+B’+C’) (A’+B+C’)

Solution:

A \ BC	$B+C$ 00	$B+\overline{C}$ 01	$\overline{B}+\overline{C}$ 11	$\overline{B}+C$ 10
A 0	1 (0)	1 (1)	1 (3)	1 (2)
$\overline{A}$ 1	1 (4)	0 (5)	0 (7)	1 (6)

Fig. 4.72

F = (A’ + C’)

Ex 84: F = ΠM (1, 2, 5)

Solution:

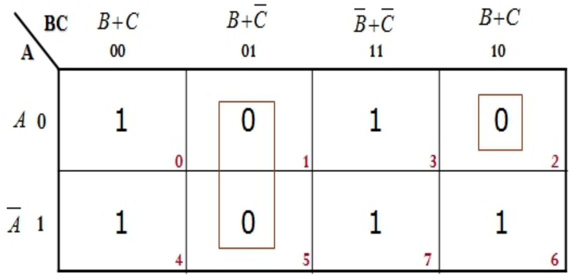


Fig. 4.73

F = (B + C’) (A + B + C)

Ex 85: F = $M_0 \cdot M_3 \cdot M_7$

Solution:

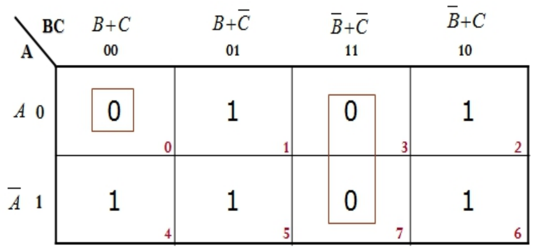


Fig. 4.74

F = (A + B + C) (B’ + C’)

Ex 86: F = (A+B+C) (A+B’+C’) (A’+B+C)

Solution:

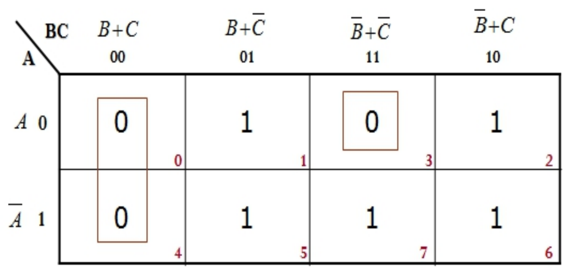


Fig. 4.75

F = (B + C) (A + B’ + C’)

Ex 87: F = ΠM (5, 7, 0, 3, 2, 4, 6, 1)

Solution:

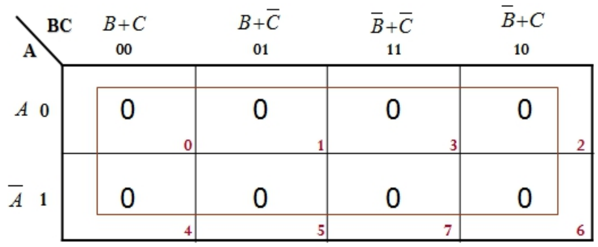


Fig. 4.76

$$F = 0$$

4.7.4 4 variable k-map

➢ ***Reduce following SOP expression:***

- For 4 variables, 2^4 = 16 combinations are available;

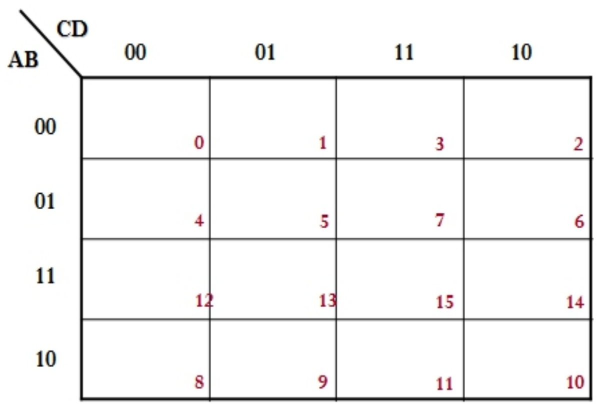
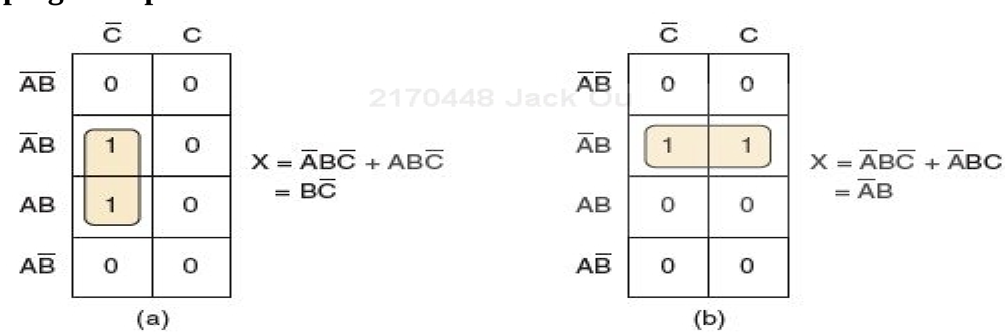


Fig. 4.77: Mapping of SOP form for 4 variable

➢ ***Looping:***

- **Looping Groups of Two:**



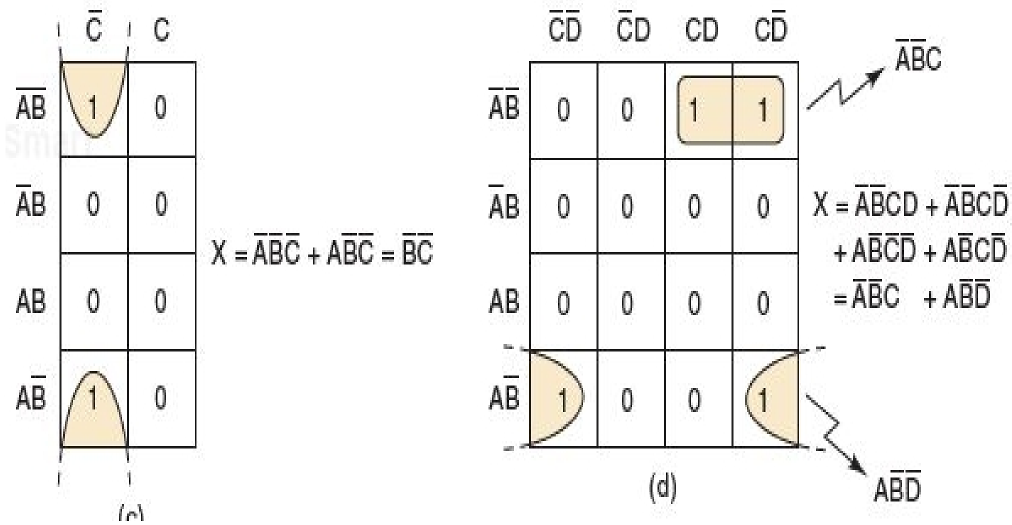


Fig. 4.78: Looping groups of two

- **Looping Groups of Four:**

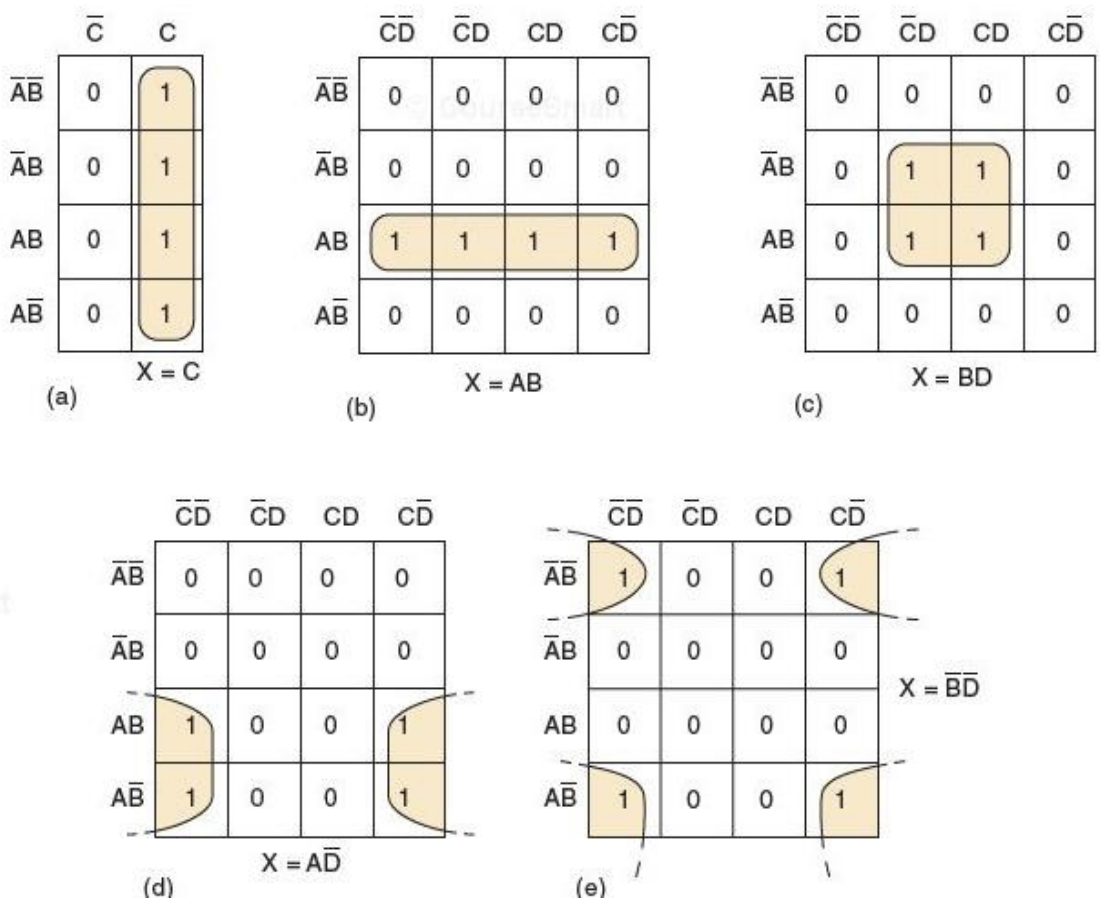


Fig. 4.79: Looping groups of four

➢ **Looping Groups of Eight:**

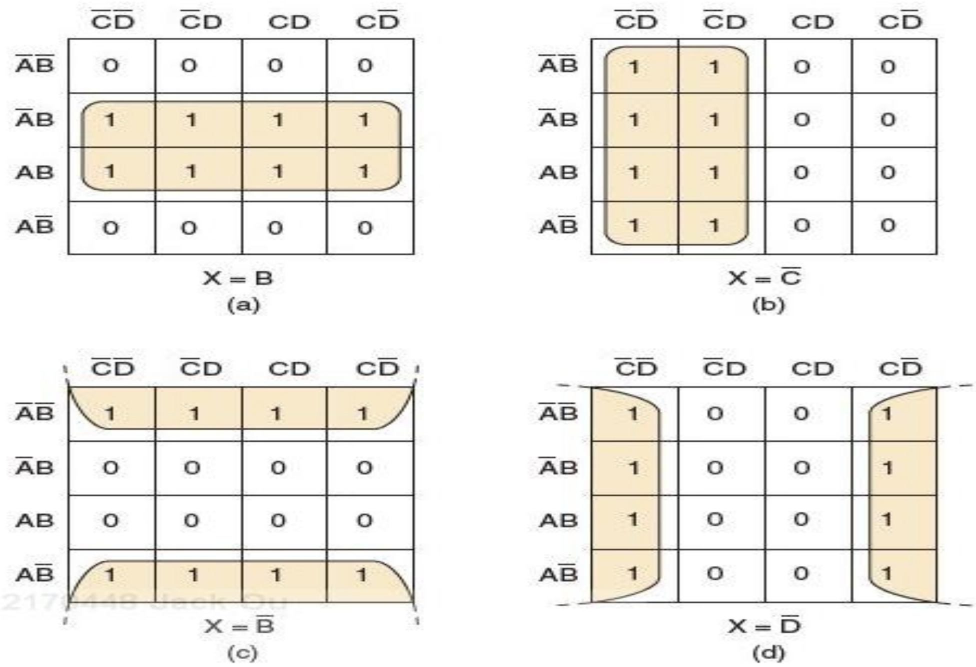
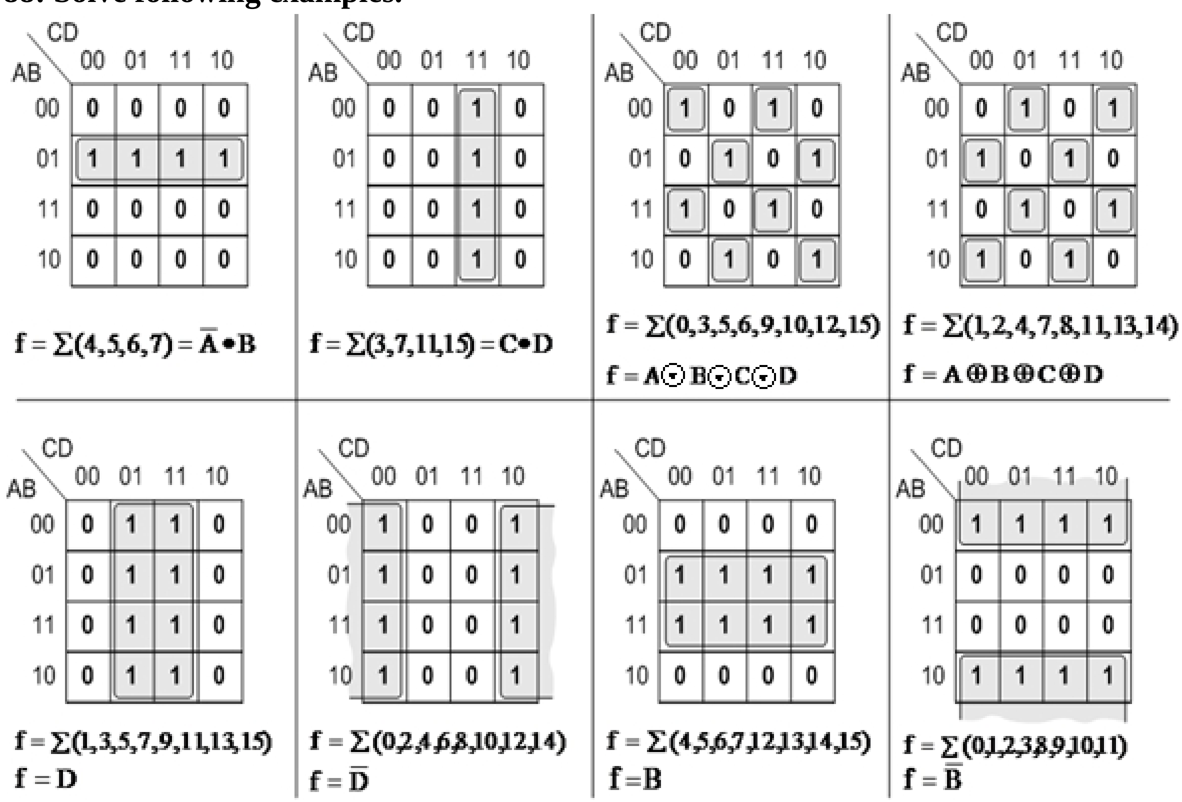


Fig. 4.80: Looping groups of eight

Ex 88: Solve following examples.



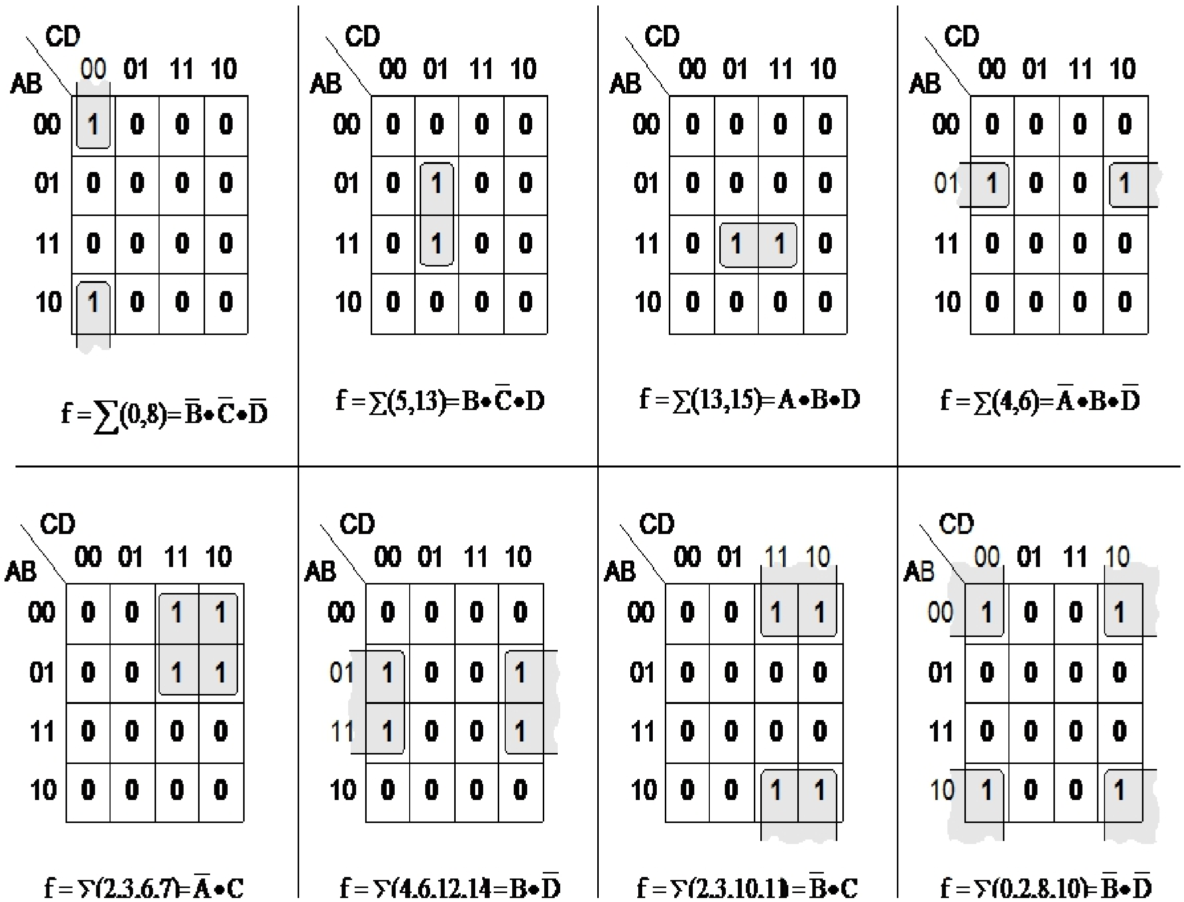


Fig. 4.81

Ex 89: ∑ (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)

Solution:

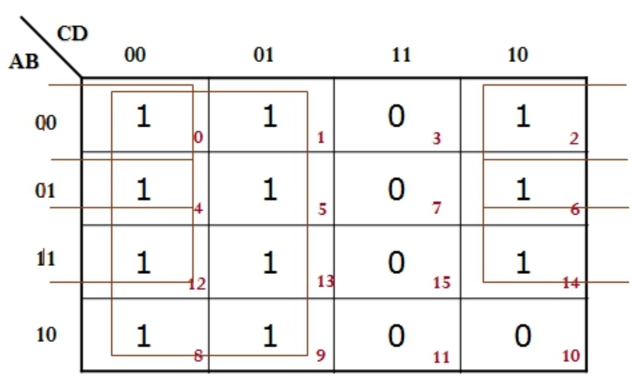


Fig. 4.82

F = C’ + A’D’ + BD’

Ex 90: A’B’C’ + B’CD’ + A’BCD’ + AB’C’

Solution:

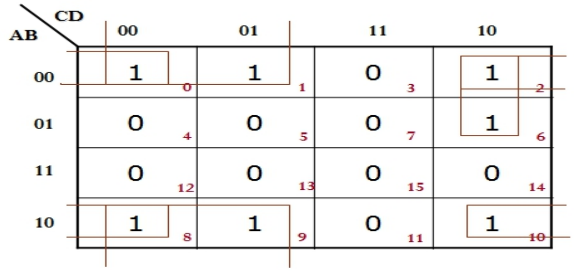


Fig. 4.83

F = B’C’ + B’D’ + A’CD’

Ex 91: $\sum$ **(0, 1, 2, 3, 5, 7, 8, 9, 12, 13)**

Solution:

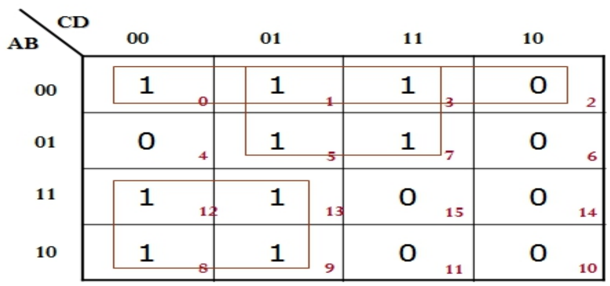


Fig. 4.84

F = A’B’ + AC’ + A’D

Ex 92: $\sum$ **(0, 1, 3, 4, 5, 6, 7, 13, 15)**

Solution:

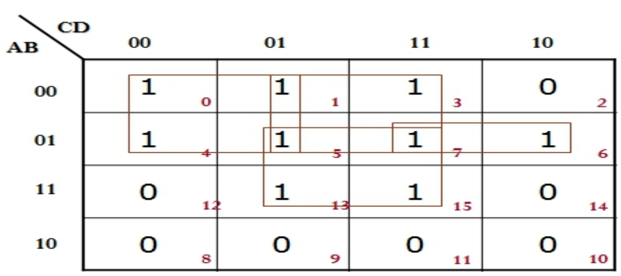


Fig. 4.85

F = A’C’ + A’D + BD + A’B

Ex 93: $\sum m$ **(5, 6, 7, 9, 10, 11, 13, 14, 15)**

Solution:

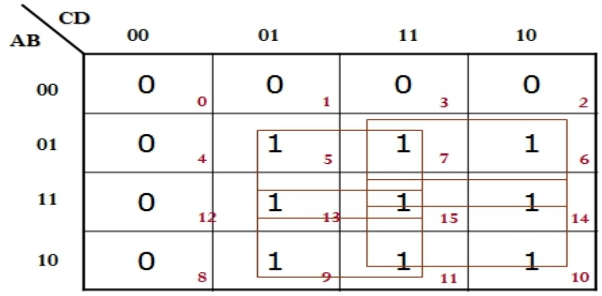


Fig. 4.86

F = BD + BC + AD + AC

- ***Reduce following POS expression:***
- For 4 variables, $2^4 = 16$ combinations are available;

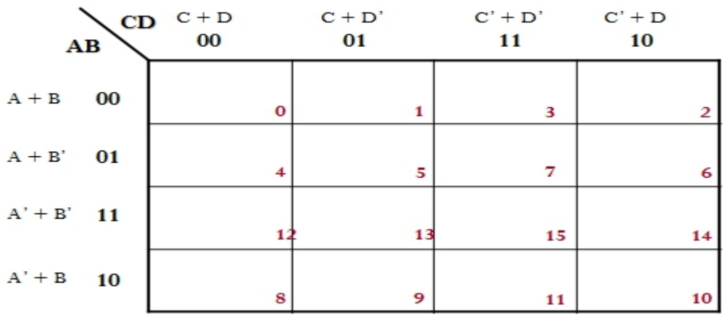


Fig. 4.87: Mapping of POS form for 4 variable K-Map

Ex 94: ΠM (0, 1, 2, 5, 7, 8, 9, 10, 14, 15)

Solution:

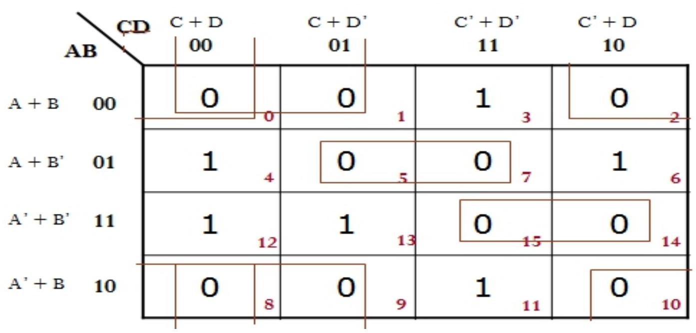


Fig. 4.88

F = (B + D) (B + C) (A + B’ + D’) (A’ + B’ + C’)

Ex 95: M_1 M_3 M_4 M_7 M_9 M_{11} M_{12} M_{14} M_{15}

Solution:

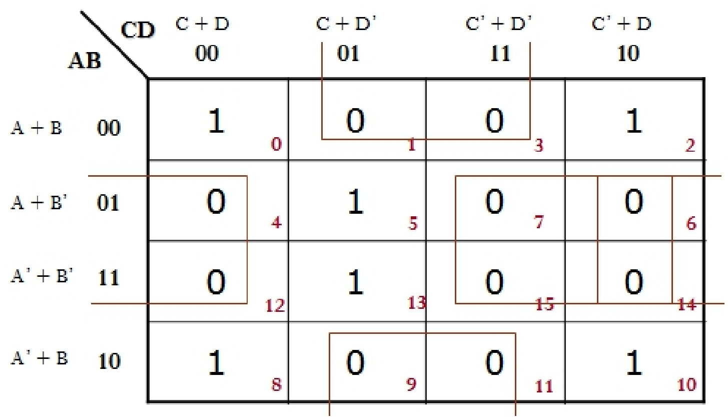


Fig. 4.89

F = (B' + D) (C' + B') (D' + B)

➢ ***Don't Care Combinations:***

Ex 96: ∑m (1, 5, 6, 12, 13, 14) + d (2, 4)

Solution:

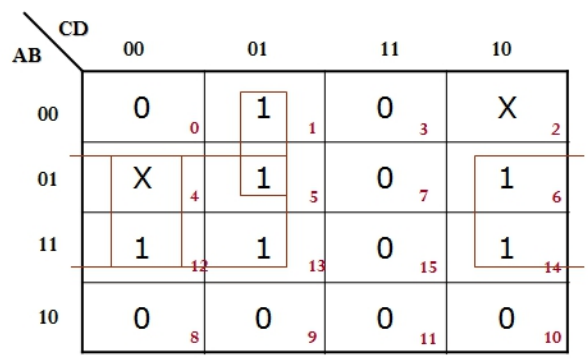


Fig. 4.90

F = BC' + BD' + A'C'D

Ex 97: ΠM (4, 7, 10, 11, 12, 15) · d (6, 8)

Solution:

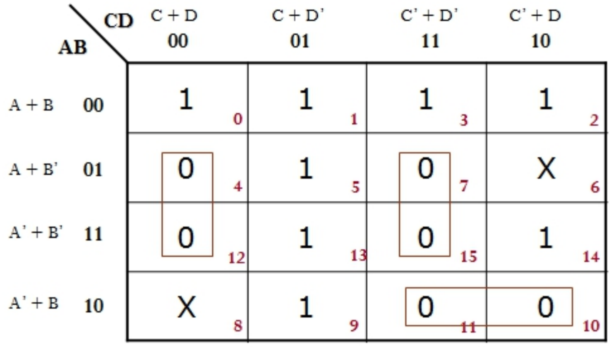


Fig. 4.91

F = (B’ + C + D) (B’ + C’ + D’) (A’ + B + C’)

4.8 BINARY ADDERS:

- Binary adders are of two types;
  1. Half adder
  2. Full adder

4.8.1 Half adder

- A binary adder adds two binary bits. Block diagram of half adder is shown in fig. 4.92.

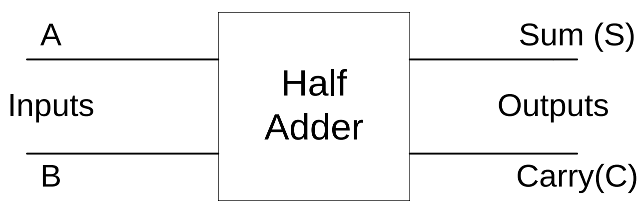


Fig. 4.92: Block diagram of half adder

- There are two input terminals, which are marked as A and B. Binary numbers, the sum of which has to be made are applied here. There are two output terminals. One terminal is for sum bit S and the other is the carry bit C. Truth table of half adder is shown in table 4.13

Table 4.13: Truth table for half adder

Inputs		**Outputs**	
A	**B**	**S (Sum)**	**C (Carry)**
0	0	0	0
1	0	1	0
0	1	1	0
1	1	0	1

- From truth table we can write the expression for sum S and carry C.
- For sum and carry summing up input combinations for which the output is 1.

$$S = A'B + AB'$$
$$C = AB$$

- It is seen that the sum S can be realized by EX-OR gate and carry C can be realized by an AND gate. Such circuit is shown in Fig. 4.93

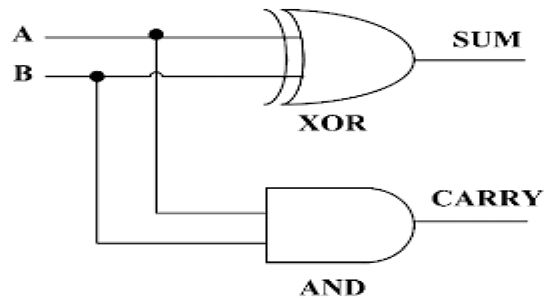


Fig. 4.93: Circuit of half adder

4.8.2 Full adder

- Full adder is made up of two half adder and OR gate.
- It has three inputs and two output.
- It can able to add 3 digit at a time.
- Fig. 4.94 shows the full adder logic circuit using half adder and OR gate.

Fig. 4.94: Block diagram of full adder

Table 4.14: Truth table for full adder

Input bit for number A	Input bit for number B	Carry bit input C_{IN}	Sum bit output S	Carry bit output C_{OUT}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

- For sum and carry summing up input combinations for which the output is 1.

S = A XOR **B** XOR **C$_{in}$**

C = (A XOR **B) C$_{in}$ + AB**

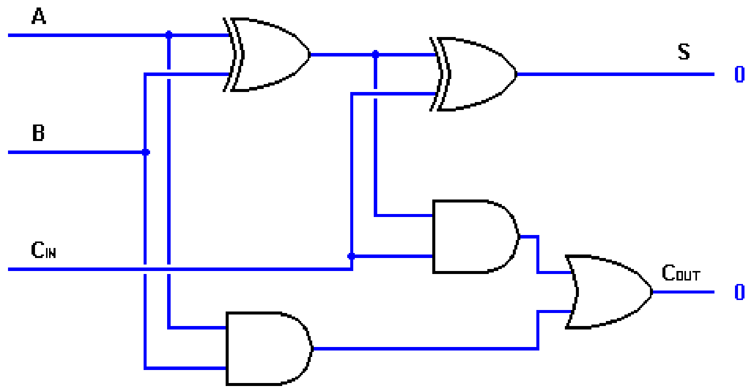


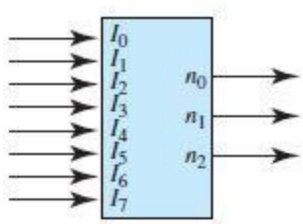
Fig. 4.95: Circuit diagram of full adder

4.8.3 Comparison between Half Adder & Full adder

Half Adder	Full Adder
It is used for 1 bit addition.	It is used for multi bit addition.
One EX-OR gate and one AND gate are used.	Two EX-OR gates, two AND gates and one OR gate are used.
Output is the sum of two signal.	Output is the sum of three signal.
Circuit is simple.	Circuit is complicated.
There are two input and output terminals.	There are three input and output terminals.
It can not be used as full adder.	It can be used as half adder.

4.9 ENCODER:

- **Encoding is the process of forming an encoded representation of a set of inputs.**
- An encoder is a combinational network that generates an n-bit binary code that uniquely identifies the one out of m activated inputs (0 ≤ m ≤ 2n − 1).



Block diagram

Inputs								Outputs		
I_7	I_6	I_5	I_4	I_3	I_2	I_1	I_0	n_2	n_1	n_0
0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	1
0	0	0	1	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	0	1	1	0
1	0	0	0	0	0	0	0	1	1	1

Truth table

Fig. 4.96: Block diagram & Truth table of the 8 to 3 encoder

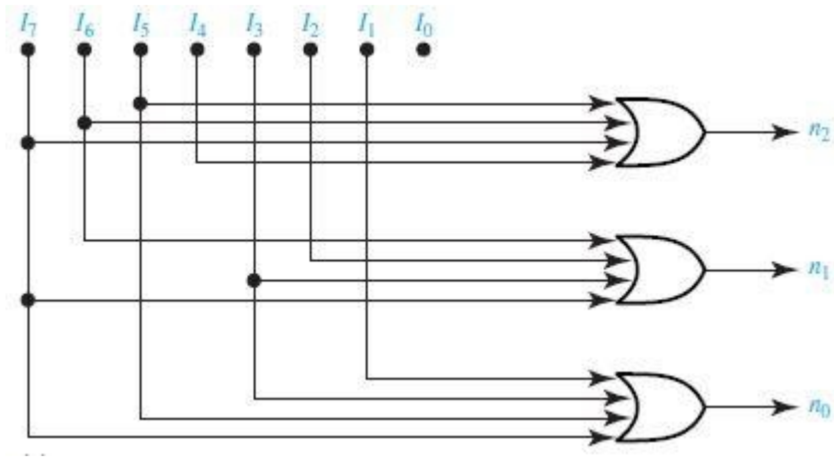
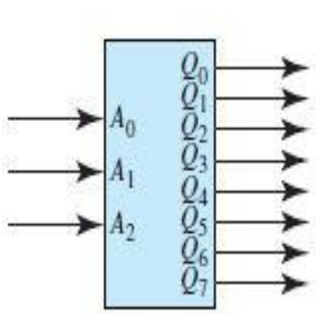


Fig. 4.97: Circuit diagram of the 8 to 3 encoder

- Notice that only one of the eight inputs is allowed to be activated at any given time. The logic diagram for the 8-to-3 encoder is shown in Fig. 4.97.

4.10 DECODER:

- An *n*-bit binary code is capable of encoding up to 2^n distinct elements of information.
- A decoder is a combinational network that decodes (converts) the *n*-bit binary-coded input to *m* outputs ($m \leq 2n$).



A_0	A_1	A_2	Q_7	Q_6	Q_5	Q_4	Q_3	Q_2	Q_1	Q_0
0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	1	0	0	0	0	0	0	1	0	0
0	1	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	1	0	0	0	0
1	0	1	0	0	1	0	0	0	0	0
1	1	0	0	1	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

Block diagram Truth table

Fig. 4.98: Block diagram & Truth table of the 3 to 8 decoder

- The block diagram of a 3-bit to 8-element decoder is shown in Fig. 4.98, where the three inputs are decoded into eight outputs, one for each combination of the input variables.
- In the truth table, observe that for each input combination, there is only one output that is equal to 1 (i.e., each combination selects only one of the eight outputs).

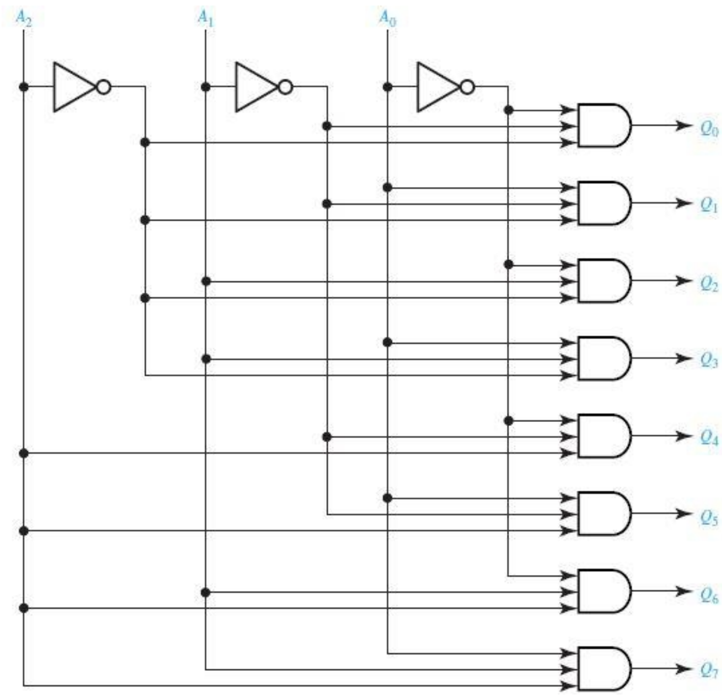
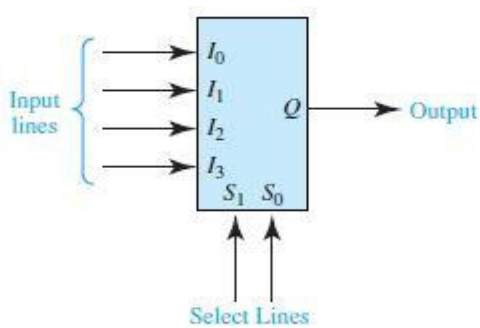


Fig. 4.99: Circuit diagram of the 3 to 8 decoder

- Decoding is so common in digital design that decoders are commercially available as MSI (medium-scale integration) packages in the form of 2-to-4, 3-to-8, and 4-to-10 decoders. Integrated circuits for decoders are available in different forms.

4.11 MULTIPLEXER:

- A multiplexer is a data selector.
- A multiplexer is a combinational network that selects one of several possible input signals and directs that signal to a single output terminal. The selection of a particular input is controlled by a set of selection variables.
- A multiplexer with n selection variables can usually select one out of 2^n input signals.



Select Lines S_1	S_0	Output Q
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

(b)

Block diagram of a 4-to-1 multiplexer Truth table

Fig. 4.100: Block diagram & Truth table of the multiplexer

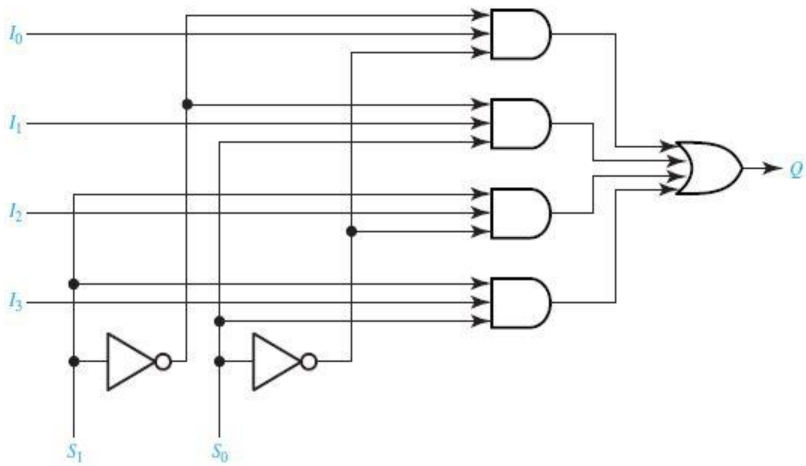


Fig. 4.101: Circuit diagram of the 4 to 1 multiplexer

- Boolean expression $\mathbf{Y = S_0' S_1' I_0 + S_0 S_1' I_1 + S_0 S_1' I_2 + S_0 S_1 I_3}$
- Notice that each of the four inputs (I_0, I_1, I_2, and I_3) is selected by S_1 and S_0, and directed to the output Q. In general, only the input whose address is given by the select lines is directed to the output. The logic diagram is shown in Fig. 4.101.
- 2 to 1, 4 to 1, 8 to 1 and 16-to-1 multiplexers are commercially available as MSI packages.

4.12 DEMULTIPLEXER:

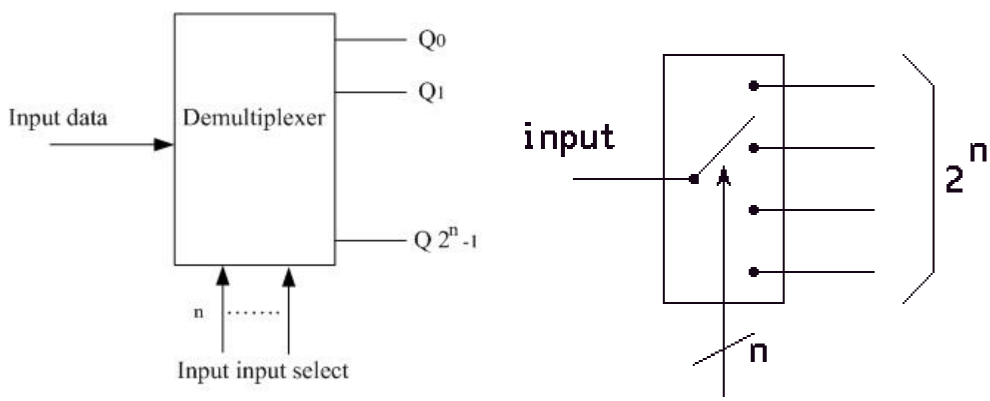


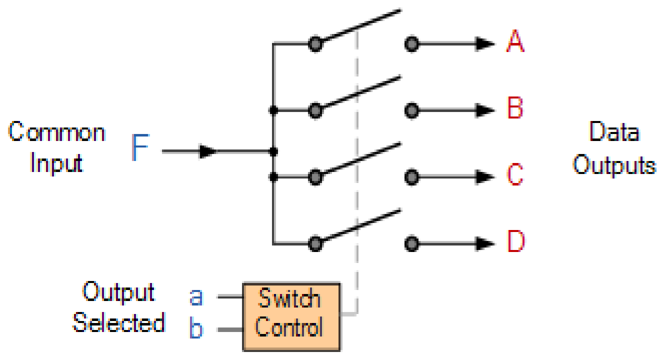
Fig. 4.102: Block diagram of the demultiplexer

- It has one input common data, ‘n’ select lines and ‘m’ output lines
- A demultiplexer performs the reverse operation of a multiplexer i.e. it receives one input and distributes it over several outputs
- At a time only one output line is selected by the select lines and the input is transmitted to the selected output line
- Relation between ‘n’ output lines and m select lines is as follows :

$$n = 2^m$$

- 1 to 4 Demultiplexer has one data input F; select line inputs a, b and four outputs A, B, C & D
- The select lines control the data to be routed. It helps in selecting the output on which the data will be routed.

- Based on the switch control, the input is routed to particular output.



Select Line		Output Line
b	**a**	
0	0	A
0	1	B
1	0	C
1	1	D

Fig. 4.103 (A): Switch representation & Truth table of 1 to 4 demultiplexer

Boolean Equation

$$A = Fb'a'; \quad B = Fb'a;$$
$$C = Fba'; \quad D = Fba;$$

Working

- When ab = "00", the input data F is routed to the output A
- When ab = "01", the input data F is routed to the output B
- When ab = "0", the input data F is routed to the output C
- When ab = "11", the input data F is routed to the output D

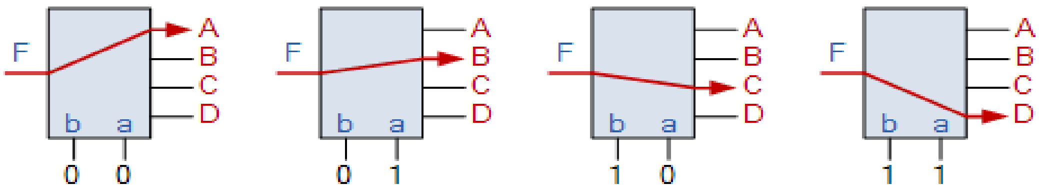


Fig. 4.103 (B): Representation of working of the 1 to 4 demultiplexer

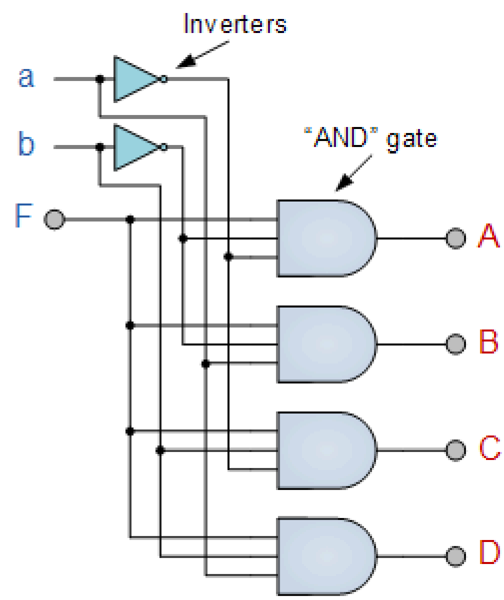


Fig. 4.104: Circuit diagram of 1 to 4 demultiplexer

Ex 4.98: A table of minterms for three variables is as follows:

A	B	C	i	Minterm m_i
0	0	0	0	$\bar{A}\cdot\bar{B}\cdot\bar{C}$
0	0	1	1	$\bar{A}\cdot\bar{B}\cdot C$
0	1	0	2	$\bar{A}\cdot B\cdot\bar{C}$
0	1	1	3	$\bar{A}\cdot B\cdot C$
1	0	0	4	$A\cdot\bar{B}\cdot\bar{C}$
1	0	1	5	$A\cdot\bar{B}\cdot C$
1	1	0	6	$A\cdot B\cdot\bar{C}$
1	1	1	7	$A\cdot B\cdot C$

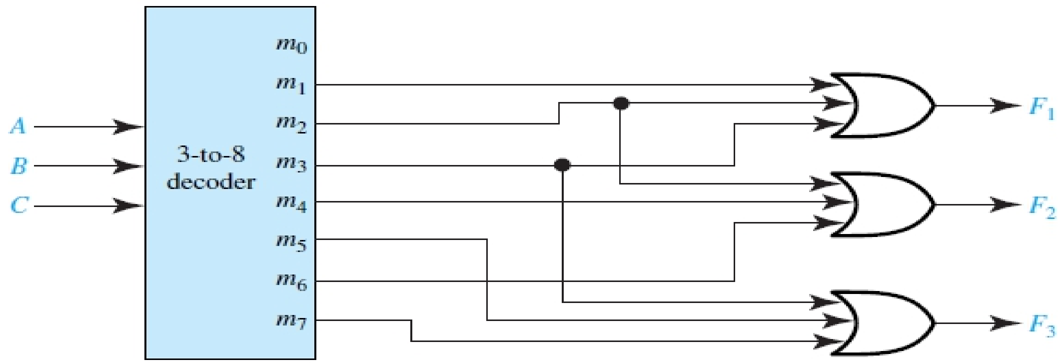
Implement the following Boolean functions by using one 3-to-8 decoder and three three-input OR gates:

$$F_1(A,B,C) = \sum m_i(1,2,3) = \bar{A}\cdot\bar{B}\cdot C + \bar{A}\cdot B\cdot\bar{C} + \bar{A}\cdot B\cdot C$$

$$F_2(A,B,C) = \sum m_i(2,4,6) = \bar{A}\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot\bar{C} + A\cdot B\cdot\bar{C}$$

$$F_3(A,B,C) = \sum m_i(3,5,7) = \bar{A}\cdot B\cdot C + A\cdot\bar{B}\cdot C + A\cdot B\cdot C$$

Solution:

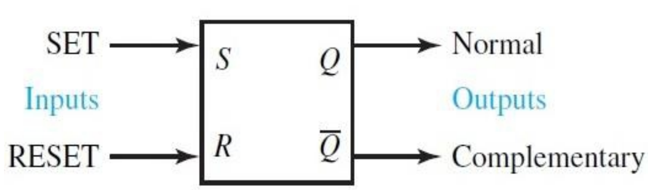


4.13 SEQUENTIAL BLOCKS:

- The output of a *sequential* block depends not only on the present inputs but also on inputs at earlier times.
- Sequential blocks have this kind of memory, and some of them are used as computer memories.
- Most sequential blocks are of the kind known as ***multivibrators***, which can be
  1. *Monostable* - the switch remains in only one of its two positions.
  2. *Bistable - the switch will remain stable in either of its two positions.*
  3. *Unstable - the switch changes its position continuously as a kind of oscillator,* being unstable in both of its two states.
- The most common sequential block is the ***flip-flop***, which is a **bistable** circuit that remembers a single binary digit according to instructions.
- Flip-flops are the basic sequential building blocks. Various types of flip-flops exist, such as the SR flip flop (SRFF), D flip-flop (or latch), and JK flip-flop (JKFF), which differ from one another in the way instructions for storing information are applied.

4.13.1 SR FLIP-FLOP (SRFF)

- The symbol for the SRFF is shown in Fig. 4.105 (a), in which S stands for “set,” R stands for “reset” on the input side, and there are two outputs, the normal output Q and the complementary output $\overline{Q}$. The operation of the SRFF can be understood by the following four basic rules.



(a) Block diagram

Inputs		Outputs	
S	R	Q_{n+1}	$\overline{Q}_{n+1}$
0	0	Q_n	$\overline{Q}_n$
0	1	0	1
1	0	1	0
1	1	Not allowed	

(b) Truth table

Fig. 4.105: Block diagram & Truth table of the SR flip-flop

- If S = 1 and R = 0, then Q = 1 regardless of past history. This is known as the set condition.
- If S = 0 and R = 1, then Q = 0 regardless of past history. This is known as the reset condition.
- If S = 0 and R = 0, then Q does not change and stays at its previous value. This is a highly stable input condition.
- The inputs S = 1 and R = 1 are not allowed (i.e., forbidden) because Q $\overline{Q}$ = 11; $\overline{Q}$ is no longer complementary to Q. This is an unacceptable output state. Such a meaningless instruction should not be used.
- Fig. 4.106 summarizes the specification for an SRFF in terms of a truth table, in which Q_n is the state of the circuit before a clock pulse and Q_{n+1} is the state of the circuit following a clock pulse.

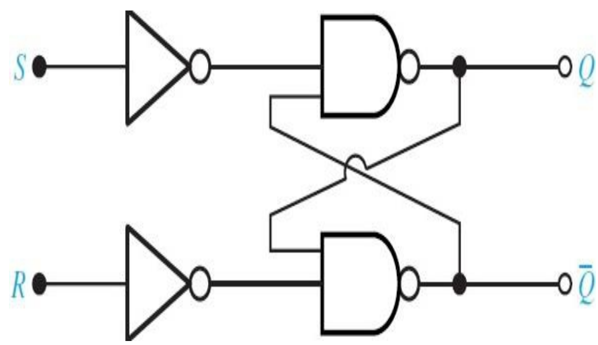
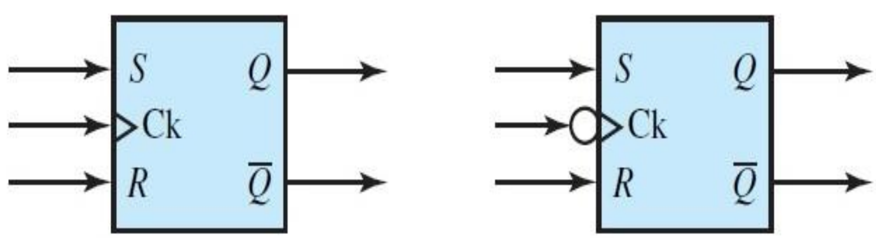


Fig. 4.106: Circuit diagram of the SR flip-flop

- Flip-flops can be constructed using combinations of logic blocks. The *realization* of an SRFF can be achieved from two NAND gates (plus two inverters), as shown in Fig.4.106.
- Edge triggered SRFF symbols are illustrated in Fig. 4.107. The triangle on the Ck (clock) input indicates that the flip-flop is triggered on the edge of a clock pulse.

- **Edge triggered SRFF:**



(a) **(b)**

Fig. 4.107: (a) Triggered on positive edge of clock pulse. (b) Triggered on negative edge of clock pulse

Ex 4.99: The inputs to an SRFF are shown in Fig. 4.108 Determine the value of Q at times t_1, t_2 and t_3.

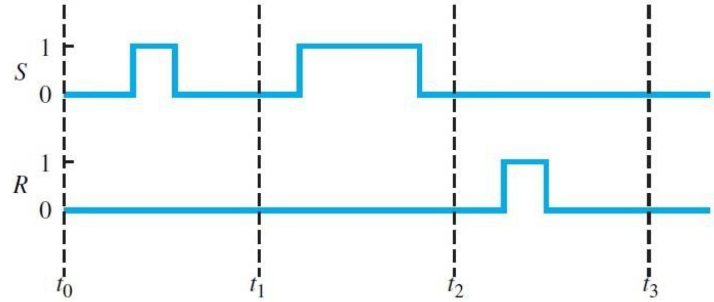


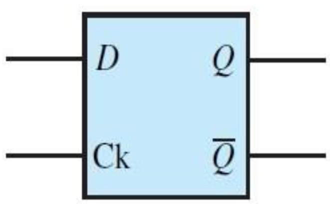
Fig. 4.108

Solution:

Notice that the value of Q at time t_0 is not given; however, it is not necessary to have this information. The first pulse of S sets the SRFF in the state $Q = 1$. Thus at $t = t_1$, $Q = 1$. While the second pulse of S tries again to set the SRFF, there will be no change since Q was already 1. Thus at $t = t_2$, $Q = 1$. The pulse of R then resets the SRFF and then at $t = t_3$, $Q = 0$.

4.13.2 D FLIP-FLOP (LATCH OR DELAY ELEMENT)

- The symbol and circuit diagram for the clocked D flip-flop is shown in Fig. 4.109, in which the two output terminals Q and $\overline{Q}$ behave just as in the SRFF, and the input terminals are D and Ck (clock).
- The term ***clocked*** flip-flop indicates that this device cannot change its state (i.e., Q cannot change) unless a specific change instruction is given through the clock (Ck) input.



(a) Symbol

D	Q(before)	Q(after)
0	0	0
0	1	0
1	0	1
1	1	1

(b) Q for various inputs D

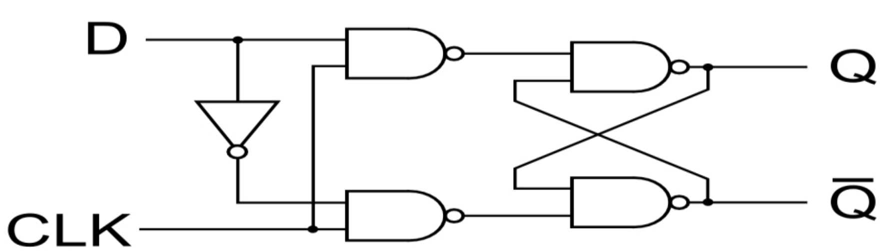


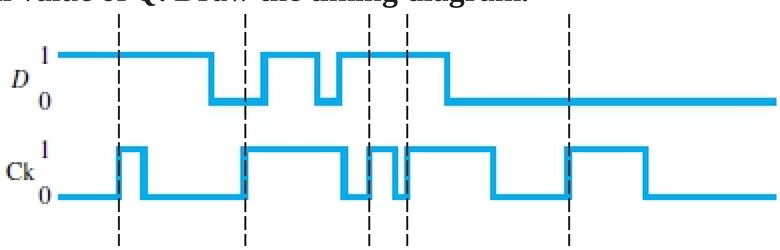
Fig. 4.109: (a) Symbol of D flip-flop (b) Truth table of D flip-flop (c) Circuit Diagram of D flip-flop

- The value of Q after the change instruction is equal to the value of D at the time the change instruction is received. The value of Q before the change instruction does not matter. Fig. 4.109 (b) illustrates the values taken by Q after the change instruction for various inputs D and prior values of Q.
- While there are several variations of the device, in the rising-edge triggered flip-flop a change instruction is effected whenever the Ck input makes a change from 0 to 1.

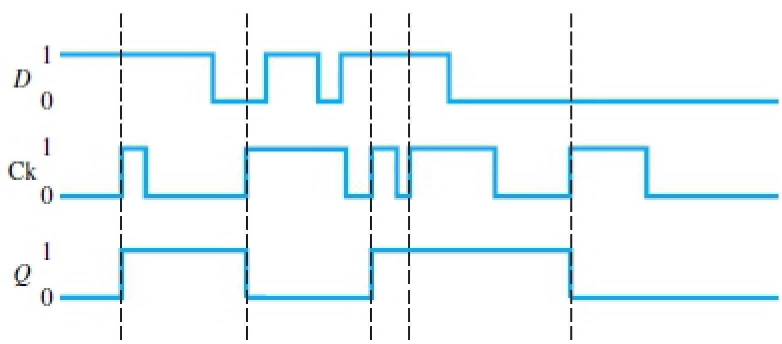
- **Propagation delay:** which means that there is a small delay (about 20 ns) between the change instruction and the time Q actually changes. The value of D that matters is its value when the change instruction is received, not its value at the later time when Q changes.

- **Timing diagram**: A timing diagram depicts inputs and outputs (as a function of time) of the flip-flop (or any other logic device) showing the transitions that occur over time.

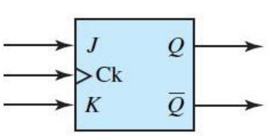
Ex 4.100: The positive-edge triggered D flip-flop is given the inputs shown in below fig. 4.110, with a zero initial value of *Q*. Draw the timing diagram.



Solution:



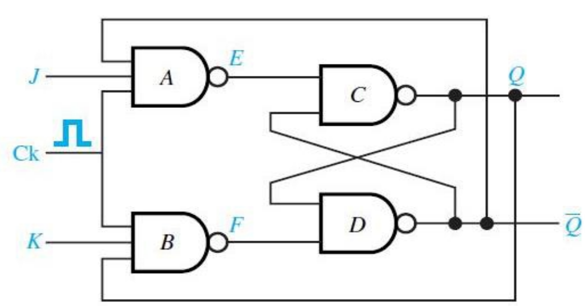
4.13.3 JK Flip-Flop (JKFF)



(a)

Inputs		Outputs	
J	K	Q_{n+1}	$\overline{Q}_{n+1}$
0	0	Q_n	$\overline{Q}_n$
0	1	0	1
1	0	1	0
1	1	$\overline{Q}_n$	Q_n

(b)



(c)

Fig. 4.110: (a) Block diagram (b) Truth table & (c) Circuit diagram for JK flip-flop

- The JKFF differs from an SRFF in that output Q is fed back to the K-gate input and, $\overline{Q}$ to the J-gate input. Assuming $Q\overline{Q}$ = 01, gate B is disabled by Q = 0 (i.e., F =1).
- The only way to make the circuit changeover is for gate A to be enabled by making J = 1 and $\overline{Q}$ = 1 (which it is already). Then when Ck = 1, all inputs to gate A are 1 and E goes to zero, which makes Q = 1. With Q and F both equal to 1, $\overline{Q}$ = 0, so the flip-flop has changed state.
- Note that the input condition JK = 11 is allowed, and in this condition, when the flip-flop is clocked, the output always changes state; thus it is said to *toggle*.
- If the clock pulse is short enough to permit the flip-flop to change only once, the JKFF operates well. However, with modern high-speed ICs a *race* is more likely to occur.

- **Race around Condition**: It is a condition, in which two pulses are intended to arrive at a destination gate in some specific order, but due to each one racing through different paths in the logic with a different number of gates, the propagation delays stack up differently and the timing order is lost.
- This can be eliminated by introducing ***delays in the feedback paths*** between outputs (Q and $\overline{Q}$) and inputs (J and K). A better solution to the problem is the master–slave JKFF.

➢ MASTER–SLAVE JK FLIP-FLOP

- Fig. 4.111 illustrates a master–slave JKFF, in which gates A, B, C, and D form the master flip-flop and T, U, V, and W form the slave.

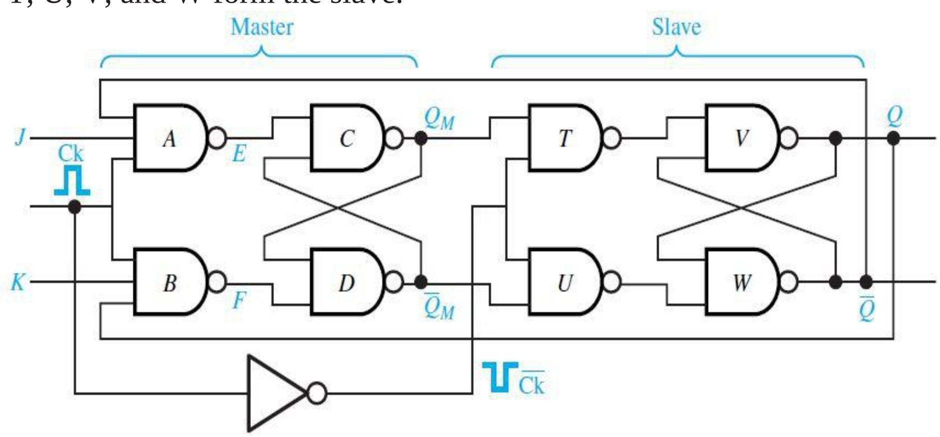


Fig. 4.111: Master–slave JK flip-flop

- The output of the master–slave JKFF can be predicted for all combinations of J and K and for any duration of clock pulse. Thus it is the most versatile and universal type of flip-flop. SRFFs are also available in master–slave configuration.

  - Ck = 1 Enables the Master & Disables the Slave.
  - Ck = 0 Disables the Master & Enables the Slave.

- Ck = 1 enables the master; $\overline{Ck}$ = 0 disables the slave. Let $Q\overline{Q}$ = 10 and JK = 11 before the occurrence of a clock pulse. B is enabled by Q = 1 so that when the clock pulse arrives (i.e., Ck = 1), F goes to zero and $\overline{Q_M}$ to 1. Now with E = 1 and $\overline{Q_M}$ = 1, QM goes to zero so that the master has been reset.
- However, the slave remains disabled until Ck goes to zero. Note that the slave, at this stage, is essentially an SRFF with inputs S and R equal to Q_M and $\overline{Q_M}$, respectively. Thus, when Ck goes to zero, $\overline{Ck}$ goes to 1 and the slave is now reset by its inputs Q_M $\overline{Q_M}$= 01. But the feedback to J and K cannot cause a race because, with Ck = 0, the master is disabled.
- Using this configuration race around condition will be not taking place because at any clock duration only one (master or slave) drives output.

4.14 SHIFT REGISTERS:

- A register (Shift register) is a collection of flip flops (and some basic combinational gates to perform different binary arithmetic and logic operations), where each flip-flop is used to store 1 bit of information.
- JKFFs and SRFFs are also used in shift-register construction.

4.14.1 Types of Shift Registers

- Fig. 4.112 shows the block diagram of a 4-bit shift-right register that uses D flip-flops. JKFFs and SRFFs are also used in shift-register construction.
- Observe in the timing diagram, each successive clock pulse transfers (or shifts) the data bit from one flip-flop to next one to right and appear at the output in serial form. The shift register is then known as a **serial-in serial-out (SISO) shift register.**

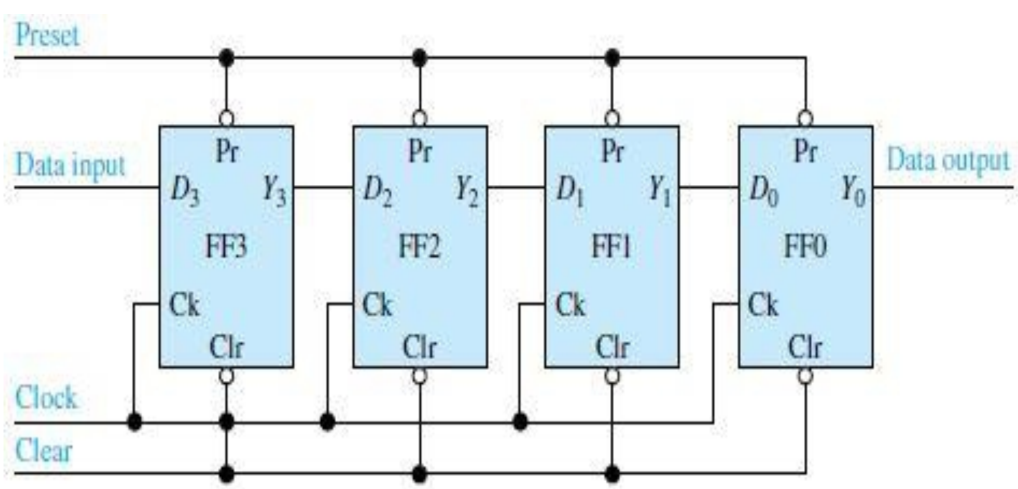


Fig. 4.112: Block diagram of a 4-bit SISO shift-right register

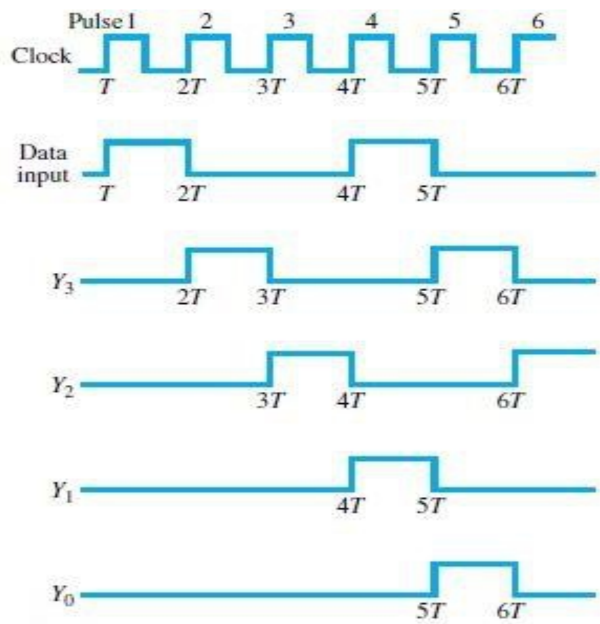


Fig. 4.113: Timing diagram

- **PISO (parallel-in serial-out), SIPO (serial-in parallel-out), and PIPO (parallel-in parallel-out) shift registers** are also often used to read in the input data and read out the output data in a convenient way that is needed for the operations involved.

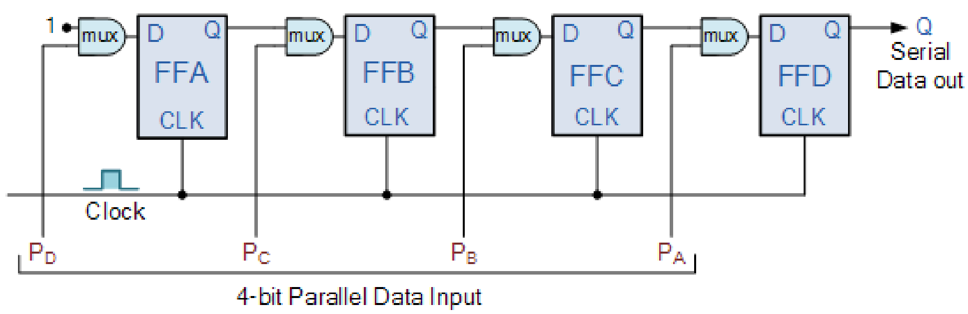


Fig. 4.114: Parallel in Serial out (PISO) shift register

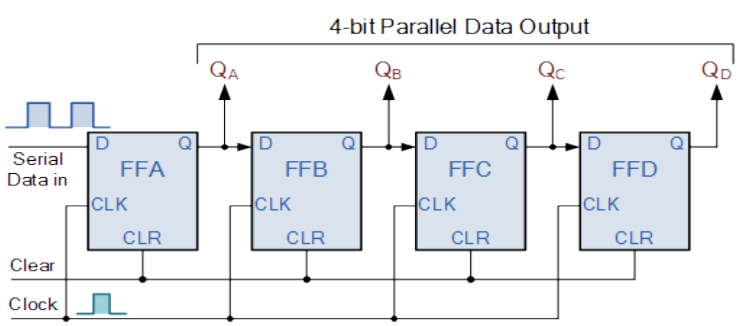


Fig. 4.115: Serial in parallel out (SIPO) shift register

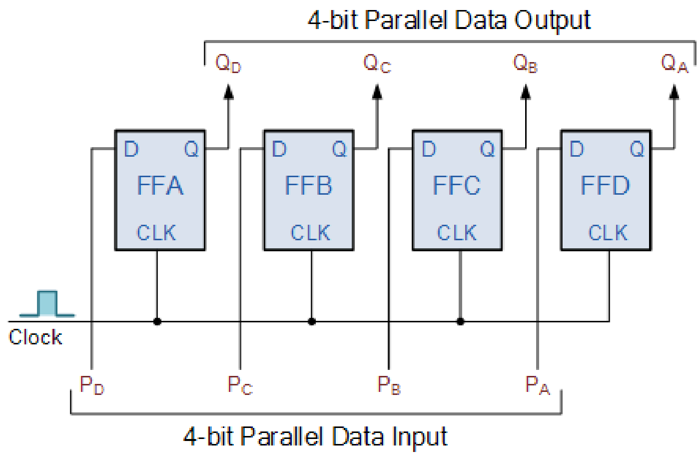


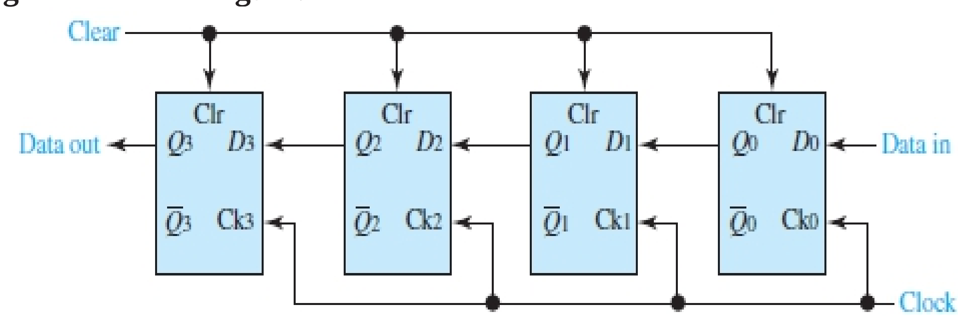
Fig. 4.116: Parallel-in parallel-out (PIPO) shift registers

- Right-shifting registers are employed in multiplication algorithms, whereas left-shifting registers are utilized in division algorithms.

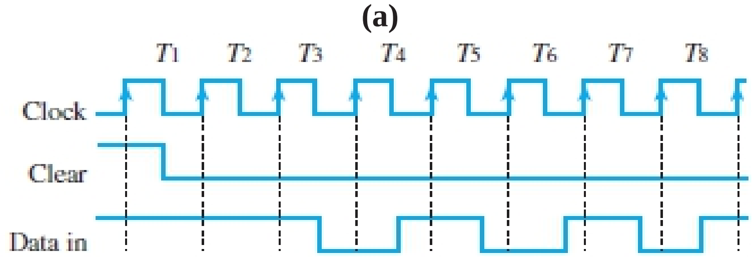
- **Bidirectional shift registers: -** Registers that are capable of shifting the data to the left or right are known as bidirectional shift registers.

- **Universal register: -** The register along with additional gates on a single chip forms an IC component known as the universal register, which usually includes the shift-left, shift-right, parallel-input, and no-change operations.

Ex 4.101: Given the block diagram for a 4-bit shift-left register shown in below Figure, draw the output (Q_0, Q_1, Q_2, Q_3, and data out) as a function of time for the clock, clear, and data-in signals given in below Figure.

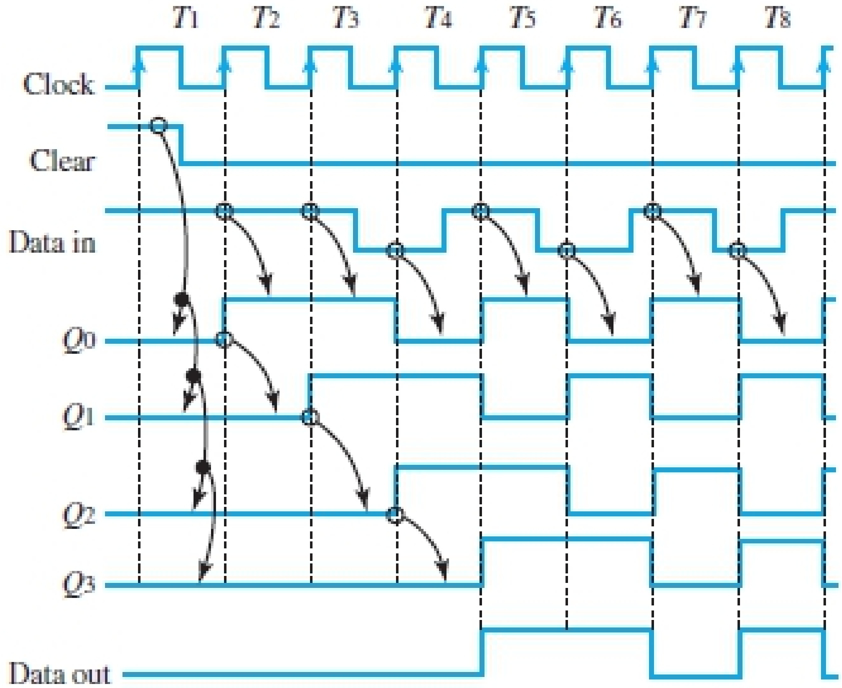


(a)



(b)

Solution:



4.15 COUNTERS:

- The shift register can be used as a counter because the data are shifted for each clock pulse. A counter is a register that goes through a predetermined sequence of states when input pulses are received. Besides, computers, timers, frequency meters, and various other digital devices contain counters for counting events. There are mainly types discussed below.

4.15.1 Asynchronous/Ripple counters

- Output of each flip-flop activates the next flip-flop throughout the entire sequence of the counter's states.

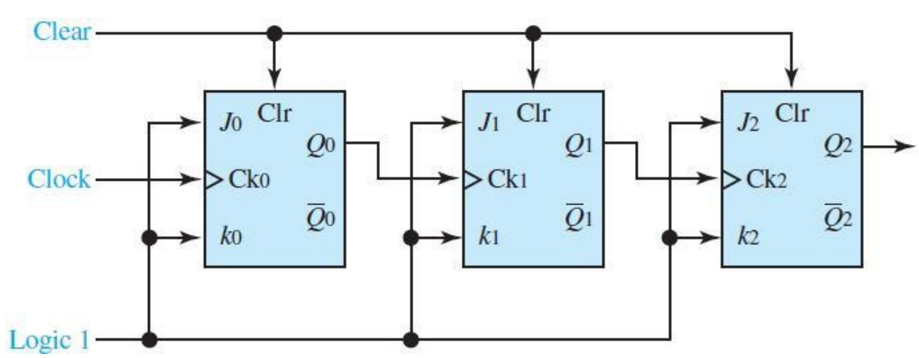
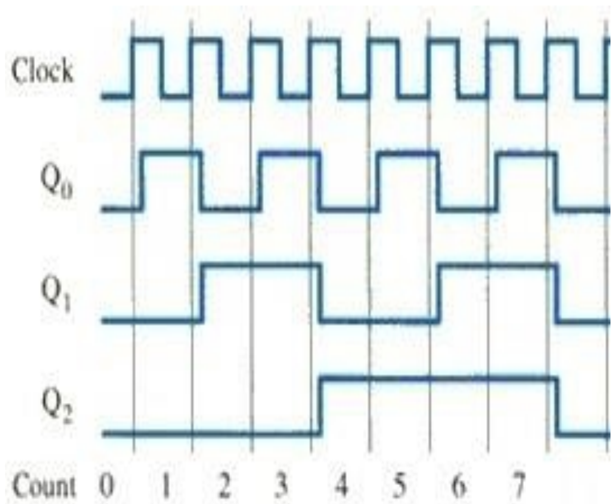


Fig. 4.117: Block diagram of a 3-bit Asynchronous /ripple counter using JKFFs



Clock	Q_2	Q_1	Q_0
	0	0	0
T_1	0	0	1
T_2	0	1	0
T_3	0	1	1
T_4	1	0	0
T_5	1	0	1
T_6	1	1	0
T_7	1	1	1
T_8	0	0	0

Fig. 4.118: Timing Diagram & Output for 8 clock pulses

- Notice from the timing diagram that the output Q_0 of the leftmost flip-flop will change its state at every clock pulse if the clear signal equals zero.
- The output Q_1, controlled by Q_0, will change its state every time Q_0 changes from 0 to 1. Similarly Q_2 is controlled by Q_1.
- Outputs for the first 8 clock pulses shown in fig. 4.118 Observe that a 3-bit counter will cycle through 8 states, 000 through 111. An n-bit ripple counter, in general, will cycle through 2^n states; it is known as a **divide-by-2^n counter** or **modulo-2^n binary counter**.
- Taking the outputs from Q_2 Q_1 Q_0, the counter becomes an **up-counter**; taking the outputs from $\bar{Q}_2$ $\bar{Q}_1$ $\bar{Q}_0$ the counter becomes a **down-counter**, which counts down from a preset number.
- The slow speed of operation, caused by the long time required for changes in state to ripple through the flip-flops, is a disadvantage of ripple counters.

4.15.2 Synchronous/Parallel counter

- In **Synchronous Counter**, the external clock signal is connected to the clock input of each individual flip-flop within the Counter so that all of the flip-flops are clocked together simultaneously and in parallel at the same time giving a fixed time relationship. In other words, changes in the output occur in "synchronisation" with the **clock signal**.
- The result of this synchronisation is that all the individual output bits changing state at exactly the same time in response to the common clock signal with no ripple effect and therefore, no propagation delay.
- Slow speed of operation problem is overcome by using synchronous converters. However, additional control logic is needed to determine which flip-flops, if any, must change state, since flip-flops are triggered simultaneously.

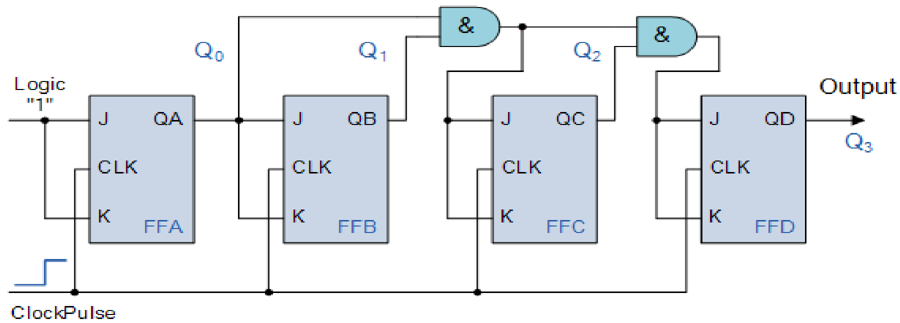


Fig. 4.119: 4-bit binary synchronous counter

- It can be seen that the external clock pulses (pulses to be counted) are fed directly to each J-K flip-flop in the counter chain and that both the J and K inputs are all tied together in toggle mode, but only in the first flip-flop, flip-flop A (LSB) are they connected HIGH, logic "1″ allowing the flip-flop to toggle on every clock pulse. Then the synchronous counter follows a predetermined sequence of states in response to the common clock signal, advancing one state for each pulse.
- The J and K inputs of flip-flop B are connected to the output "Q" of flip-flop A, but the J and K inputs of flip-flops C and D are driven from AND gates which are also supplied with signals from the input and output of the previous stage.
- If we enable each J-K flip-flop to toggle based on whether or not all preceding flip-flop outputs (Q) are "HIGH" we can obtain the same counting sequence as with the asynchronous circuit but without the ripple effect, since each flip-flop in this circuit will be clocked at exactly the same time.
- Then as there is no inherent propagation delay in synchronous counters, because all the counter stages are triggered in parallel at the same time, the maximum operating frequency of this type of frequency counter is much higher than that for a similar asynchronous counter circuit.

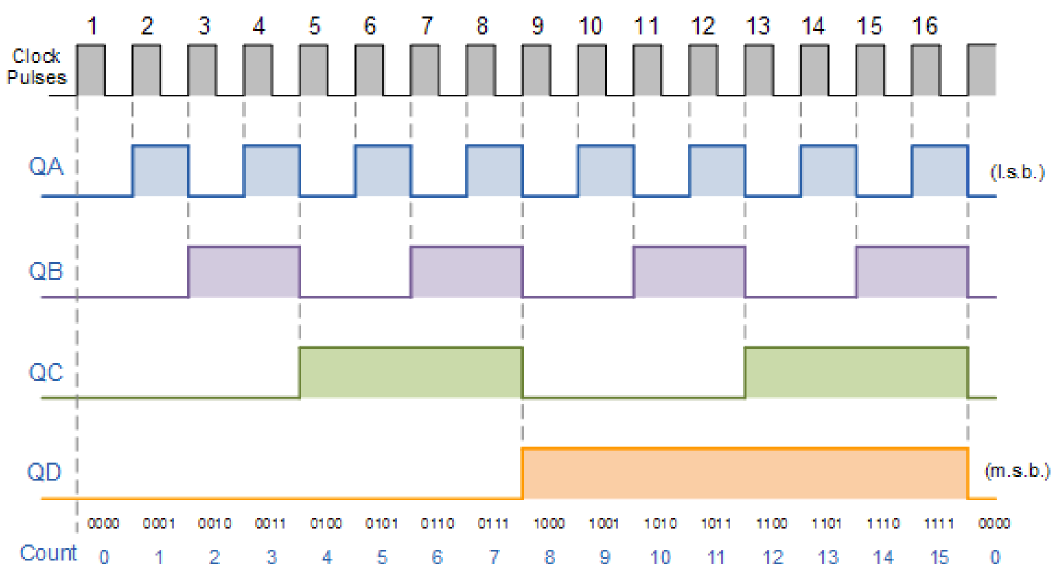


Fig. 4.120: Timing Diagram of 4-bit binary synchronous counter

- Because this 4-bit synchronous counter counts sequentially on every clock pulse the resulting outputs count upwards from 0 ("0000") to 15 ("1111"). Therefore, this type of counter is also known as a 4-bit Synchronous Up Counter.

4.15.3 Ring counter

- As in a synchronous counter, all flip-flops are triggered simultaneously. However, the output of each flip-flop drives only an adjacent flip-flop. A single pulse propagates through the ring in a ring counter, whereas all remaining flip-flops are at the zero state.

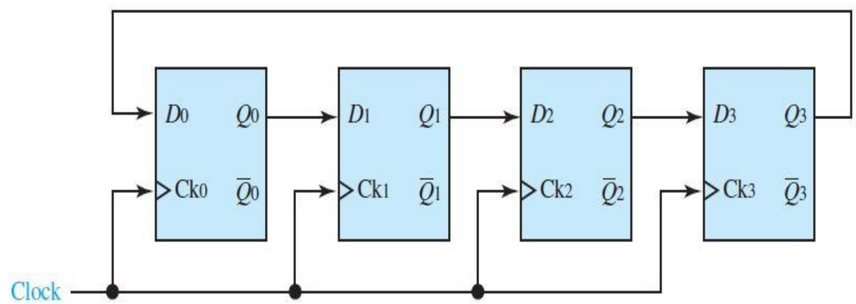


Fig. 4.121: 4-bit (modulo-4) ring counter

- Some counters are also programmable. Fig. 4.121 shows a 4-bit (modulo-4) ring counter using D flip-flops; its timing diagram is given in Fig. 4.122.

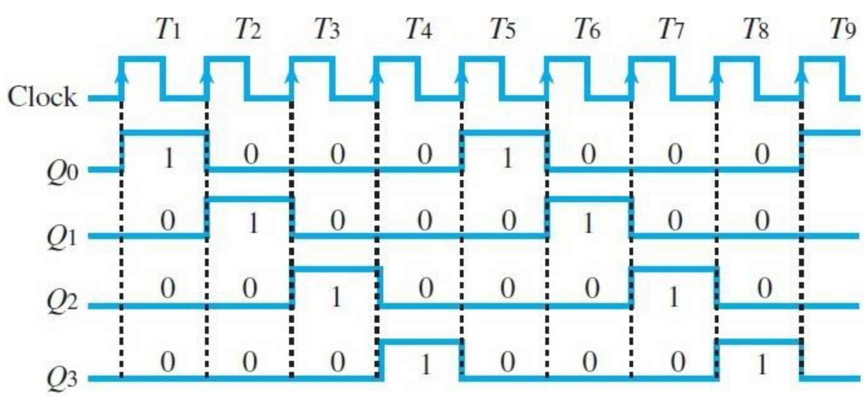
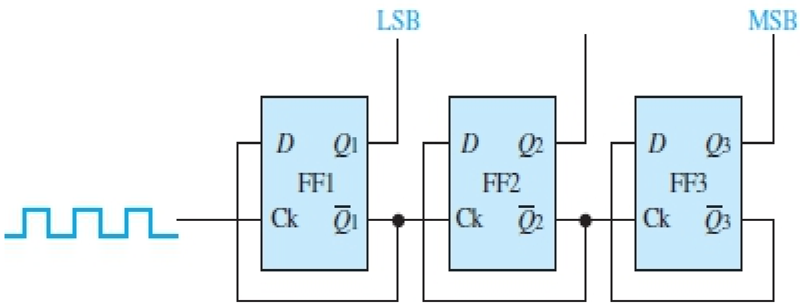


Fig. 4.122: Timing diagram 4-bit (modulo-4) ring counter

- A modulo-*n* ring counter requires *N* flip-flops and no other gates, whereas modulo-*N* ripple and synchronous counters need only $\log_2 N$ flip-flops. However, ripple and synchronous counters generally use more components than ring counters.

Ex 101: The block diagram for a 3-bit ripple counter is shown in Figure. Obtain a *state table* for the number of pulses *N* = 0 to 8, and draw a *state diagram* to explain its operation.



(a)

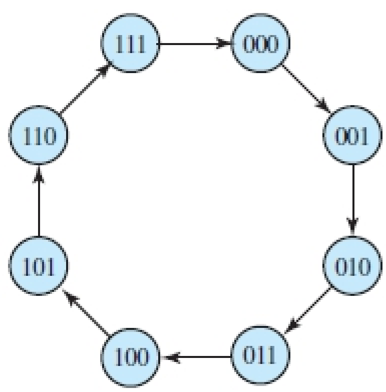
Solution:

- **State table:**

N	Q_{1N}	Q_{2N}	Q_{3N}	$(Q_3Q_2Q_1)_N$
0	0	0	0	000
1	1	0	0	001
2	0	1	0	010
3	1	1	0	011
4	0	0	1	100
5	1	0	1	101
6	0	1	1	110
7	1	1	1	111
8	0	0	0	000

(b)

- **State diagram:**



(c)

The state table and the state diagram are given in Figures (b) and (c). The horizontal arrows indicate the times when clock inputs are applied to FF2 and FF3. These times are located by noting that every time Q_1 makes a transition from 1 to 0, FF2 is clocked, and when Q_2 goes from 1 to 0, FF3 is clocked. In the state diagram, the eight states of the system are indicated by the values of the three-digit binary number Q_3 Q_2 Q_1.

4.16 DIGITAL-TO-ANALOG & ANALOG-TO-DIGITAL CONVERTER:

- For the results of digital computations to be used in the analog world, it becomes necessary to convert the digital values to proportional analog values.
- Unlike analog signals, digital data can be transmitted, manipulated, and stored without degradation. But a DAC is needed to convert the digital signal to analog to drive an earphone or loudspeaker amplifier in order to produce sound (analog air pressure waves).

4.16.1 Digital-to-analog (D/A) converters

- Fig. 4.123 shows the block diagram of a typical digital-to-analog (D/A) converter, which accepts an *n*-bit parallel digital code as an input and provides an analog current or voltage as an output.
- For an ideal D/A converter, the analog output for an *n*-bit binary code is given by

$$V_o = -V_{ref}\,(b_0 + b_1 \times 2^{-1} + b2 \times 2^{-2} + \cdots + b_{n-1} \times 2^{-n+1})$$

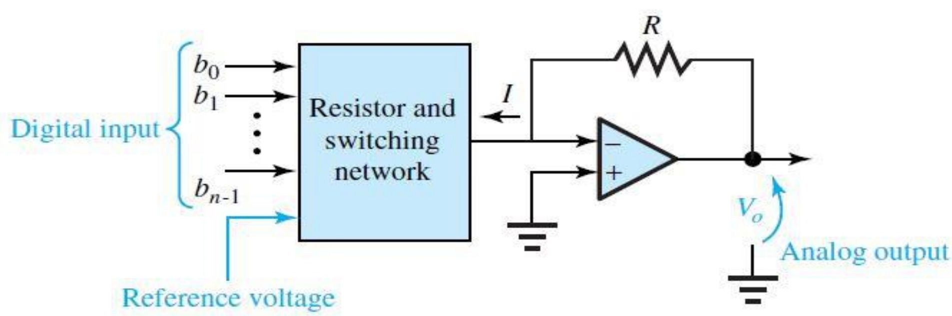


Fig. 4.123: Block diagram of D/A converter

Where;

V_0 - analog output voltage
V_{ref} - reference analog input voltage
b_0 - most significant bit of binary input code
b_{n-1} - least significant bit of binary input code

- In order to provide current-to-voltage conversion and/or buffering, an op amp is used at the output. However, in some high-speed applications where a limited output voltage range is acceptable, a resistor, instead of an op amp, is used for the current-to-voltage conversion, thereby eliminating the delay associated with the op amp.

1. Weighted-resistor D/A converter

- Fig. 4.124 shows a 4-bit weighted-resistor D/A converter which includes a reference voltage source, a set of four electronically controlled switches, a set of four binary-weighted precision resistors, and an Op-Amp.

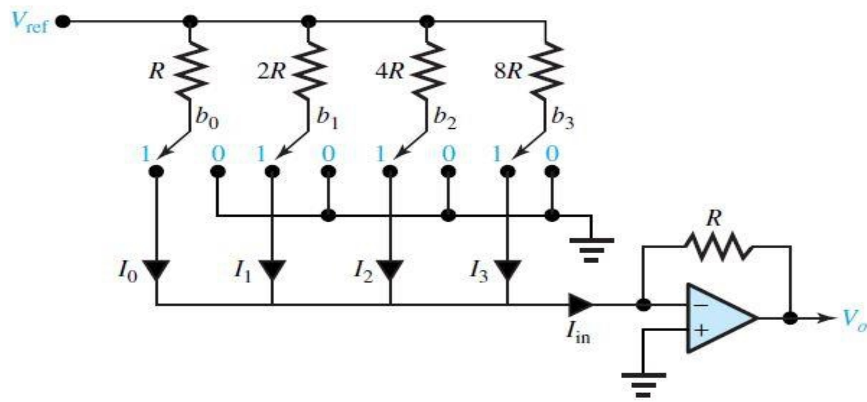


Fig. 4.124: Circuit diagram of weighted resistor D/A converter

- Each binary bit of digital input code controls its own switch. The switch closes with a bit value of 1, and the switch stays open with binary 0. The resistor connected to the most significant bit (MSB), *b*0, has a value of *R*; *b*1 is connected to 2*R*, *b*2 to 4*R*, and *b*3 to 8*R*.
- Thus, each low-order bit is connected to a resistor that is higher by a factor of 2. For a 4-bit D/A converter, the binary input range is from 0000 to 1111.
- An important design parameter of a D/A converter is the *resolution*, which is the smallest output voltage change, ΔV, which for an *n*-bit D/A converter is given by

$$\Delta V = \frac{V_{ref}}{2^{n} - 1}$$

- The range of resistor values becomes impractical for binary words longer than 4 bits. Also, the dynamic range of the op amp limits the selection of resistance values.
- To overcome these limitations, the *R*–2*R* ladder D/A converter is developed.

2. R–2R Ladder D/A Converter

- Fig. 4.125 shows a 4-bit *R*–2*R* ladder D/A converter, which contains a reference voltage source, a set of four switches, two resistors per bit, and an op amp.

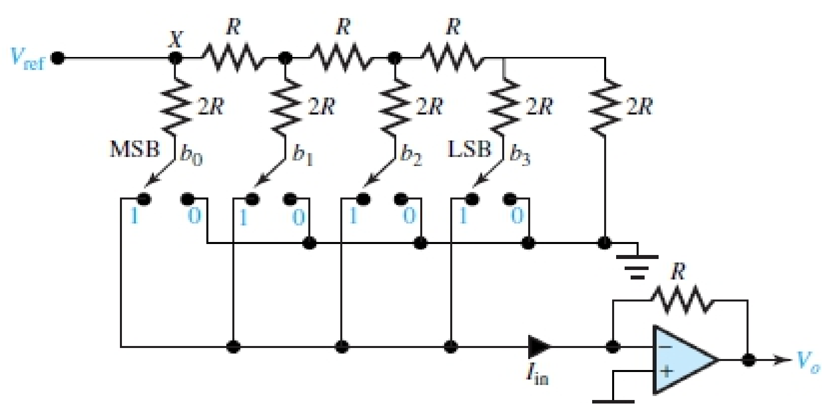


Fig. 4.125: Circuit diagram of 4-bit *R*–2*R* ladder D/A converter

- The analog output voltage can be shown to be because only two resistor values (*R* and 2*R*) are used, the *R*–2*R* ladder converter networks are relatively simple to manufacture, fast, practical, and reliable.
- The commercially available AD558, which is an 8-bit *R*-2*R* D/A converter, is an example.

3. 2^n - R D/A CONVERTER

- An *n*-bit 2^n –*R* D/A converter needs 2^n resistors of equal value *R* and (2^{n+1} −2) analog switches.
- A 3-bit 2^n −*R* D/A converter is shown in Fig. 4.126, which includes the eight resistors connected in series to form a voltage divider providing eight analog voltage levels, as well as 14 analog switches controlled by the digital input code such that each code creates a single path from the voltage divider to the converter output.

$$V_o = -V_{ref}\,(b_0 \times 2^{-1} + b_1 \times 2^{-2} + \cdots + b_{n-1} \times 2^{-n})$$

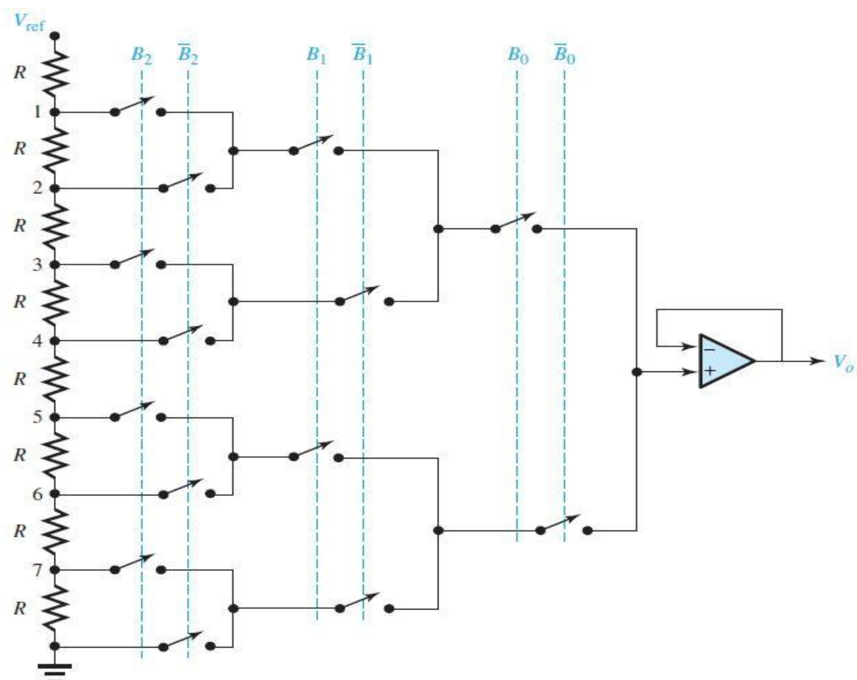


Fig. 4.126: Circuit diagram of 4-bit 2^n– *R* D/A converter

- A unit-gain amplifier is connected to the output in order to prevent loading of the voltage divider.
- 2^n − *R* D/A converters are economically manufactured as LSI packages in spite of the large number of components needed.

Ex 102: For the 4-bit D/A converter of Fig. 4.124 with *V*ref = −5 V, determine the range of analog output voltage and the smallest increment.

Solution:

$V_o = -V_{ref} (b_0 + b_1 \times 2^{-1} + b2 \times 2^{-2} + \cdot\cdot\cdot + b_{n-1} \times 2^{-n+1})$
So; $5 \times (1 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}) = 5 \times 1.875 = 9.375$ V

The smallest increment is given by $5 \times 1 \times 2^{-3} = 5/8 = 0.625$ V

Ex 103: For the 3-bit 2^n–*R* D/A converter of Fig. 4.126, calculate the analog output voltage when the input is (a) 100, and (b) 010.

Solution:

(a) For the binary input 100, switches controlled by *B*0, .*B*1, and .*B*2 will be closed. A path is then produced between the output *Vo* and point 4, where the voltage is equal to *V*ref /2. The analog output voltage is therefore *V*ref*/*2.
(b) For the binary input 010, switches controlled by *B*0, *B*1, and .*B*2 will be closed. A path exists between *Vo* and point 6, where the voltage is equal to *V*ref*/*4. The analog output voltage is thus *V*ref*/*4.

4.16.2 ANALOG-TO- DIGITAL (D/A) CONVERTERS

- The A/D converter converts analog input signals into digital output data in many areas such process control, aircraft control, and telemetry. Being the interface between analog systems and digital systems, it plays a key role in many industrial, commercial, and military systems.
- Several types of A/D converters exist: counter-controlled, successive-approximation, and dual-ramp (dual-slope) converters.
- The commercially available LM311 is an example that is widely used by designers.

1. Counter-Controlled A/D Converter

- Resetting the binary counter to zero produces D/A output voltage $V_2 = 0$ and initiates the analog-to-digital conversion.
- When the analog input V_1 is larger than the DAC (D/A converter) output voltage, the comparator output will be high, thereby enabling the AND gate and incrementing the counter.
- V_2 is increased as the counter gets incremented; when V_2 is slightly greater than the analog input signal, the comparator signal becomes low, thereby causing the AND gate to stop the counter. The counter output at this point becomes the digital representation of the analog input signal.
- The relatively long conversion time needed to encode the analog input signal is the major disadvantage of this method.

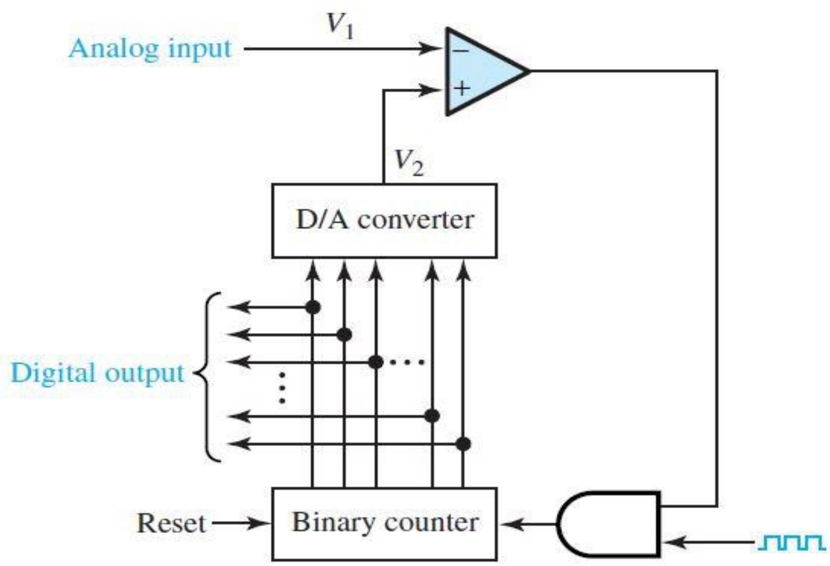


Fig. 4.127: Circuit diagram of counter-controlled A/D converter

2. Successive-Approximation A/D Converter

- This converter, shown in Fig. 4.128, also contains a D/A converter, but the binary counter is replaced by a successive-approximation register (SAR), which makes the analog-to-digital conversion much faster.

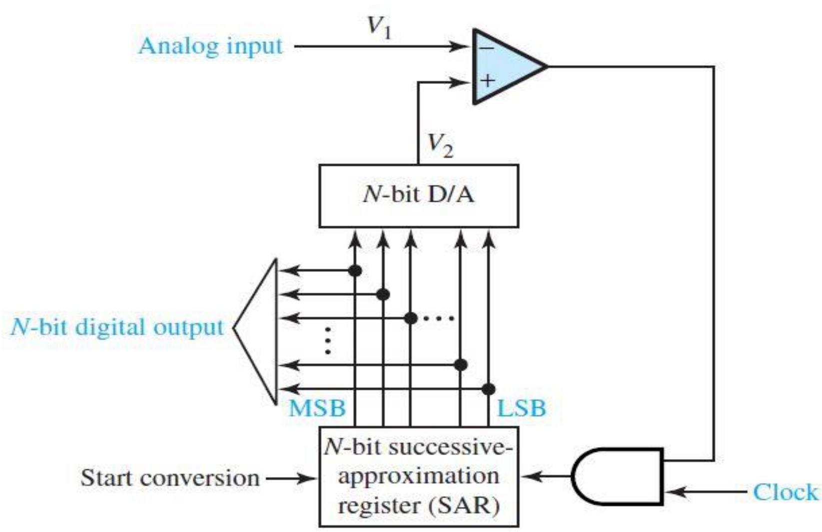


Fig. 4.128: Circuit diagram of successive-approximation A/D converter

- The SAR sets the MSB to 1 and all other bits to 0, after a start-of conversion pulse. If the comparator indicates the D/A converter output to be larger than the signal to be converted, then the MSB is reset to 0 and the next bit is tried as the MSB.

- On the other hand, if the signal to be converted is larger than the D/A computer output, then the MSB remains 1. This procedure is repeated for each bit until the binary equivalent of the input analog signal is obtained at the end.
- This method requires only n clock periods, compared to the 2n clock periods needed by the counter-controlled A/D converter, where n is the number of bits required to encode the analog signal.
- The National ADC 0844 is a popular 8-bit A/D converter based on the SAR.

3. Dual-Ramp (Dual-Slope) A/D Converter

- After a start-of-conversion pulse, the counter is cleared and the analog input V_{in} becomes the input of the ramp generator (integrator). When the output of the ramp generator V_o reaches zero, the counter starts to count.

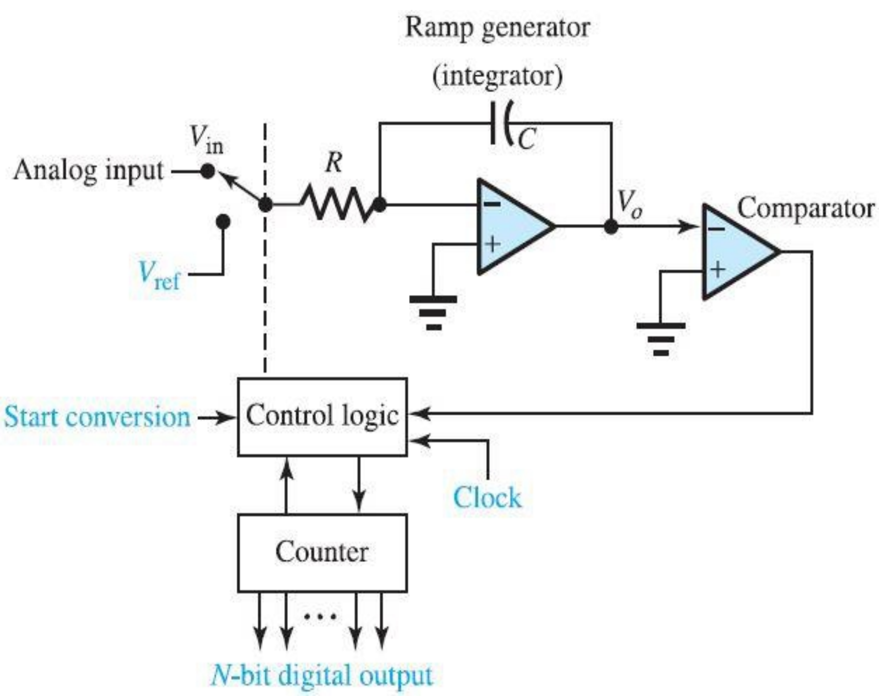


Fig. 4.129: Circuit diagram of dual-ramp A/D converter

- After a fixed amount of time T, as shown in Fig. 4.130, the output of the ramp generator is proportional to the analog input signal. At the end of T, the reference voltage V_{ref} is selected, when the integrator gives out a ramp with a positive slope. As V_o increases, the counter is incremented until Vo reaches the comparator threshold voltage of 0 V, when the counter stops being incremented again. The value of the counter becomes the binary code for the analog voltage V_{in}, since the number of clock pulses passing through the control logic gate for a time t is proportional to the analog signal Vin. Dual-ramp A/D converters can provide accuracy at low cost, even though the process is slow because a double clock pulse count is an inherent part of the process.

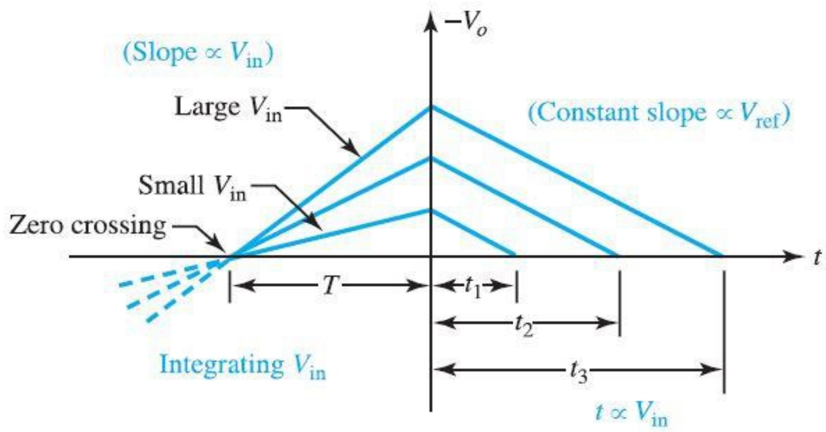


Fig. 4.130: Output of ramp generator in dual-ramp A/D converter.

Ex 104: The speed of an 8-bit A/D converter is limited by the counter, which has a maximum speed of 40 × 10^6 counts per second. Estimate the maximum number of A/D conversions per second that can be achieved.

Solution:
The rate of the clock will be constant, independent of the analog input. It must be slow enough to allow the counter to count up to the highest possible input voltage. This will require 255$(= 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0)$ counts, which will take $255 / (40 \times 10^6) = 6.375\ \mu s$.

Thus, the process can be repeated $10^6/6.375$ = 156,863 times per second.

4.17 MEMORY:

- For a digital computer which stores both programs and data, memory can be divided into three types:
  1. Random-access memory
  2. Mass storage, and
  3. Archival storage.

1. **Random-access memory**
- It includes read-and-write memory (RAM), read-only memory (ROM), programmable read-only memory (PROM), and erasable programmable read-only memory (EPROM), in which any memory location can be accessed in about the same time, the time required to access data in a mass-storage device is relative to its location in the device.

2. **Mass storage**
- Such as magnetic disk memory, has a relatively large storage capacity and is lower in cost per bit than random access memory.

3. **Archival storage**

- Such as magnetic tape, is long-term storage with a very large capacity, but with a very slow access time, and may need user intervention for access by the system.

4.17.1 Read-and-write memory (RAM)

- Writing is the same as storing data into memory and reading is the same as retrieving the data later.
- RAM is said to be volatile because its contents are retained only as long as power is present.
- A RAM device is a collection of 2^n addressable storage locations, each of which contains k bits.

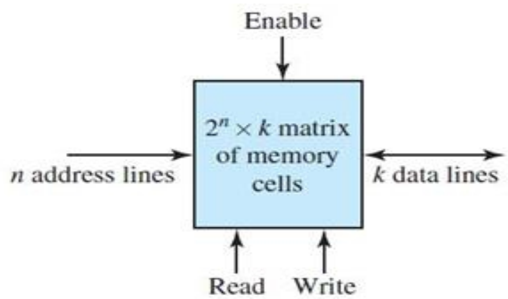


Fig. 4.131: Block diagram of 2^n X k bit RAM

- Its block diagram is shown in Fig. 4.131, in which each cell may be a flip-flop or a capacitor, and n address lines are decoded to select k cells.
- A static RAM, in which each cell is a flip-flop, is the read-and-write memory that retains its data so long as the power is applied, without any further action needed from the computer. Static RAM is used in microprocessor-based systems requiring small memory. Common static-RAM sizes are 2K × 8, 8K × 8, and 32K × 8, where K stands for $2^{10} = 1024$.
- A dynamic RAM, in which each cell is a capacitor (which leaks charges and therefore requires continuous refreshing from the computer to maintain its value), is the read-and-write memory that is used in large memory systems due to its lower cost and greater density. Common dynamic-RAM packages are available in 16K-bit, 64K-bit, 256K-bit, and 1M-bit (where M stands for mega = 2^{20}) sizes.

4.17.2 Read-only memory (ROM)

- ROM is nonvolatile (because it maintains its contents even when its power is shut off) and is used to store data and programs that do not change during the operation of the system.

1. **The mask-programmed ROM**

- It is read-only devices that are programmed for data storage during the manufacturing of the chip itself. These are generally less expensive devices for mass production. Character-font memory for laser printers is a good example.

2. Programmable read-only memory (PROM)

- It is a field-programmable memory that is fabricated by the manufacturer containing all 0s and is programmed irreversibly by the user by electrically changing appropriate 0s to 1s. PROMs are quite economical in small quantities.

3. Erasable programmable read-only memory (EPROM)

- It is nonvolatile and widely used in microprocessor systems for program storage. It can be erased by shining an ultraviolet light and reprogrammed if necessary. These are produced in low to moderate volumes.

4. Electrically erasable programmable ROM (EEPROM)

- It is used for remote-area applications. The device is provided with special pins which, when activated electrically, alter the rewriting of selected memory locations.

➢ **Difference between ROM, EPROM & EEPROM**

ROM/PROM	EPROM	EEPROM/E^{2}PROM
One time programmable so erasing is not possible	Technique used for erasing is UV light	A voltage of 20 V to 25 V is applied to erasing
Erasing is not possible	Selective erasing is not possible. All the locations get erased	Selective erasing is possible. A particular locations can be erased
Erasing is not possible	Erasing can be done in 10 to 15 min.	Erasing can be done in 10 ms
Less expensive	Moderate cost	More expensive

4.17.3 Magnetic storage devices

- ***Magnetic disk memory*** is nonvolatile and provides large storage capabilities with moderate access times.
- The data are stored on one or more rigid aluminum circular disks coated with iron oxide.
- The most common disks have 11-in diameters and 200 tracks (concentric rings of data) per surface, numbered from 0 to 199, starting with the outside perimeter of the disk, with a typical track packing density of 4000 bits per inch.
- Disks are mounted on a common spindle, and all disks rotate at a typical speed of 3600 revolutions per minute (rpm).
- A typical disk has 17 sectors of fixed size per track and 512 bytes (1 byte = 8 bits) of information per sector. Any desired sector can be quickly accessed.

1. Floppy disks

- Floppy disks, also known as flexible disks, are the low-cost, medium-capacity, nonvolatile memory devices made of soft flexible mylar plastic with magnetically sensitive iron-oxide coating.
- The original 8-in standard floppy is no longer in popular use. The $5\frac{1}{4}$ - in mini floppy has a disk and a disk jacket covering the mylar media for protection, along with a write-protect notch and index hole.
- The present-day mini floppy disks are either double-sided/double-density (DS/DD) with 9 sectors per track or 40 tracks per side or double-sided/quad-density (DS/QD) with 9 or 15 sectors per track and 80 tracks per side.
- In a DS/DD mini floppy disk, about 720 kbytes of data can be stored; whereas in a DS/QD mini-floppy disk, about 1–2 Mbytes of data can be stored. The $3\frac{1}{2}$-in microfloppy disk, also known as a microdiskette, is enclosed in a rigid protective case and is provided with a write-protect notch.
- Micro diskettes are recorded in quad-density format with a capacity of 2 Mbytes; 4- to 16-Mbyte $3\frac{1}{2}$-in diskettes are being developed. Also, 2-in diskettes are introduced in electronic cameras and portable personal computers.

2. Magnetic tapes

- Magnetic tapes are ideal devices for storing vast quantities of information inexpensively. However, the access time is slow because the entire tape must be read sequentially.
- The most commonly used tapes are ½-in wide, 2400 or 3600 ft long, and contained in a long 10½-in reel. Tape densities of 200, 556, 800, 1600, 6250, and 12,500 bits per inch (BPI) are standard.
- In addition to these magnetic storage devices, two newer types of secondary storage have come into use: Winchester disks and videodisks (also known as optical disks).
- The former are sealed modules that contain both the disk and the read/write mechanism, requiring little maintenance and allowing higher-density recording. The latter have been introduced recently, with high reliability and durability and a storage capacity of 1 Gbyte of data (equivalent to almost 400,000 typewritten pages of information).
- A typical 14-in optical disk has 40,000 tracks and 25 sectors per track, with each sector holding up to 1 Kbyte of information. While a write-once optical-disk drive is currently available, a read-and-write drive is being developed.

4.18 DISPLAY DEVICES:

- Display devices can be categorized as on/off indicators, numeric, alphanumeric, or graphical displays. They may also be classified as active and passive devices.
- Active display devices emit light, such as light-emitting diodes (LEDs), whereas passive display devices, such as liquid-crystal displays (LCDs), reflect or absorb light.

1. Light-emitting diode (LED)

- This is a reliable, rugged, and inexpensive semiconductor display device requiring about 10 mA of current flow for full illumination. An LED is shown in Fig. 4.132.

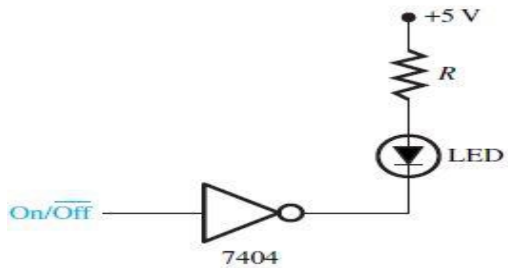


Fig. 4.132: Light emitting diode (LED)

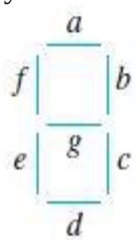
- It is available in red, yellow, or green color; Two-color LEDs are also available in common size.
-

2. Liquid-crystal display (LCD)

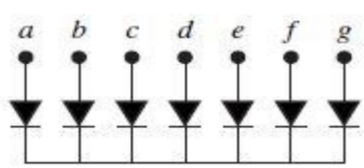
- This display needs only microwatts of power (over a thousand times less than a LED) and is used in such devices as electronic wrist watches.
- With the application of an electric field the molecules of the liquid-crystal material are straightened out, absorbing the light and the display appears black.
- With no electric field applied, the display appears as a silver mirror because the light is reflected.

3. Segment displays

- Seven-segment displays are the most commonly used numeric display devices, while 10- and 16-segment display devices are alphanumeric which are used to display number and character.



(a)



(b)

Fig. 4.133: (a) Seven segments (b) Shows its internal structure, consisting of a single LED for each of the segments

(a)

(b)

Fig. 4.134: (a) Displays the digits by an appropriate combination of lighted segments (b) The 16-segment display commonly used for alphanumeric data

4. Cathode-ray tube (CRT)

- While the CRT display is one of the oldest, it still remains one of the most popular display Technologies. The raster-scanned display devices work similarly to commercial television sets, whereas graphics display devices use different technologies to achieve extremely high resolution.
- A CRT video signal has only two levels, with a 0 level causing a dark spot and a 1 level causing a bright spot. The appropriate combination of 1s and 0s displays data on a CRT screen, with each Character displayed by dot-matrix displays, typically in 5 × 7 or 7 × 9 display fonts.

4.19 COMPUTER SYSTEMS:

- Digital computers, in general, are automatic machines that accept data and instructions, perform predefined operations very quickly on the data, and have the results available to the user in various forms. They can be classified as microcomputers, minicomputers, mainframes, and supercomputers.

1. Microcomputers:

- Microcomputer has become a common part of everyday life. Today's 16- and 32-bit microcomputer systems are also dedicated for real-time applications in a distributed system. A microprocessor is an LSI device, which is a realization of the computer central-processor unit (CPU) in IC form. The microprocessor is the CPU of the microcomputer system.

2. Minicomputers:

- Minicomputer developed in the early 1960s, are high-performance, general-purpose multiuser Computers. These are also designed for real-time dedicated applications. The PDP-11 series from Digital Equipment Corporation (DEC) have been the most prominent 16-bit minicomputers, and are now obsolete.
- The 32-bit minicomputers, known as superminis, were developed in the 1970s, the most prominent one being the VAX 8600 from DEC, which was capable of executing about 5 million instructions per second (MIPS). New VAX lines with larger MIPS have been developed since.

3. Mainframes

- Mainframes capable of executing in excess of 53 MIPS, are high-performance, general-purpose Computers supporting very large databases, ranging in price from one to ten million dollars.
- These are used by many universities, large businesses, and government agencies, and are supplied mainly by IBM. Examples include IBM 360, CDC 7600 of Control Data Corporation, and Texas Instrument Advanced Scientific Computer (TI–ASC).

4. **Supercomputers:**

- Supercomputers capable of executing in excess of one billion floating-point operations per second (FLOPS), are very powerful, extremely high-performance computers for applications that are beyond the reach of the mainframes, and cost more than ten million dollars.
- These are used for weather prediction, image processing, and nuclear-energy studies that require high-precision processing of ordered data achieved by a speed advantage due to parallel processors. Cyber 205, Cray X-MP, and Cray 2 are some examples of supercomputers. In the 1980s, supercomputing centers were developed at six American universities for high performance computing.
- By 1990 it was possible to build chips with a million components; semiconductor memories became standard in all computers; widespread use of computer networks and workstations had occurred. Explosive growth of wide area networking took place with transmission rates of 45 million bits per second.

4.19.1 Organization / Architecture of computer system & Introduction to microprocessor

- There are two principal components: hardware and software.
- The hardware refers as physical components such as memory unit (MU), arithmetic and logic unit (ALU), control unit (CU), input/output (I/O) devices, etc.
- Software refers as the programs (collections of ordered instructions) that direct the hardware operations.
- Microprocessors are classified by word size in bits, such as 1-bit, 4-bit, 8-bit, and 16-bit microprocessors; generally speaking, the larger the word size, the more powerful the processor.

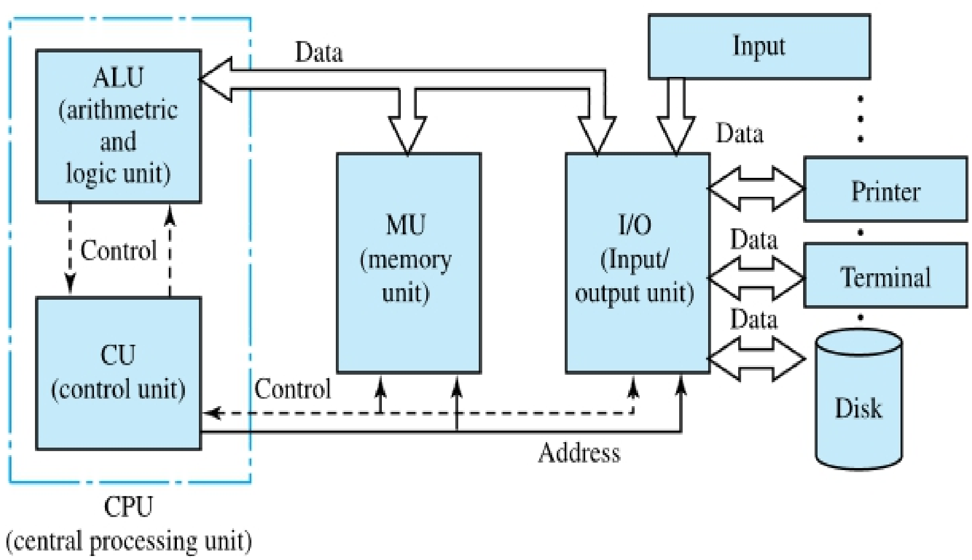


Fig. 4.135: Block diagram of Organization / Architecture

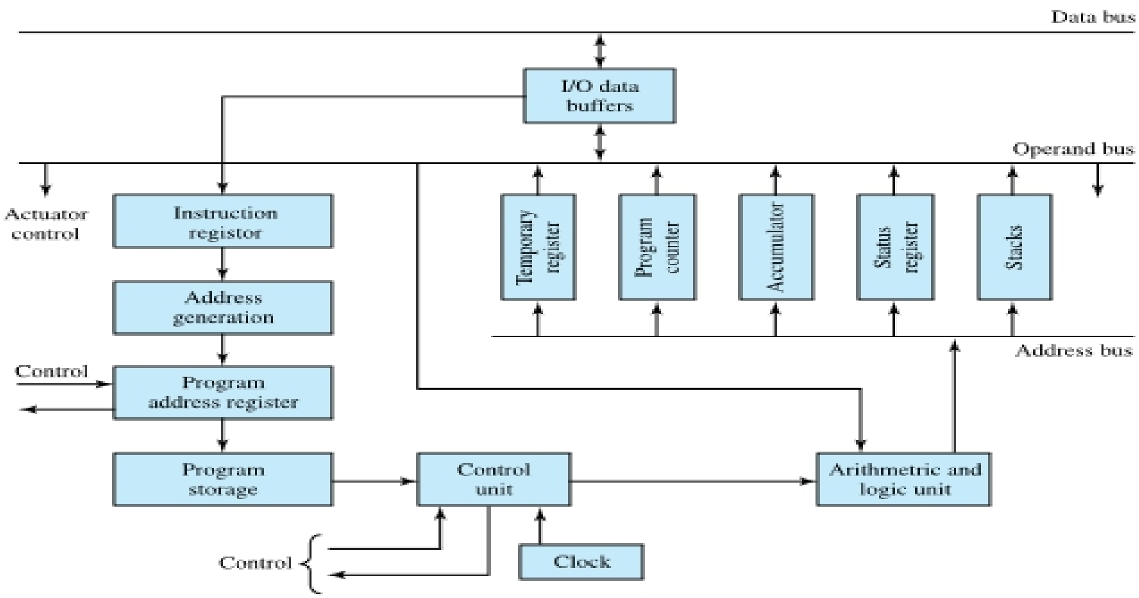


Fig. 4.136: Block diagram of typical microprocessor

➢ **CPU**

- This is the brain of the computer as it perform all the calculating, organizing and control functions.

➢ **BUS**

- A **bus**, which is a set of wires carrying address, data, and control signals, is employed for interconnecting the major components of a microcomputer system.

1. **Address lines**
   - These are unidirectional signals that specify the address of a memory location of an I/O device. With a typical 24-bit address bus, the microprocessor can access 224 (over 16 million) memory locations. Memory is generally organized in blocks of 8, 16, or 32 bits.

2. **Data bus**
   - It is a bidirectional bus, varying in size from 8 to 32 bits, which carries data between the CPU, MU, and I/O units.

3. **Control bus**
   - It provides signals to synchronize the memory and I/O operations, select either memory or an I/O device, and request either the read or the write operation from the device selected.

➢ **Arithmetic logic unit (ALU)**

- It accepts data from the data bus, processes the data as per program-storage instructions and/or external control signals, and feeds the results into temporary storage, from which external control and actuator control functions can be performed.

- **Accumulators**
- Accumulators are parallel storage registers used for processing the work in progress, temporarily storing addresses and data, and general functions.

- **Stacks**
- It provide temporary data storage in a sequential order and are of use during the execution of subroutines. A subroutine is a group of instructions that appears only once in the program code, but can be executed from different points in the program.

- **Program counter**
- It is a register/counter that holds the address of the memory location containing the next instruction to be executed.

- **Status register**
- It contains condition-code bits or flags (set to logic 1 or logic 0, depending on the result of the previous instruction) that are used to make decisions and redirect the program flow.

- **Control unit (CU)**
- It consists of the timing and data-routing circuits, decodes the instruction being processed and properly establishes data paths among the various elements of the microprocessor.

- **Instructions**
- It generally consist of a series of arithmetic and logic type operations, and also include directions for fetching and transferring data.

4.20 COMPUTER NETWORKS:

- Computer network is defined as two or more computers are connected in such a way that they can share their data, information as well as their resources.
- Computer network is the interconnected collection of autonomous computer and peripheral devices.
- The links connecting the devices are called communication channels.
- The concepts of connecting computers for sharing resources or data are called networking.
- Two computers are said to be interconnected if they are able to exchange information.

- **Networks are used by people because;**

1. Access to remote information
   Eg. WWW, E-commerce, E-shopping
2. Person to person communication
   Eg. E-mail, Chat, Video conferencing
3. Interactive environment
   Eg. Games, Songs, Video on demand

➢ **Advantages / Need of computer network:**

1. Efficient management of resources
2. Faster data sharing
3. Keeping information reliable and up to date
4. High reliability
5. Efficient communication

4.20.1 Types of Networks

1. LAN (Local Area Network)

- LAN is a group of computer and network communication devices interconnected within the geographically limited area. Such as office, building, computer lab or campus.
- LAN tends to use only one type of transmitting medium was cabling.

➢ **Characteristics of LAN**

1. It allows users to share storage devices like printer, application data and other network resources.
2. It transfers data at high speed (more than 1 mbps)
3. Multiple accesses
4. It exists in limited geographically area (up to few kilometers)
5. Its technology is less expensive.
6. It having a lower error rate.

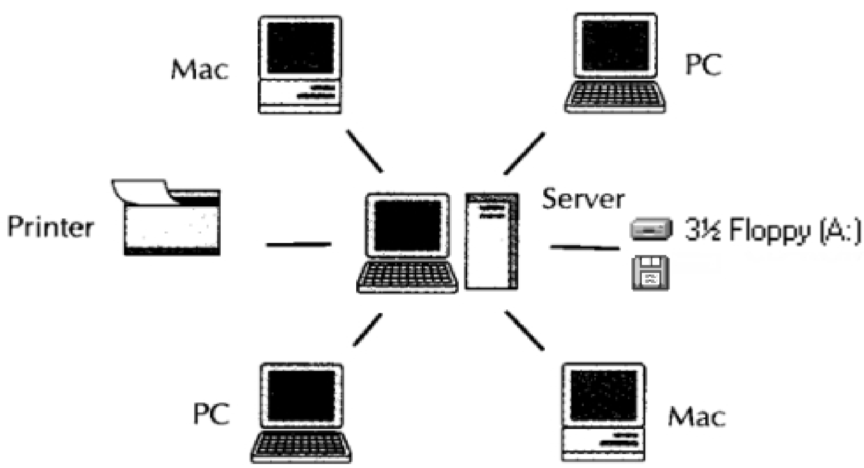


Fig. 4.137: Illustration of LAN

➢ **Advantages of LAN**

1. LAN are the best means provide a cost effective multi-user computer environment.
2. LAN can fit any site requirement.
3. Any number of users can be accommodated.

4. Allows sharing of mass center storage and printers.
5. It is flexible and growth oriented.
6. It provides data integrity.
7. Transfer data at high speed.

2. MAN (Metropolitan Area Network)

- A MAN covers a much larger area and might cover an entire city but uses the LAN technology.
- It may be a single network which as able television network or it may be means of connecting a number of LAN's together into a larger network. So that resources may be shared LAN to LAN as well as device to device.
- For example, a company can use a MAN to connect the LAN's in all of its offices throughout a city.
- A special category or standard has been adopted for MAN, and this standard is now being implemented and it is called DQDB (Distributed Queue Dual Bus).
- A MAN can support both data and voice.

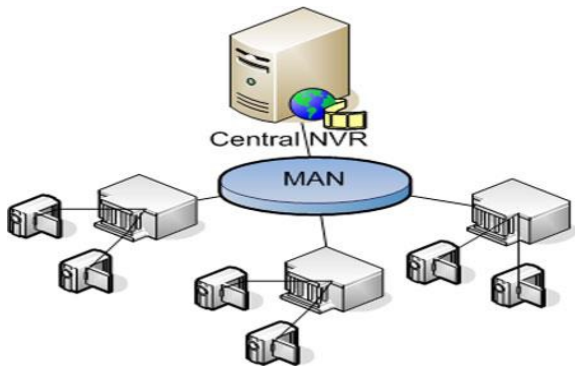


Fig. 4.138: Illustration of MAN

3. WAN (Wide Area Network)

- When a network is spread over wide areas, such as cities, states, countries or continent it is called a WAN.
- Communication on a WAN takes place via telephone lines, satellite or microwave transmission rather than physical cable.
- Most WAN are combination of LANs and other types of communication.

Fig. 4.139: Illustration of WAN

- **Characteristics of WAN**

1. They exists in unlimited geographical area.
2. They are more susceptible to error due to the distance the data can travel.
3. They interconnect multiple LAN.

4.20.2 Network Architecture

- Computer network architecture refers to the convention used to define how the different protocols of the system interact with each other support the end users.
- The ISO 7-layer model for an OSI is shown in fig. 4.140.

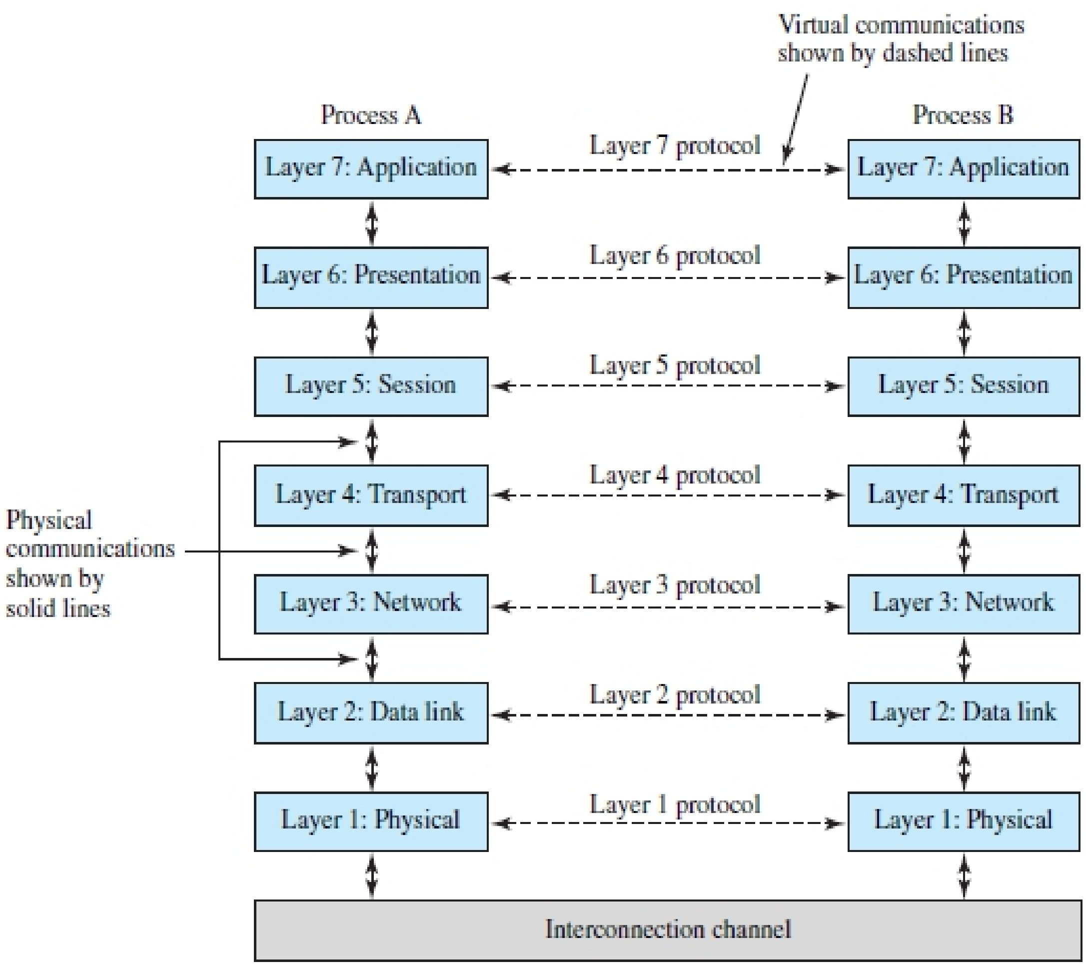


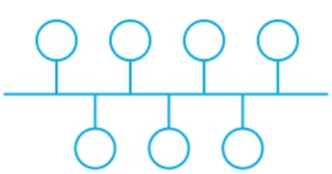
Fig. 4.140: ISO 7-layer model for an OSI

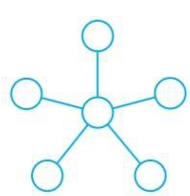
- All layers need be implemented; the more layers that are used, the more functionality and reliability are built into the system. Starting from the bottom layer, the functions of the layers are as follow.

1. **Physical**— Defines the type of medium, the transmission method, and the transmission rates available for the network; provides the means for transferring data across the interconnection channel and controlling its use.
2. **Data Link**— Defines how the network medium is accessed, which protocols are used the packeting/framing methods, and the virtual circuit/connection services; responsible for the transfer of data across the link; provides for the detection and correction of data transmission errors.
3. **Network**— Standardizes the way in which addressing is accomplished between linked networks; performs networking functions and internetworking.
4. **Transport**— Handles the task of reliable message delivery and flow control between applications on different stations; provides source-to-destination data integrity.
5. **Session**— Establishes two-way communication between applications running on different stations on the network; provides the user interface into the transport layer.
6. **Presentation**— Translates data formats so that computers with different "languages" can communicate; provides the syntax (rules) of representation of data between devices.
7. **Application**— Interfaces directly with the application programs running on the stations; provides services such as file access and transfer, peer-to-peer communication among applications, and resource sharing; provides support to process end users' applications such as electronic mail, database management, and file management.

4.20.3 Network Topology

- **Topology which refers to the way interconnection path between many users.**
- The five basic network topologies are Bus, Star, Ring, Mesh, Tree topologies as illustrate in Fig. 4.141.
- **Bus topology** is used predominantly by LANs, whereas star topology is commonly used by private-branch exchange (PBX) systems.
- **Ring topology** may have centralized control (with one node as the controller) or decentralized control (with all nodes having equal status).
- **Tree topology** is used in most of the remote-access networks, whereas distributed topology is common in public and modern communications networks.
- **Star topology** has a dedicated point to point link only to a center controller usually called HUB. There are not direct linked to each other.
- **A fully distributed network / mesh topology** allows every set of nodes to communicate directly with every other set through a single link and provides an alternative route between nodes.


(a)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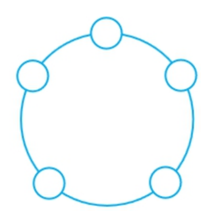

(c)

Fig. 4.141: (a) Bus topology (b) Star topology (c) Ring topology (d) Tree topology (e) mesh topology

➢ **Structure of typical TCP/IP Internet**

- The Internet is physically a collection of packet switching networks interconnected by gateways along with protocols that allow them to function logically as a single, large, virtual network.
- Gateways (often called IP routers) route packets to other gateways until they can be delivered to the final destination directly across one physical network. Fig. 4.142 shows the structure of physical networks and gateways that provide interconnection.

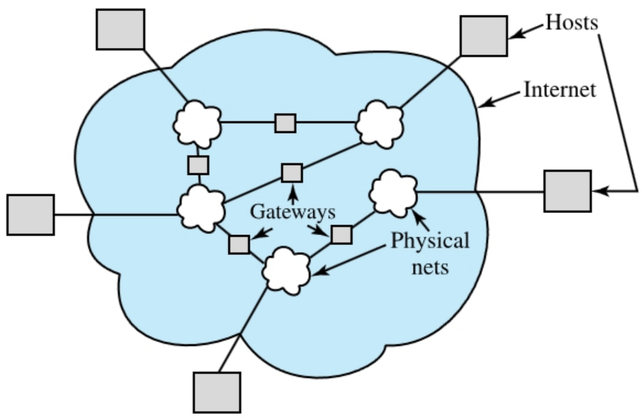


Fig. 4.142: Structure of typical TCP/IP Internet

- Gateways do not provide direct connections among all pairs of networks. The TCP/IP is designated to provide a universal interconnection among machines, independent of the particular network to which they are attached.
- Besides gateways that interconnect physical networks, as shown in Fig. 4.142, Internet access software is needed on each host (any end-user computer system that connects to a network) to allow application programs to use the Internet as if it were a single, real physical network. Hosts may range in size from personal computers to supercomputers.

4.20.4 Transmission Media

- These, also known as physical channels, can be either bounded or unbounded.

- **Bounded media**, in which signals representing data are confined to the physical media, are twisted pairs of wires, coaxial cables, and optical-fiber cables, used in most LANs.
- **Unbounded media**, such as the atmosphere, the ocean, and outer space in which the transmission is wireless, use infrared radiation, lasers, microwave radiation, radio waves, and satellites.

- Data are transmitted from one node to another through various transmission media in computer communications networks.
- **Twisted pairs** are used in low-performance and low-cost applications with a data rate of about 1 Mbit per second (Mbps) for a transmission distance of about 1 km.
- **Baseband coaxial** cables used for digital transmission are usually 50Ω cables with a data rate of about 10 Mbps over a distance of about 2 km.
- **Broad-band coaxial** cables used for analog transmission (cable TV) are usually 75 Ω cables with a data rate of about 500 Mbps over a distance of about 10 km.
- **Optical fiber cable** is the lighter and cheaper fiber-optic cables support data transmission of about 1 Gbps over a distance of about 100 km.

4.20.5 Types of Data Transmission

- Data can be transferred between two stations in either serial or parallel transmission.
- **Parallel data transmission**, in which a group of bits moves over several lines at the same time, is used when the two stations are close to each other (usually within a few meters), as in a computer–printer configuration.
- **Serial data transmission**, in which a stream of bits moves one by one over a single line, is used over a long distance. Serial data transfer can be either asynchronous or synchronous.
- **Asynchronous data communication** is most commonly applied in low-speed terminals and small computers. Large-scale integration (LSI) devices known as UARTs (universal asynchronous receivers/transmitters) are commercially available for asynchronous data transfer.
- **Synchronous data communication** is used for transferring large amounts of data at high speed. USARTs (universal synchronous/asynchronous receivers/transmitters) are commercially available LSI devices.
- **Frequency-division multiplexing (FDM)** is a technique for data transmission widely used in telephone, radio, and cable TV systems in which the transmission frequency spectrum (i.e., bandwidth) is divided into smaller bands known as sub-channels.

- Data transmission between two stations can be achieved in either simplex, half-duplex, or full-duplex mode.
- In a **simplex mode**, mainly used in radio and TV broadcasts, information travels only in one direction. This mode is rarely used in data communications.
- In a **half-duplex mode,** used by radio communications, information may travel in both directions, but only in one direction at a time. The transmitter becomes the receiver and vice versa. E.g. Walkie – Talkie.

- In a **full-duplex mode**, information may travel in both directions simultaneously. This mode, used in telephone systems, adopts two different carrier frequencies.

4.20.6 Modem

- A modem (modulator/demodulator) is an electronic device that takes digital data as a serial stream of bits and produces a modulated carrier signal as an output. That is to say, the digital signals are converted to an analog form with a relatively narrow bandwidth.
- The carrier signal is then transmitted over the telephone line to a similar modem at the receiving end, where the carrier signal is demodulated back into its original serial stream of bits, as shown in Fig. 4.143

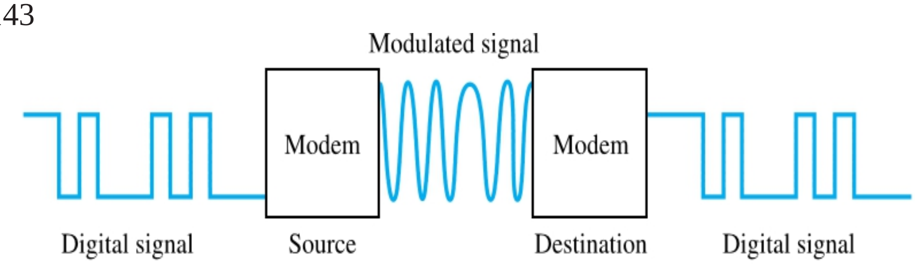


Fig. 4.143: Input/output signals of a modem

- The serial digital data to be transmitted are modulated, filtered, and amplified for analog transmission; the analog data received at the receiving end are amplified, filtered, and demodulated to produce serial digital signals.
- There are four different types of modems: half-duplex, full-duplex, synchronous, and asynchronous.
- With half-duplex modems data can be transmitted in only one direction at a time. Fullduplex modems transmit data in both directions at the same time; one modem is designated as the originating modem and the other as the answering modem, while transmitting and receiving data are done at different frequencies.
- Asynchronous modems are low-data-rate modems transmitting serial data at a rate of about 1800 bits per second (bps).
- Synchronous modems are high-data-rate modems transmitting serial data at a rate of about 10,800 bps.

- Modems can also be classified as voice-band or wide-band modems.
- **Voice-band modems** are low-to-high speed modems designed for use on dial-up, voice-grade, and standard telephone lines up to a rate of about 10,800 bps.
- **Microprocessor-controlled modems** are known as smart modems, such as the Hayes modem, manufactured by Hayes Microcomputer Products.
- **The portable acoustic-coupler device**, which is a different type of voice-band modem, is a low-speed modem with a rate of about 600 bps that is connected acoustically to a standard telephone.
- **Wide-band modems** are very high-speed modems with rates of 19,200 bps and above, designed for use with dedicated telephone lines. These are currently used mostly on private communications systems.

4.21 PRACTICAL APPLICATION: A CASE STUDY: MICROCOMPUTER CONTROLLED BREAD MAKING MACHINE:

- Fig. 4.144 shows a simplified schematic diagram of a microcomputer-controlled bread making machine.

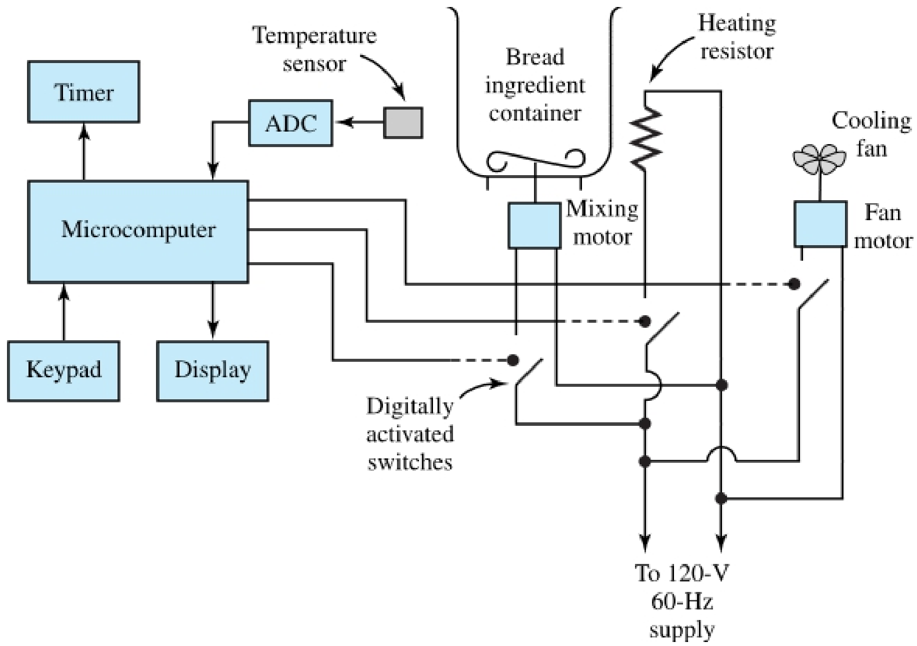


Fig. 4.144: Schematic diagram of Microcomputer-Controlled Bread making Machine

- A microcomputer along with its timing circuit, keypad, and display unit controls the heating resistor, fan motor, and bread-ingredient mixing motor by means of digitally activated switches.
- An analog temperature sensor, through an A/D converter, provides the status of temperature to the microcomputer.
- A digital timer circuit counts down, showing the time remaining in the process.
- The control programs are stored in ROM and determine when and how long the machine should mix the ingredients added to the bread pan, when and how long the heating resistor should be turned on or off for various parts of the cycle, and when and how long the fan should be on to cool the loaf after baking is finished. The parameters such as light, medium, or dark bread crust are entered through the keypad into RAM.
- According to the programs stored and the parameters entered, the machine initially mixes the ingredients for several minutes.
- The heating resistor is turned on to warm the yeast, causing the dough to rise while a temperature of about 90°F is maintained. The time remaining and the temperature are continually checked until the baked loaf is cooled, and the finished bread is finally ready in about 4 hours.
- Microprocessors and computers in various forms are used extensively in household appliances, automobiles, and industrial equipment.

QUESTIONS

4.1 Introduction
1. Define analog and digital signal.
2. Give comparison of analog and digital systems.

4.2 Introduction to Digital System
1. Give advantages and disadvantages of digital systems.
2. Define positive, negative and mixed logic systems.

4.3 Number systems
1. Convert following binary number in to decimal number
   i. 11001.011 [Ans: $(25.375)_{10}$]
   ii. $(110011)_2 = (?)_{10}$ [Ans: $(51)_{10}$]
2. Convert following decimal number in to binary number
   i. 81 [Ans: $(1010001)_2$]
   ii. $(163)_{10} = (?)_2$ [Ans: $(10100011)_2$]
   iii. 537 [Ans: $(1000011001)_2$]
   iv. $(111)_{10} = (?)_2$ [Ans: (1101111)2]
   v. 0.875 [Ans: $(0.111)_2$]
   vi. 35.17 [Ans: $(100011.0010101)_2$]
3. Convert following octal number in to decimal number
   i. $(37246)_8 = (?)_{10}$ [Ans: $(16038)_{10}$]
   ii. 103.45 [Ans: $(67.578)_{10}$]
4. Convert following decimal number in to octal number
   i. 654 [Ans: $(1216)_8$]
   ii. $(890)_{10} = (?)_8$ [Ans: $(1572)_8$]
   iii. 0.23 [Ans: $(0.165)_8$]
5. Convert following binary number in to octal number
   i. $(111101001)_2 = (?)_8$ [Ans: $(751)_8$]
6. Convert following octal number in to binary number
   i. 611 [Ans: $(110001001)_2$]
5. Convert following hexadecimal number in to decimal number
   i. 10 [Ans: $(16)_{10}$]
   ii. $(AF)_{16} = (?)_{10}$ [Ans: $(175)_{10}$]
   iii. $(3A1.4)_{16} = (?)_{10}$ [Ans: $(929.25)_{10}$]
6. Convert following decimal number in to hexadecimal number
   i. 246 [Ans: $(F6)_{16}$]
   ii. $(0.56)_{10} = (?)_{16}$ [Ans: $(8F)_{16}$]
7. Convert following binary number in to hexadecimal number
   i. 10001100 [Ans: $(8C)_{16}$]
   ii. 00110111 [Ans: $(37)_{16}$]
8. Convert following hexadecimal number in to octal number
   i. BC05 [Ans: $(136005)_8$]
   ii. $(35AC.90F)_{16} = (?)_8$ [Ans: $(32654.4417)_8$]

9. Convert following octal number in to hexadecimal number
   i. 65302 [Ans: $(6AC2)_{16}$]
   ii. $(3764.670)_8 = (?)_{16}$ [Ans: $(7F4.DC0)_{16}$]
10. Convert following number in to defined radix r conversion
    i. $(1543)_{10} = (?)_5$
    ii. $(1420)_{10} = (?)_4$
    iii. $(10111)_2 = (?)_{12}$
11. Find the value of unknown base
    i. Determine b if $(34)_{10} = (202)_b$

4.4 Logic gates

1. Explain various logic gates with its logic symbol and truth table.
2. Design all basic gates using universal gates.

4.5 Boolean algebra

1. Give difference between sequential and combinational circuits.
2. Explain various laws of Boolean algebra.
3. Explain De-Morgan's theorem.

4.6 Different forms of boolean algebra

1. Explain standard and canonical form.

OR

1. Define SOP and POS form.

4.7 Simplification of logical function using karnaugh map (k-map)

1. What is k-map?

4.8 Binary adders

1. Explain half adder and full adder in detail.
2. Give comparison between half adder and full adder

4.9 Encoder

1. What is encoder? Explain it in detail with necessary diagrams.
2. Explain octal to binary (8 to 3) encoder.

4.10 Decoder

1. What is decoder? Explain it in detail with necessary diagrams.
2. Implement full adder using decoder.
3. Implement following function using decoder: $F = \Sigma(4, 6, 7, 9, 10, 13)$

4.11 Multiplexer

1. What is multiplexer? Explain it in detail.
2. Implement the function $F(x, y) = \sum(0, 1, 3)$ using multiplexer.
3. Implement the function $F(x, y, z) = \sum(1, 2, 6, 7)$ using 4 X 1 multiplexer.

4.12 Demultiplexer

1. What is demultiplexer? Explain it in detail.
2. Explain 1 to 8 demultiplexer in detail.

4.13 Sequential blocks

1. Explain SR flip-flop with necessary diagrams.
2. Explain D flip-flop with necessary diagrams.
3. Explain JK flip-flop with necessary diagrams.
4. Explain master-slave JK flip-flop in detail.
5. Explain race around condition in JK flip-flop.

4.14 Shift registers

1. Explain serial-in serial-out shift register with timing diagram.
2. Explain PISO, SIPO and PIPO shift registers with diagram.

4.15 Counters

1. Explain asynchronous or Ripple counter.
2. Explain synchronous counter.
3. Explain ring counter.

4.16 Digital-to-analog & analog-to-digital converter

1. What is digital to analog converter? Explain various types of digital to analog converter with its applications.
2. What is analog to digital converter? Explain various types of analog to digital converter with its applications.

4.17 Memory

1. What is memory? List down applications of it. Also explain ROM, RAM, PROM, EPROM and EEPROM.
2. Give comparison of ROM, EPROM and EEPROM.

4.18 Display Devices

1. Explain various display devices.

4.19 Computer Systems

1. What is computer system? Explain organization / Architecture of computer system.
2. Draw block diagram of basic microprocessor system and explain all the block of it in brief.

4.20 Computer Networks

1. What is computer network? Give advantages of it. Also explain various types of computer networks.
2. Explain computer architecture.

OR

2. Explain ISO 7-layer model for an OSI.

3. Explain various network topologies (Mesh, Ring, Star, Bus, Tree topology) with its advantages.
4. Write short note on transmission media.
5. Explain various types of data transmission modes.
6. What is modem? Explain it in detail with block diagram.

4.21 A Case Study – Microcomputer – Controlled Bread making Machine

1. Write short note on Microcomputer – Controlled Bread making Machine with diagram.

Chapter 5

SIGNAL PROCESSING

- Signal and signal classification
- Spectral analysis
- Modulation
- Sampling and Multiplexing
- Interference and noise
- Practical application: case study

SIGNAL

- Signal is the representation of a physical quantity that conveys information about that quantity.
- As electric quantities like voltage or current are relatively easy in processing and transmitting.
- Signals are generally represented in form of voltage or current. Thus electric signal is a voltage or current waveform whose variation with respect to time or frequency represents desired information about physical quantity of interest.
- For example temperature sensor gives output voltage that is proportional to the temperature being measured.

Classification of Signals

Signals as per their characteristics can be classified as follows.

1. Continuous time and Discrete time signals
2. Continuous value and Discrete value signals
3. Periodic and Aperiodic (non-periodic) signals
4. Deterministic and Random signals
5. Even and Odd signals
6. Causal and Non-causal signals
7. Energy and Power signals

1) <u>Continuous time and Discrete time Signals</u>:

   - A signal whose value is defined at all instances of time is called continuous time signal.
   - On the other hand signal whose values are defined only at discrete instances of time is called discrete time signal.
   - Most of the signals in nature are continuous time signals. A discrete time signal can be obtained from continuous time signal by process called **sampling**. This has been illustrated in Figure 1.

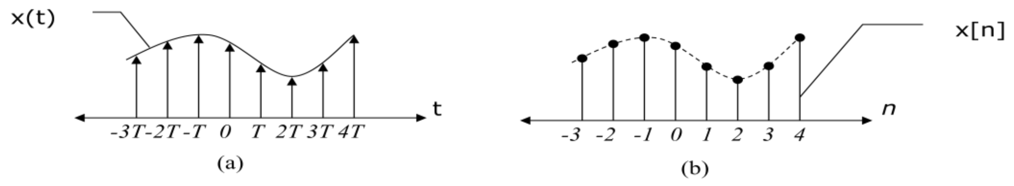


Figure 1: (a) Continuous time signal x(t) sampled at every T interval, (b) Resulting discrete time signal x(n).

2) Continuous value and Discrete value signals:

- If a signal can take any value in a give range between some minimum and maximum value then the signal is called continuous value signal.

- On the other hand if a signal takes only certain fixed values in a given range then it is called discrete value signal. For processing the signal on digital systems these discrete values are represented in terms of binary.

- The process of converting a continuous value signal to a discrete value signal is called **quantization**. This is illustrated in Figure 2.

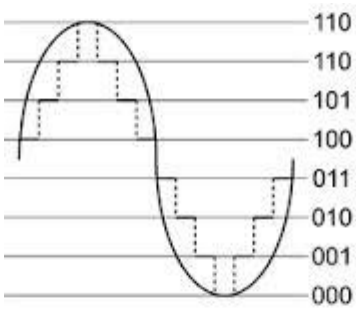


Figure 2: Continuous value signal (solid line) and discrete value signal (dotted line).

3) Periodic and Aperiodic (non-periodic) signals:

- Periodic signals are one that repeats itself after a regular interval of time. A periodic signal x(t) satisfies following property.

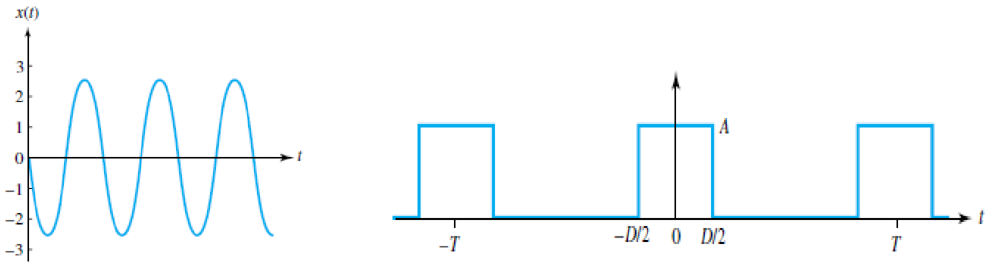
$$x(t+kT)=x(t)$$

where, k is any integer and T is the smallest interval after which the signal repeats itself, called the period of the signal.

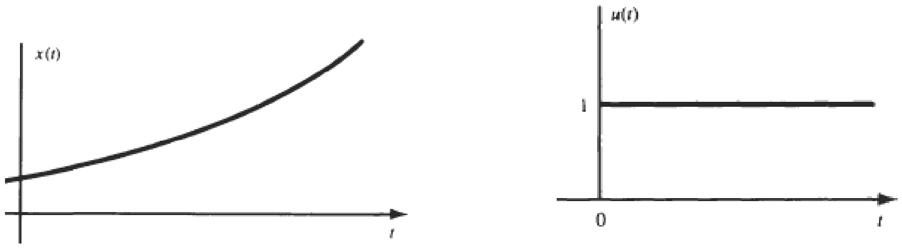
- Number of times a periodic signal repeats itself in 1 second is called **frequency (f)**. f = 1/T.

- Signals that are not repetitive in nature are aperiodic or non-periodic signals.

- Periodic and aperiodic signals are illustrated in Figure 3.



(a) (b)



(c) (d)

Figure 3: (a) sinusoidal signal & (b) pulse train are periodic signals; (c) exponential signal and (d) unit step signal are aperiodic signals.

4) <u>Deterministic and Random signals:</u>

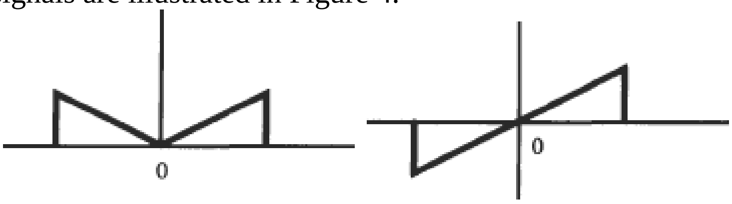
- Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a mathematical function of time**.** For example x(t) = Asin(100πt).
- Random signals are those signals that take random values at any given time and hence they cannot be represented as function of time. Such signals are characterized statistically.

5) <u>Even and Odd signals</u>

- A signal ***x (t)*** is referred to as an even signal if ***x (-t) = x (t).*** They are symmetric around time t = 0. Example x(t) = Acos(2πft).

- A signal ***x (t)*** is referred to as an odd signal if ***x (- t) = - x (t).*** They are not symmetric around time t = 0. Example x(t) = Asin(2πft).

- These signals are illustrated in Figure 4.



(a) (b)

Figure 4. (a) Even signal (b) Odd signal.

- Any signal *x*(*t*), in general, can be expressed as the sum of its even and odd parts as expressed below.

$$x(t) = x_e(t) + x_0(t)$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

<u>Example</u>

Discuss the nature of evenness and oddness of:
(a) The sinusoidal signal x(t) = A cos($2\pi f_0 t + \theta$).
(b) The complex exponential signal x(t) = $e^{j2\pi fot}$.

<u>Solution</u>

a) The signal is, in general, neither even nor odd.
However,
for the special case of $\theta = 0$,it is even;
for the special case of $\theta = \pm\pi/2$, it is odd.
In general,

$$x(t) = A \cos \theta \cos 2\pi f_0 t - A \sin \theta \sin 2\pi f_0 t$$

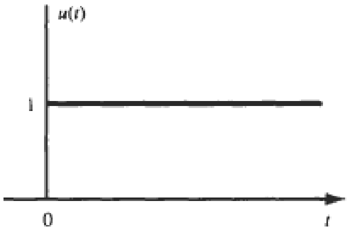
Since $\cos 2\pi f_0 t$ is even and $\sin 2\pi f0t$ is odd, it follows that

$$xe(t) = A \cos \theta \cos 2\pi f_0 t \text{ and}$$
$$xo(t) = -A \sin \theta \sin 2\pi f_0 t$$

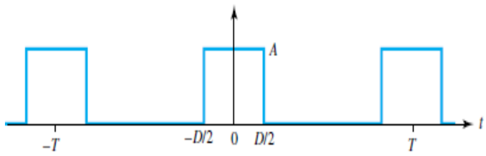
b) For $\theta = 0$, $x(t) = A\, e^{j2\pi fot}$, the real part and the magnitude are even; the imaginary part and the phase are odd. Noting that a complex signal x(t) is called hermitian if its real part is even and its imaginary part is odd, the signal and symmetry are then said to be hermitian.

6) <u>Causal and Non-causal signals</u>:

- A signal x(t) is said to be causal if, for all $t < 0$, $x(t) = 0$.
- A signal whose values are defined for both $t < 0$ and $t > 0$ are called non-causal signals.



Causal signal



Non-Causal signal.

7) <u>Energy and Power signals</u>:

- Energy and power of a signal x(t) is defined as

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \; dt = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 \; dt$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

- A signal is Energy signal if and only if the energy Ex of the signal, is well defined and finite i.e. $0 < Ex < \infty$.
- A signal is a power-type signal if and only if the power Px of the signal, is well defined and finite i.e. $0 < Px < \infty$.
- Typically for energy signal average power is 0 and for power signal energy is ∞.

- Typically periodic signals are power signals. For example x(t) = Acos(2πft).

Example

Classify each of the following signals as an energy signal or a power signal, by calculating the energy E, or the power P (A, θ, ω and τ are real positive constants).

1) x1(t) = A | sin(ωt + θ)|
2) x2(t) = Aτ / $\sqrt{\tau + jt}$, j=$\sqrt{-1}$
3) x3(t) = $At^2e^{-t/\tau}$ u(t).

Solution

1) Power signal.

   The signal is periodic, with period π/ω, and

$$P_1 = \frac{\omega}{\pi}\int_0^{\pi/\omega} A^2|\sin(\omega t+\theta)|^2\,dt = \frac{A^2}{2}.$$

2) Neither energy nor power

$$E_2 = \lim_{T\to\infty}\int_{-T}^{T}\frac{(A\tau)^2}{\sqrt{\tau+jt}\sqrt{\tau-jt}}\,dt \to \infty,$$

and

$$P_2 = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\frac{(A\tau)^2}{\sqrt{\tau^2+t^2}}\,dt = 0.$$

3) Energy signal

$$E_3 = \int_0^{\infty} A^2\,t^4\,\exp(-2t/\tau)\,dt = \frac{3A^2\tau^5}{4}.$$

4) Real and Complex signals:

   - A signal x(t) is a real signal if its value is a real number.

   - A signal x(t) is a complex signal if its value is a complex number. A complex signal x(t) is represented as

     x (t) = x1 (t) + j x 2 (t)

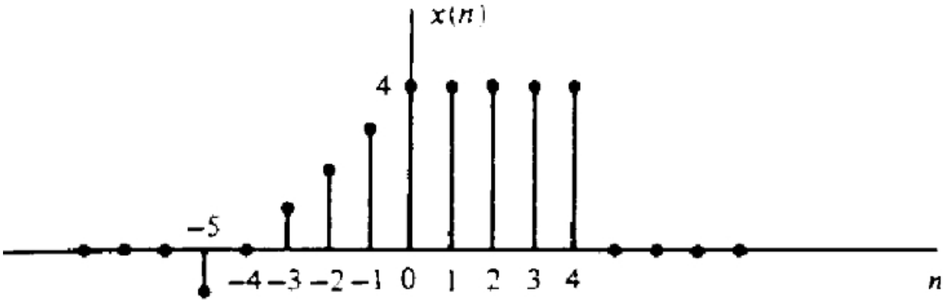
   - where x1(t) and x2(t) are real signals and j = $\sqrt{-1}$.

   - Popular example of complex signal is complex exponential signal $x(t) = e^{j2\pi ft}$

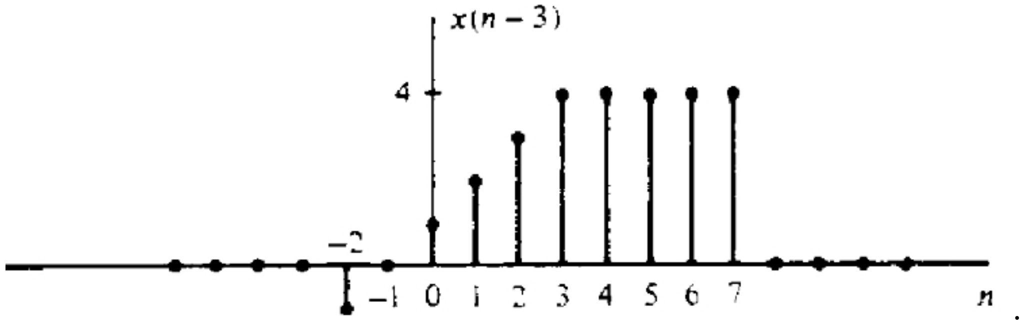
     x(t) = $e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$

Example

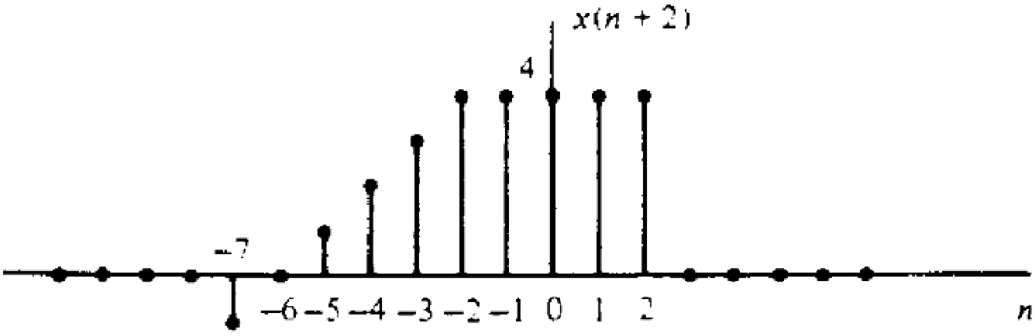
A signal x(n) is graphically illustrated in Figure given below. Show a graphical representation of the signals x(n - 3) and x(n + 2).



Solution
The signal x(n -3) is obtained by delaying x(n) by three units in time. The result is illustrated in Figure



On the other hand, the signal x(n + 2) is obtained by advancing x(n) by two units in time. The result is illustrated in Figure.



Note: The Delay corresponds to shifting a signal to the right, whereas advance implies shifting the signal to the left on the time axis.

SIGNAL PROCESSING

Any operation performed on the signal to achieve the desired purpose is called signal processing. Simple examples of signal processing can be amplification of a signal so as to increase the strength of the signal or modulation of a signal so as to communicate it over a longer distance, etc.

<u>Example of Signal processing in Analog communication system</u>

Simple block diagram of an analog communication system representing various signal processing operation at different stages is shown in Figure 5.

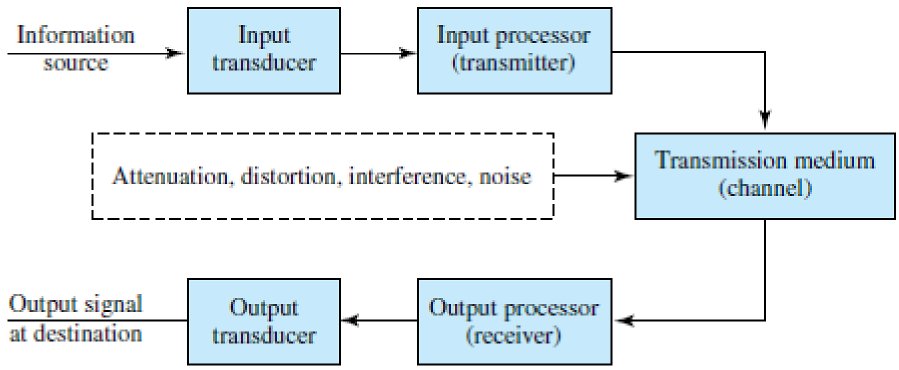


Figure 5. Block diagram of analog communication system showing need of signal processing operations.

Transducer:

- A transducer is usually required to convert the output of a source into an electrical signal that is suitable for transmission.
- Typical examples include a microphone converting an acoustic speech or a video camera converting an image into electric signals.

Transmitter:

- The transmitter (input processor) converts the electric signal into a form that is suitable for transmission through the physical channel or transmission medium.
- For example, in radio and TV broadcasts, the signal is appropriately modulated before transmission.

Communication channel:

- The communication channel (transmission medium) is the physical medium that is utilized to send the signal from the transmitter to the receiver.
- The channel can be wired (land line telephones) or wireless (radio, mobile phones).
- The transmission medium is the most vulnerable part of a communication system and the signal while traveling over the transmission medium may get corrupted (get error) due to phenomena like attenuation, distortion, interference, noise, etc.

Receiver:

- The receiver is to recover the message signal contained in the received signal. For example a modulated signal when received has to be demodulated to get the desired information.
- The received signal can appropriately converted back to the desired form using output transducer. For example, loudspeaker to convert electrical signals to sound.

This system to successfully work may require number of signal processing operations like

- *Amplification* to compensate for attenuation
- *Filtering* to reduce interference and noise, and/or to obtain selected facets of information
- *Equalization* to correct some types of distortion
- *Frequency translation* or *sampling* to get a signal that better suits the system characteristics
- *Multiplexing* to permit one transmission system to handle two or more information-bearing signals simultaneously

Example of Signal processing Digital Communication System

Block diagram of a digital signal processing system is shown in Figure 6. For each function performed on transmitter side, there is an inverse operation on the receiver side.

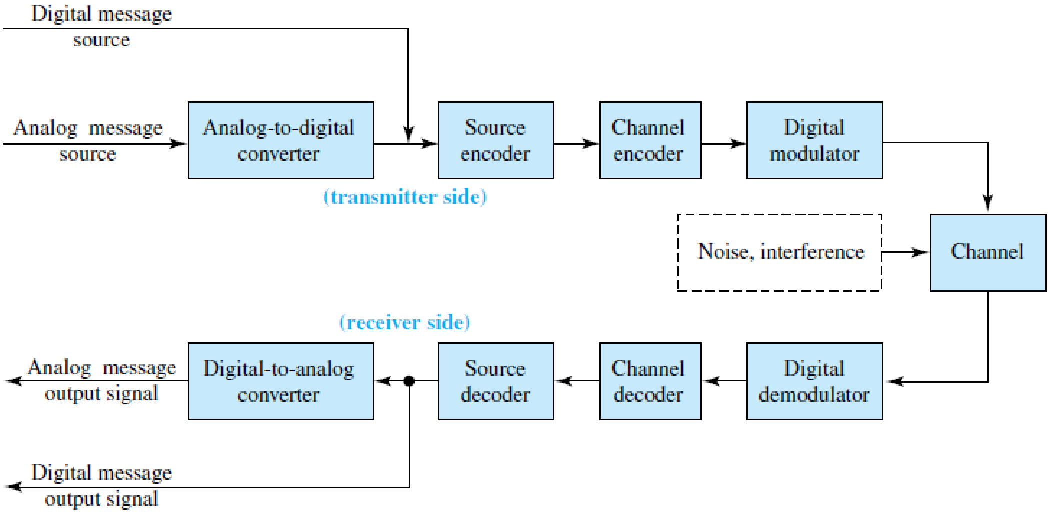


Figure 6. Digital signal processing system showing need of signal processing operations.

Analog to digital convertor(A/D):

- The analog input signal (such as an audio or video signal) must first be converted to a digital signal by an analog to digital (A/D) converter.

Source encoder:

- Source encoder works to convert the output of A/D converted in more efficient representation so that less number of bits has to be transmitted over the channel.
- For example: data compression.

Channel encoder:

- The purpose of the channel encoder is to introduce some redundancy in a controlled manner in the binary information sequence, so that the redundancy can be used at the receiver to overcome the effects of noise and interference encountered in the transmission of the signal through the channel.
- The binary sequence generated by the channel encoder is modulated before transmission over the channel. Example of digital modulation can be amplitude shift keying (ASK), frequency-shift keying (FSK), etc.

- On the receiver side opposite signal processing operations are performed starting with demodulation, channel decoding and source decoding and digital to analog conversion.

Frequency domain representation of a sinusoidal signal

$$x(t) = A\cos(2\pi ft + \theta)$$

- Till now a sinusoidal signal has been represented as a function of time t. Such representation is called time domain representation.

- However such sinusoidal signal can be completely characterized by three quantities: its amplitude, phase and frequency.
- Hence instead of representing it as a function of time we can also represent it as a function of frequency. Such representation is called frequency domain representation.
- Such frequency domain representation of a signal x(t) consists of two plots amplitude vs. frequency and phase vs. frequency. Such representation is called signal spectrum (or line spectrum).
- Spectrum of x(t) is shown in Figure 7. The advantage of the frequency domain representation is that it can display clearly the various frequency components that exist in a signal.

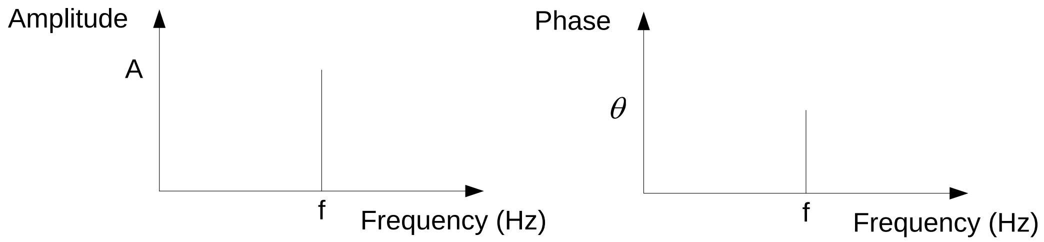


Figure 7 Spectrum of $x(t) = A\cos(2\pi f t + \theta)$.

- In spectrum representation Sinusoidal signals are important because they have unique (single) frequencies.
- An arbitrary signal does not have a unique frequency. The signal shown in Figure 8 is periodic but not pure sinusoidal. It actually is sum of two sinusoidal signals of different frequencies represented as follows.

$$x(t) = 3\sin(2\pi f_1 t) + \sin(2\pi(3f_1)t)$$

- The magnitude spectrum is also shown in the Figure.

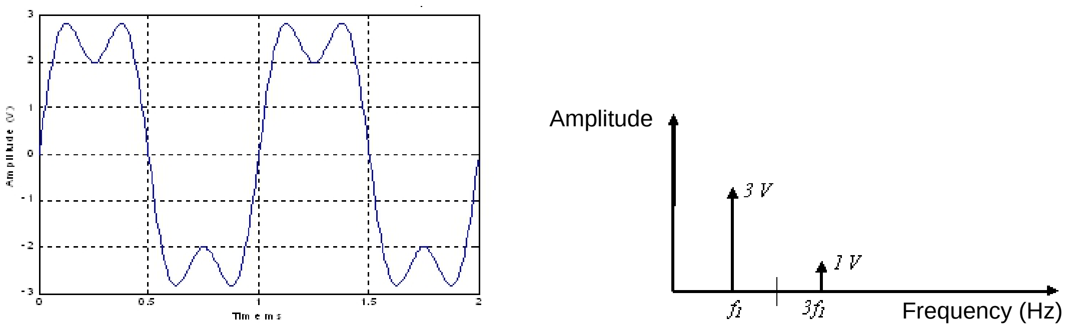


Figure 8. Arbitrary periodic signal and its amplitude spectrum.

FOURIER SERIES

- A Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and cosines.
- Fourier series make use of the orthogonality relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful in signal processing.

- Fourier series representation of an arbitrary periodic signal x(t) with period T, is given by as

$$x(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

Where, $\omega = 2\pi/T$ and

$$a_0 = \frac{1}{T}\int_0^T x(t)dt \, ,$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega t\, dt, \qquad \text{for } n = 1, 2, \ldots$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega t\, dt, \qquad \text{for } n = 1, 2, \ldots$$

- From this representation the magnitude and phase of n^{th} frequency component (called n^{th} harmonics) can be calculated using

$$A_n = \sqrt{a_n^2 + b_n^2} \qquad \text{and} \qquad \phi_n = -\arctan\left(\frac{b_n}{a_n}\right)$$

(arctan = $\tan^{-1}$)

Example

Find the Fourier series for the square 2π-periodic wave defined on the interval $[-\pi, \pi]$:

$$f(x) = \begin{cases} 0, & if \quad -\pi \le x \le 0 \\ 1, & if \quad 0 < x \le \pi \end{cases}.$$

Solution

First we calculate the constant a_0:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_0^{\pi} 1dx = \frac{1}{\pi}\cdot\pi = 1.$$

Find now the Fourier coefficients for $n \neq 0$:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx = \frac{1}{\pi}\int_0^{\pi} 1\cdot\cos nx dx = \frac{1}{\pi}\left(\frac{\sin nx}{n}\right)\Bigg|_0^{\pi} = \frac{1}{\pi n}\cdot 0 = 0,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx = \frac{1}{\pi}\int_0^{\pi} 1\cdot\sin nx dx = \frac{1}{\pi}\left(-\frac{\cos nx}{n}\right)\Bigg|_0^{\pi} = -\frac{1}{\pi n}\cdot(\cos n\pi - \cos 0) = \frac{1-\cos n\pi}{\pi n}.$$

Since , $\cos n\pi = (-1)^n$

we can write:

$$b_n = \frac{1-(-1)^n}{\pi n}.$$

Thus, the Fourier series for the square wave is

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)^n}{\pi n}\sin nx.$$

We can easily find the first few terms of the series. By setting, for example, $n = 5$, we get

$$f(x) = \frac{1}{2} + \frac{1-(-1)}{\pi}\sin x + \frac{1-(-1)^2}{2\pi}\sin 2x + \frac{1-(-1)^3}{3\pi}\sin 3x + \frac{1-(-1)^4}{4\pi}\sin 4x + \frac{1-(-1)^5}{5\pi}\sin 5x + \ldots$$

$$= \frac{1}{2} + \frac{2}{\pi}\sin x + \frac{2}{3\pi}\sin 3x + \frac{2}{5\pi}\sin 5x + \ldots$$

The graph of the function and the Fourier series expansion for $n = 10$ is shown in Figure 9.

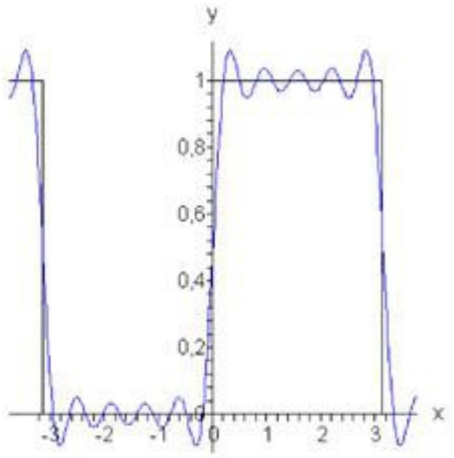


Figure 9.1 $n = 10$

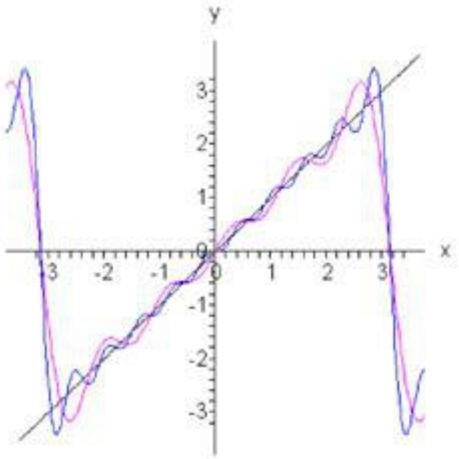


Figure 9.2 $n = 5$, $n = 10$

Example

Find the Fourier series for the function

$$f(x) = \begin{cases} 0 & \text{if} \quad -\pi \le x \le 0 \\ \sin x & \text{if} \quad 0 < x \le \pi \end{cases},$$

defined on the interval $[-\pi, \pi]$.

Solution

First we find the constant a_0:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi}\sin x\,dx = \frac{1}{\pi}(-\cos x)\Big|_0^{\pi} = \frac{1}{\pi}(-\cos\pi + \cos 0) = \frac{2}{\pi}.$$

Now we calculate the coefficients a_n:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi}\sin x\cos nx\,dx = \frac{1}{2\pi}\int_0^{\pi}\left[\sin(x+nx) + \sin(x-nx)\right]dx$$

$$= \frac{1}{2\pi}\int_0^{\pi}\left[\sin(n+1)x - \sin(n-1)x\right]dx = \frac{1}{2\pi}\left(-\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right)\Bigg|_0^{\pi}$$

$$= \frac{1}{2\pi}\left(\frac{\cos(n-1)\pi}{n-1} - \frac{\cos(n+1)\pi}{n+1} - \frac{1}{n-1} + \frac{1}{n+1}\right) = \frac{1}{2\pi}\left(\frac{\cos(n-1)\pi}{n-1} - \frac{\cos(n+1)\pi}{n+1} - \frac{2}{n^2-1}\right).$$

Notice that

$$\cos(n+1)\pi = \cos(\pi n + \pi) = \cos(\pi n - \pi + 2\pi) = \cos((n-1)\pi + 2\pi) = \cos(n-1)\pi.$$

Since $\cos(n-1)\pi = (-1)n-1$, we get the following expression for the coefficients an:

$$a_n = \frac{1}{2\pi}\cdot\frac{2}{n^2-1}\cdot\left[(-1)^{n-1}-1\right].$$

It's seen that an = 0 for odd n. For even n, when n = 2k (k = 1,2,3,...), we have

$$a_{2k} = \frac{1}{2\pi}\cdot\left(-\frac{4}{k^2-1}\right) = -\frac{1}{\pi}\cdot\frac{2}{k^2-1}.$$

Calculate the coefficients bn. Start with b1:

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin x\,dx = \frac{1}{\pi}\int_{0}^{\pi}\sin x\sin x\,dx = \frac{1}{\pi}\int_{0}^{\pi}\sin^2 x\,dx = \frac{1}{2\pi}\int_{0}^{\pi}(1-\cos 2x)\,dx = \frac{1}{2\pi}\left[x-\frac{\sin 2x}{2}\right]\Bigg|_0^{\pi} = \frac{1}{2\pi}\cdot\pi =$$

The other coefficients bn for n > 1 are zero. Indeed,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{0}^{\pi}\sin x\sin nx\,dx = \frac{1}{2\pi}\int_{0}^{\pi}\left[\cos(x-nx)-\cos(x+nx)\right]dx$$

$$= \frac{1}{2\pi}\int_{0}^{\pi}\left[\cos(n-1)x-\cos(n+1)x\right]dx = \frac{1}{2\pi}\left(\frac{\sin(n-1)x}{n-1}-\frac{\sin(n+1)x}{n+1}\right)\Bigg|_0^{\pi} = 0.$$

Thus, the Fourier series of the given function is given by

$$f(x) = \frac{1}{\pi}+\frac{1}{2}\sin x-\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{1}{4k^2-1}\cos(2kx).$$

Graphs of the function and its Fourier expansions for n = 2 and n = 8 are shown in Figure 10.

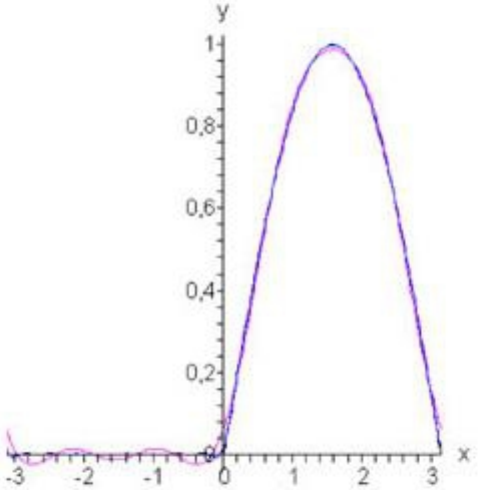


Figure 10.1 *n* = 2, *n* = 8

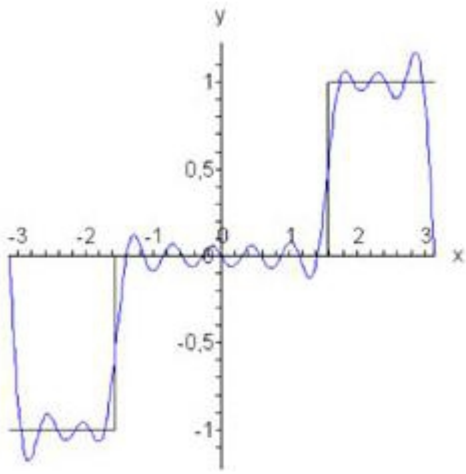


Figure 10.2 *n* = 10

SPECTRUM ANALYSIS

- The frequency spectrum of a time-domain signal is a representation of that signal in the frequency domain.
- The frequency spectrum can be generated via a Fourier transform of the signal, and the resulting values are usually presented as amplitude and phase, both plotted versus frequency.
- The process of obtaining the spectrum of a given signal using the basic mathematical tools is known as frequency or spectral analysis.

SIGNAL BANDWIDTH

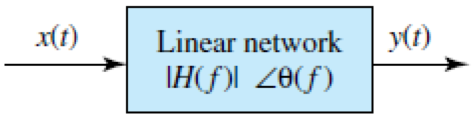
- The range of frequency over which frequency components are present in spectrum of a signal is called signal bandwidth.
- In other words signal bandwidth is defined as the difference between the maximum and minimum values of frequencies present in signal spectrum.
- Bandwidth of a voice signal is 3 kHz, high quality audio is 20 kHz and that of a video signal is 5 MHz.

FILTERING

- A filter is a frequency selective network which allows certain frequency components to pass through it without attenuation (i.e reducing their value) and does not allow other frequency components to pass through it (i.e. heavily attenuate).
- For example, a low pass filter designed with cutoff frequency f_c will allow all frequency components below f_c to pass through it and heavily attenuate frequency components above f_c.
- Similarly a high pass filter designed with cutoff frequency f_c will allow all frequency components above f_c to pass through it and heavily attenuate frequency components below f_c.
- When as signal passes over the transmission line it may get contaminated with other unwanted (noise) signals. These signals if have different frequencies compared to signal frequency then they can be removed easily using a filter.

DISTORTION

- Any undesired alteration in the nature of the signal is known as distortion. Generally when a signal is communicated over a transmission line they may be subjected to distortion.
- Distortionless transmission line is one in which the signal obtained at the output of the transmission line is replica of the signal at the input. This means that the shape of the signal is preserved.
- Transmission line can be modeled as liner network as shown in the Figure 11 where H(f) indicates the effect on amplitude of various frequency components of signals and θ(f) indicates the effect on phase of various frequency components of signal.



Where,

K= scaled factor

t_d = delayed in time

Figure 11 Model of transmission line.

Transmission line is said to be distortion less if

$$|H(f)| = K \qquad \text{and} \qquad \theta(f) = -360^\circ(t_d f)$$

- This means that the transmission line is distortion free if amplitude and phase of all frequency components present in the signal are uniformly affected. In this case the overall amplitude of the signal may change and signal may be delayed(t_d) but still the nature of shape of the signal remains same.

- Transmission line that does not satisfy this condition will cause distortion in the signal.

Types of distortions:

- Amplitude or frequency distortion: If the amplitude of different frequency components in the input signal x(t) is affected differently i.e. H(f) ≠ K (constant) then such distortion is called amplitude or frequency distortion.
- Phase distortion: If different frequency components in the input signal x(t) are delayed by different time i.e. effect of phase at different frequency is not constant then such distortion is called phase distortion.

EQUALIZER

- Equalizer is a system that is used to compensate for the distortion that may occur in the signal when communicated over the transmission line.

- Equalizer is used at the receiver end of the transmission line. The signal received from the transmission line is given to the equalizer. Equalizer is designed such that it affects the amplitude and phase of various frequency components of the signal so as to compensate for the distortion that may have occurred in transmission line.

- Thus, as shown in Figure 2 transmission line followed by equalizer combinely acts as a distortionless transmission line.

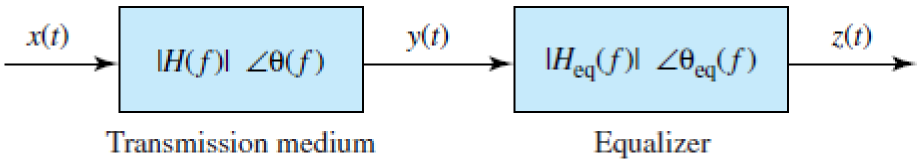


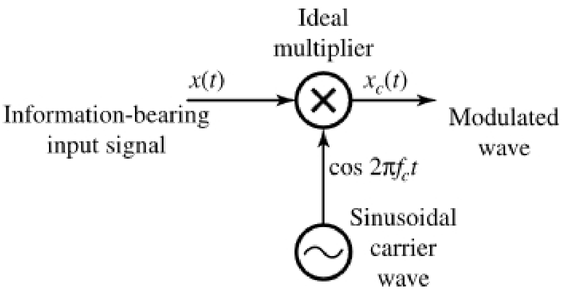
Figure 12, Transmission line with equalizer.

MODULATION

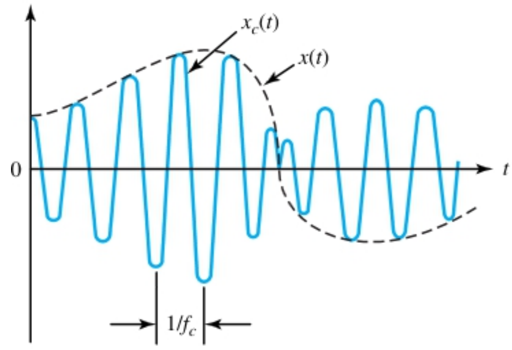
- Modulation is the process whereby the amplitude (or another characteristic) of a wave is varied as a function of the instantaneous value of another wave. The first wave, which is usually a single-frequency wave, is called the carrier wave; the second is called the modulating wave.
- Demodulation or detection is the process whereby a wave resulting from modulation is so operated upon that a wave is obtained having substantially the characteristics of the original modulating wave.
- Modulation and demodulation are then reverse processes. The information from a signal x(t) is impressed on a carrier waveform whose characteristics suit a particular application.
- If the carrier is a sinusoid, we will see that a phenomenon known as frequency translation occurs. If, on the other hand, the carrier is a pulse train, the modulating signal needs to be sampled as part of the modulation process.
- Frequency translation and sampling have extensive use in communication systems. Both of these lend to multiplexing, which permits a transmission system to handle two or more information-bearing signals simultaneously.

Frequency Translation and Product Modulation

- The basic operation needed to build modulators is the multiplication of two signals. Whenever sinusoids are multiplied, frequency translation takes place.



(a) Product modulator



(b) Waveforms

Figure 13

- Figure 13(a) shows a product modulator, which multiplies the signal x(t) and a sinusoidal carrier wave at frequency f_c to yield

$$x_c(t) = x(t) \cos 2\pi f_c t$$

- Choosing x(t) to be a low-pass signal with bandwidthW << fc, Figure 13 (b) depicts the relationship between $x_c(t)$ and x(t). The modulated wave $x_c(t)$ can now be seen to have a bandpass spectrum resulting from frequency translation.
- If x(t) contains a sinusoidal component $A_m \cos 2\pi f_m t$, multiplication by a sinusoidal carrier wave $\cos 2\pi f_c t$ with $f_c >> f_m$ yields

$$(Am \cos 2\pi f_m t) \times (\cos 2\pi f_c t) = \frac{Am}{2} \cos 2\pi(fc - f_m)t + \frac{Am}{2} \cos 2\pi(fc + f_m)\, t$$

Where,

f_c =frequency of carrier signal

f_m = frequency of modulated signal

- Waveforms of the signal, the carrier wave, and the product, as well as their respective line spectra, are shown in Figure. Notice that the low frequency fm has been translated to the higher frequencies $f_c \pm f_m$.

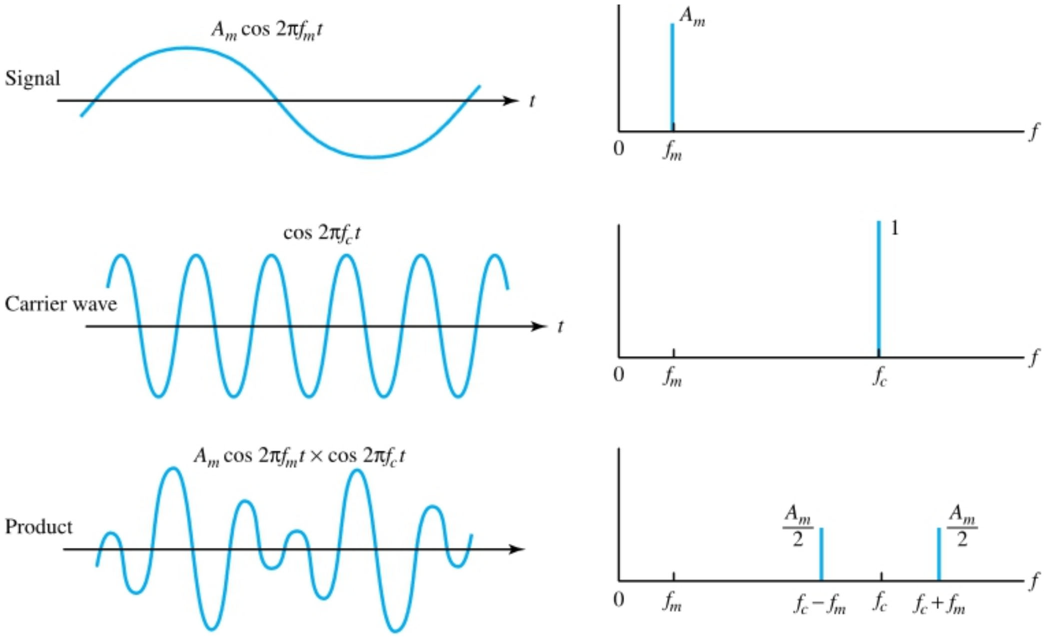


Figure 13.1

- Next, let us consider an arbitrary low-pass signal x(t) with the typical amplitude spectrum of Figure 13.1.(a). The amplitude spectrum of the modulated wave $x_c(t)$ will now have two sidebands (lower and upper sidebands), each of width W on either side of f_c, as illustrated in
- Figure 13.1(b). Thus, we have a signal that can be transmitted over a bandpass system with a minimum bandwidth of

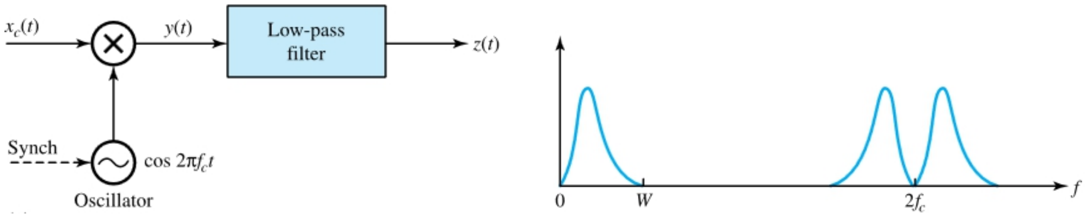
$$B = 2W$$

- which is twice the bandwidth of the modulating signal. This process is then known as double sideband modulation (DSB). Either the lower or the upper sideband may be removed by filtering so as to obtain single-sideband modulation (SSB) with B=W, if the bandwidth needs to be conserved.
- By choosing the carrier frequency f_c at a value where the system has favorable characteristics, the frequency translation by product modulation helps in minimizing the distortion and other problems in system design.
- Now, in order to recover x(t) from $x_c(t)$, the product demodulator shown in Figure 14.(a),which has a local oscillator synchronized in frequency and phase with the carrier wave, can be used. The input y(t) to the low-pass filter is given by

$$\begin{aligned} x(t)\cos 2\pi f_c t &= x(t)\cos^2 2\pi f_c t \\ &= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\pi(2f_c)t \end{aligned}$$

indicating that the multiplication has produced both upward and downward frequency translation.

- In Equation above, the first term is proportional to x(t), while the second looks like DSB at carrier frequency $2f_c$. Then, if the low-pass filter in Figure 14(a) rejects the high-frequency components and passes $f \leq W$, the filtered output z(t) will have the desired form z(t) = Kx(t).

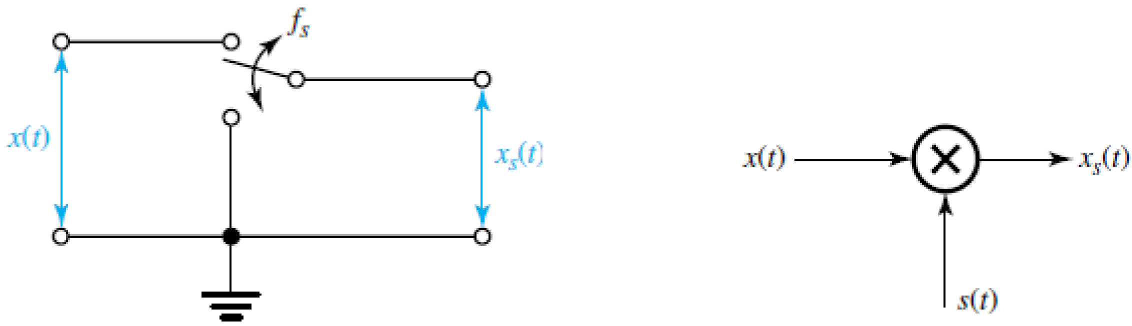


(a) Product demodulator. (b) Spectrum prior to low-pass filtering.

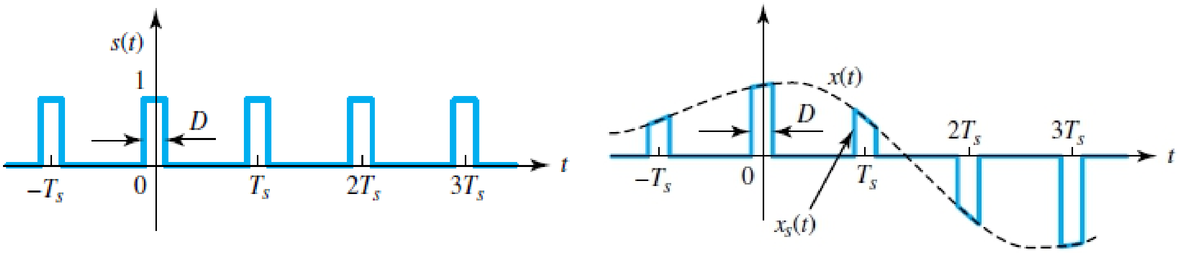
Figure 14

SAMPLING

- Sampling is a process of converting a continuous time signal to discrete time signals.
- Thus a signal which was described at all instances of time, after sampling is described only by its values at discrete instances of time commonly called samples.
- Number of samples taken per second is called the sampling frequency (f_s). Hence time between two samples called sampling interval can be defined as $T_s = 1/f_s$
- The process of sampling can be understood as shown in Figure 15 (a).
- The switch is closed for a small interval of time at every T_s instance and for this time the input is available on the output side. This can be modeled as shown in Figure 15(b) using a switching function s(t) as shown in Figure 15(c).
- Sampled signal is multiplication of continuous time signal x(t) with the switching signal s(t).



(a) (b)



(b) (d)

Figure 15 (a) Switching sampler (b) Model of sampler (c) Train of pulses as switching function (d) sampled signal

$$x_s(t) = x(t)s(t)$$

- As signal is represented only by discrete samples, the complete information is not retained. However, sampling theorem gives the condition of proper sampling that if satisfied it is possible to reconstruct the signal back from the sample values.

- **Sampling Theorem**: Minimum sampling frequency that is necessary to reconstruct the signal back from the sampled signal should be twice that of the maximum frequency component present preset in the signal being sampled.
- Thus if the signal to be sampled has all frequency components within range of $0 \leq f \leq W$ then $f_s \geq 2W$. This sampling frequency is called Nyquist rate.
- Higher the sampling frequency better will be the reconstruction of original signal from the sampled signal.
- If the signal is sampled as sampling frequency less than the Nyquist rate, then it will not be possible to reconstruct the original signal back and the information will be lost. Error that occurs due to this is called aliasing.

Example

Consider the analog signal

$$x_a(t) = 3\cos 100\pi t$$

a) Determine the minimum sampling rate required to avoid aliasing.
b) Suppose that the signal is sampled at the rate F_S= 200 Hz. What is the discrete-time signal obtained after sampling?
c) Suppose that the signal is sampled a: the rate F_S = 75 Hz. What is the discrete-time signal obtained after sampling'!
d) What is the frequency $0 < F < F_S/2$ of a sinusoid that yields samples identical to those obtained in part (c)?

Solution

a) The frequency of the analog signal is F = 50 Hz. Hence the minimum sampling rate required to avoid aliasing is F_S = 100 Hz.
b) If the signal is sampled at F_S= 220 Hz. the discrete-time signal is

$$x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$$

c) If the signal is sampled at F_S= 75 Hz. the discrete-time signal is

$$\begin{aligned} x(n) &= 3\cos\frac{100\pi}{75}n = 3\cos\frac{4\pi}{3}n \\ &= 3\cos\left(2\pi - \frac{2\pi}{3}\right)n \\ &= 3\cos\frac{2\pi}{3}n \end{aligned}$$

d) For the sampling rate of F_S= 75 Hz, we have

$$F = f\,F_S = 75f$$

The frequency of the sinusoid in part (c) is f = 1/3. Hence

$$F = 25 \text{ Hz}$$

Clearly, the sinusoidal signal

$$y_a(t) = 3\cos 2\pi Ft$$
$$= 3\cos 50\pi t$$

Sampled at F, = 75 samples/s yields identical samples. Hence F = 50 Hz is an alias of F = 25 Hz for the sampling rate F, = 75 Hz.

PULSE MODULATION

- Pulse modulation systems represent a message signal by a train of pulses.
- Pulse modulation can be either analog or digital.
- In analog pulse modulation, the amplitude, the width or position of the carrier pulse is changed in accordance with the modulating signal. The three basic pulse modulation techniques are Pulse Amplitude Modulation (PAM), Pulse Width Modulation (PWM) and Pulse Position modulation (PPM).

Classification

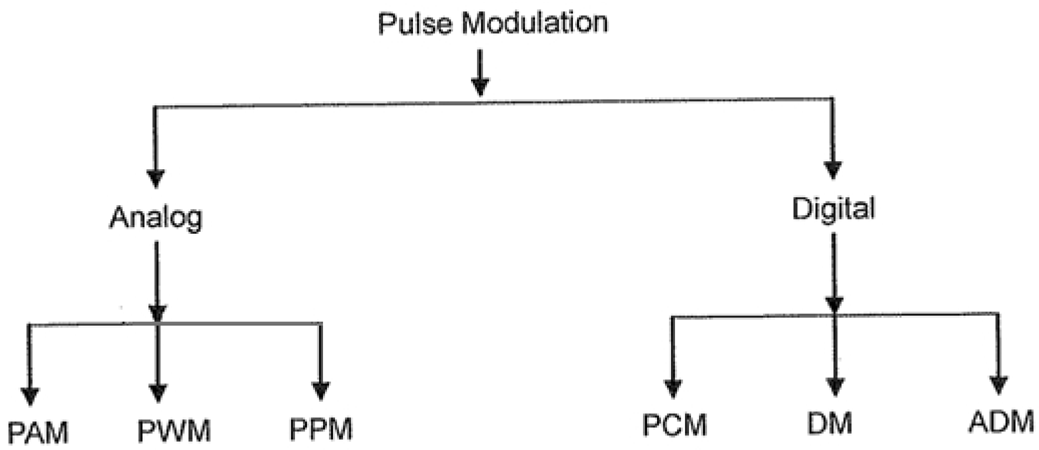


Figure 16

General block diagram

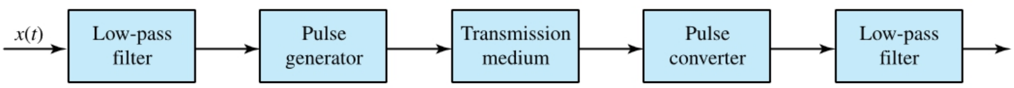


Figure 17

- The pulse generator produces a pulse train with the sampled values carried by the pulse amplitude, duration, or relative position. These are then known as pulse amplitude modulation (PAM), pulse duration modulation or pulse width modulation (PDM or PWM), and pulse position modulation (PPM), respectively.

- At the output end, the modulated pulses are converted back to sample values for reconstruction by low-pass filtering.

PULSE AMPLITUDE MODULATION (PAM)

- Pulse Amplitude Modulation (PAM) is based on a conversion the signal into a series of amplitude-modulated pulses as illustrated in Figure 18

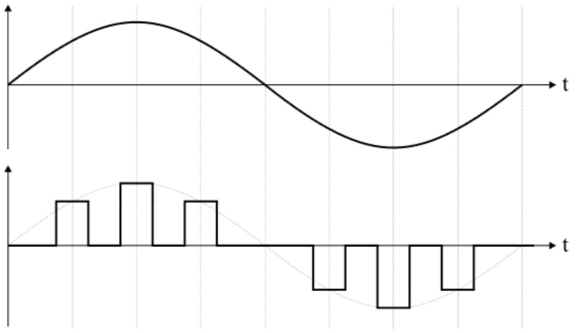


Figure 18

- In PAM the amplitude of a constant width, constant-position pulse is varied according to the amplitude of the sample of the analog signal.
- PAM waveforms resemble the original analog signal more than the waveforms for PWM or PPM.
- This is used as an intermediate form of modulation with PSK, QAM, and PCM, although it is seldom used by itself.

Pulse Width Modulation (PWM)

- It is sometimes called pulse duration modulation (PDM) or pulse length modulation (PLM).
- In PWM width of a constant amplitude pulse is varied proportional to the amplitude of the analog signal at the time the signal is sampled
- PWM used in special-purpose communications system mainly for the military but are seldom used for commercial digital transmission.
- Figure 19 shows how PWM works on analog information signal.

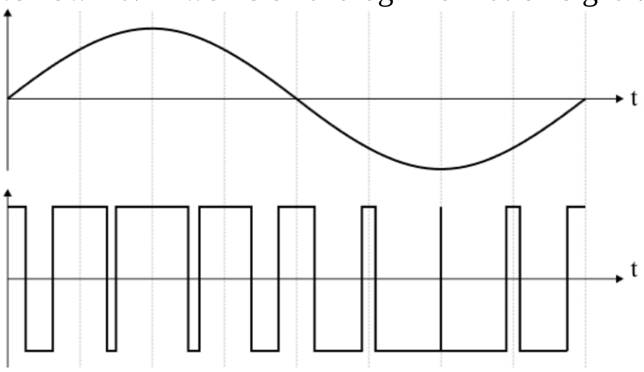


Figure 19

Pulse Position Modulation (PPM)

- In PPM the position of a constant-width pulse within a prescribed time slot is varied according to the amplitude of the sample of the analog signal.
- The higher the amplitude of the sample, the farther to the right the pulse is positioned within the prescribed time slot.
- The highest amplitude sample produces a pulse to the far right, and the lowest amplitude sample produces a pulse to the far left.
- It is used in special-purpose communications system mainly for the military but are seldom used for commercial digital transmission
- Figure 20 shows how PPM works on analog information signal

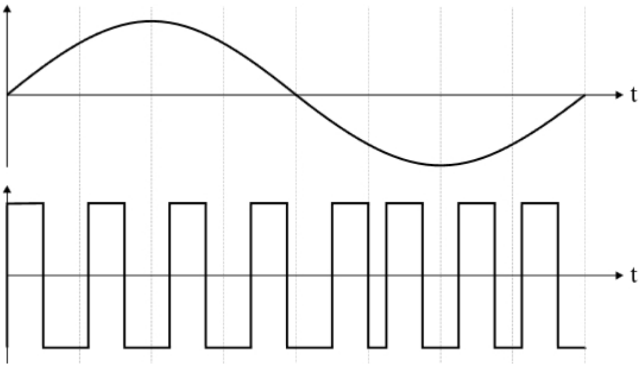


Figure 20

MULTIPLEXING

- Multiplexing is an operation where in two or more signals are transmitted jointly over the same transmission channel.
- Multiplexing helps to efficiently use resources for communication more information.

- There are two commonly used methods for multiplexing.
  1. Time division multiplexing (TDM)
  2. Frequency division multiplexing (FDM)

Time division multiplexing (TDM)

- In TDM the same transmission channel is used by more than one source of information in time shared mode. Information from each source is transmitted on the channel one-by-one. Basic model of TDM is shown in Figure 21.

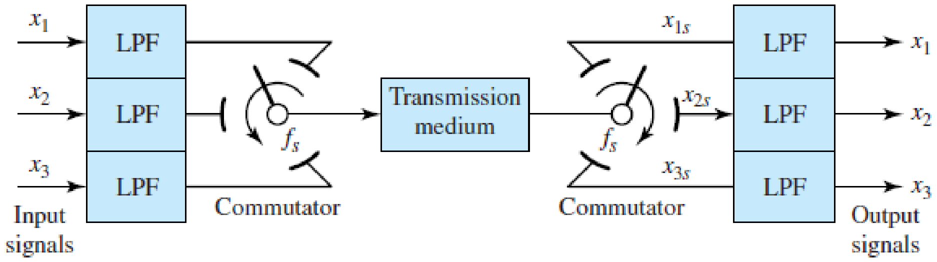


Figure 21. Basic model of TDM system.

- On the input side an electronic switch obtains samples from each input every T_s seconds.
- The sample taken is passed over the transmission line. Similarly on the output side another synchronized electronic switch distributes the samples to individual channels for reconstruction.
- Low pass filters (LPF) ensures that sampling and reconstruction occurs without errors due to sampling.

Application of TDM: Network data transmission, ISDN, WAV audio standard etc

Frequency division multiplexing (FDM)

- In FDM more than one signal are transmitted over a single communication channel at different frequencies.
- FDM system works on sharing the available bandwidth of the communication channel. Basic block diagram of FDM is shown in Figure 22.

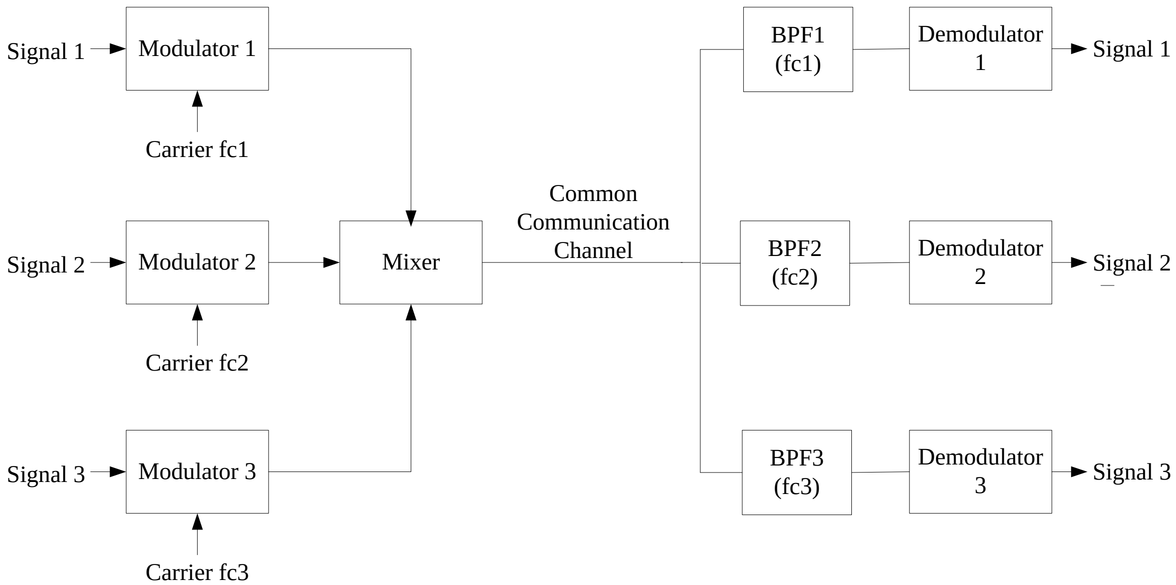


Figure 22 Block diagram of FDM.

- On the input side each signal is modulated at using different carrier frequencies. These signals of different frequencies are mixed and transmitted over the channel.
- On the receiver side the signal is given to number of bandpass filter designed with different band frequencies.
- The band frequencies are corresponding to carrier frequency on the input side. Bandpass filter will allow to pass signal corresponding to specific frequency only.
- Thus the signals that were mixed gets separated and the information signal can be received by performing demodulation.

Application of FDM: Telephone system, TV broadcasting, FM radio

INTERFERENCE

Interference refers to the effect of unwanted signals/phenomenon that contaminate (introduce error) the information signal.

Interfering signals can enter the system in following ways:

- *Capacitive coupling*, because of the stray capacitance between the system and an external voltage
- *Magnetic coupling*, because of the mutual inductance between the system and an external current
- *Radiative coupling*, because of electromagnetic radiation impinging on the system, particularly in the channel
- *Ground-loop coupling*, because of the currents flowing between different ground points

Interference can be of different types

- AC hum due to power lines
- High frequency pulses
- Erratic waveforms known as statics

Effect of interference is reduced generally through following practices.

- Shielding i.e. enclosing the system inside a conducting box which is connected to earth, so that any interfering signal will get directly by pass to ground.
- Interference is generally a common mode signal and hence this can be rejected using differential amplifier having high CMRR (Common Mode Rejection Ratio).
- Using appropriate filters that will help to separate interfering signal from information signal on basis of their frequencies.

NOISE

- Any unwanted signal that contaminates the information signal is called Noise.
- Noise may get added to the information signal from number of sources both external and internal to the system.
- Noise can be categorized into four types
  1. Thermal noise
  2. Intermodulation noise
  3. Cross talk
  4. Impulse noise

Thermal noise

- Thermal noise is due to random motion of electrons in the wire. It is present in all electronic devices and transmission line. It is proportional to temperature. It uniformly distributed over all frequencies and hence also called as white noise.

Thermal noise power = N = kTB Watts
Where k = Boltzman's constant
T = Temperature in Kelvin
B = Bandwidth

Intermodulation

- Intermodulation noise is generated when signals with different frequencies are transmitted over common transmission channel as in FDM. These produces additional components with frequencies equal to sum and difference of signal frequencies traveling on the medium

Crosstalk

- Crosstalk basically means interference between two adjacent lines carrying information signal. It occurs due to electrical coupling between nearby conductors. Very common example of crosstalk is over telephone lines.

Impulse noise

- Impulse noise is a kind of momentary noise that may be generated due to external electromagnetic disturbance, lightning etc. It thus different from the earlier three noise which are regular in nature.

SIGNAL TO NOISE RATIO (SNR)

- Signal to noise ratio is defined as

$$\text{SNR} = \frac{\text{Average signal power}}{\text{Average noise power}}$$

- SNR is ratio of desired signal power to unwanted noise power.
- SNR indicates the effect of noise over the signal. It should be as high as possible.
- SNR is a ratio of two powers. So it is often defined in decibels (dB).

$$[\text{SNR}]_{\text{dB}} = 10 \log_{10} \text{SNR}$$

ATTENUATION

- As the signal travels over a distance on the transmission line it losses some energy in form of heat due to resistance of the medium. Because of this strength of the signal reduces. This is called attenuation.
- Attenuation is expressed in decibels as

$$\text{Attenuation} = 10 \log_{10} \frac{P_{\text{out}}}{P_{\text{in}}}$$

Where P_{in} = power at sending end
P_{out} = power at receiving end

- Effect of attenuation can be reduced by introducing amplifiers (generally called repeaters) on the transmission line.

PRACTICAL APPLICATION: A CASE STUDY

Antinoise Systems - Noise Cancellation

- Traditionally sound-absorbing materials have been used quite effectively to reduce noise levels in aircraft, amphitheaters, and other locations. An alternate way is to develop an electronic system that cancels the noise.
- For passengers in airplanes, helicopters, and other flying equipment, a proper headgear is being developed in order to eliminate the annoying noise.
- For industrial workers who are facing long-term ill effects due to various noises at their workplace and for persons who are irritated by the pedestrian noise levels in certain locations, antinoise systems that nearly eliminate or nullify noise become very desirable.

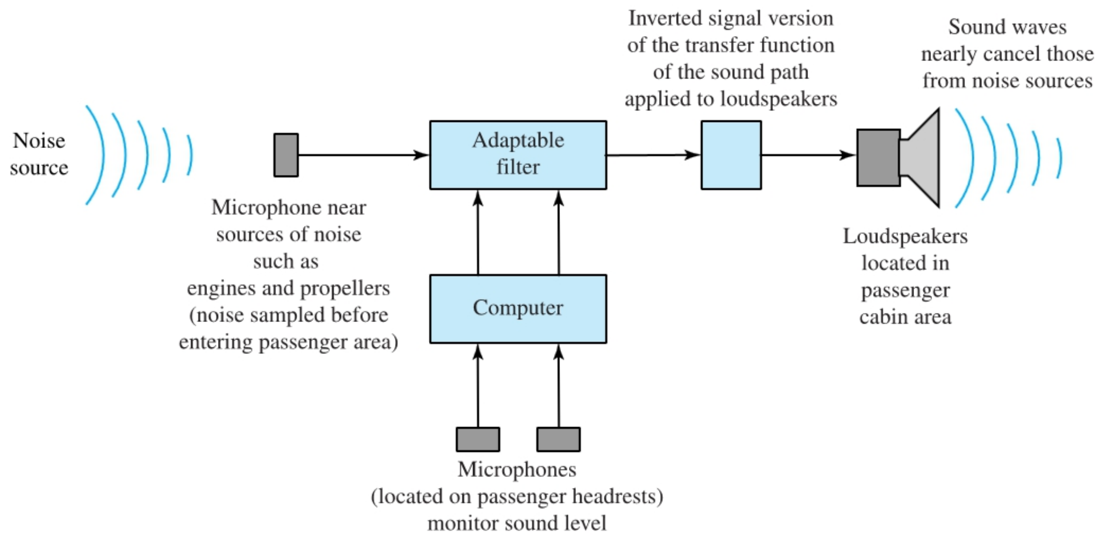


Figure 24 Block diagram of antinoise system to suppress the noise in an aircraft.

- Figure 24 illustrates in block-diagram form the principle of noise cancellation as applied to an aircraft carrying passengers.
- The electric signal resulting after sampling the noise at the noise sources is passed through a filter whose transfer function is continuously adjusted by a special-purpose computer to match the transfer function of the sound path.
- An inverted version of the signal is finally applied to loudspeakers, which project the sound waves out of phase with those from the noise sources, nearly canceling the noise. Microphones on the headrests monitor the sound experienced by the airline passengers so that the computer can determine the proper filter adjustments.
- Signal processing, which is concerned with manipulating signals to extract information and to use that information to generate other useful electric signals, is indeed an important and far reaching subject.

Example:

1) Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.
   a) $x(n) = 3\cos(5\pi + n/6)$
   b) $x(n) = 2\exp[\, j(\pi/6 - \pi)]$
   c) $x(n) = \cos(n\pi f2) - \sin(\pi n/8) + 3\cos(n\pi/4 + \pi/3)$
2) Classify the following signals into energy-type or power-type signals, and determine the energy or power content of the signal.

(a) $x_1(t) = e^{-t} \cos t \, u_{-1}(t)$

(b) $x_2(t) = e^{-t} \cos t$

(c)

$$x_3(t) = \mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \\ 0, & t = 0 \end{cases}$$

Note: $\int e^{ax} \cos^2 x \, dx = \frac{1}{4+a^2}[(a \cos^2 x + \sin 2x) + \frac{2}{a}]e^{ax}$.

3) For the power-type signals given, find the power content in each case.

(a) $x(t) = Ae^{j(2\pi f_0 t + \theta)}$.

(b) $x(t) = u_{-1}(t)$, the unit-step signal.

4) Let x(t) = 12 cos 2π100t + 8 cos 2π150t, and $x_c(t)$ = x(t) cos 2πf_ct, where f_c = 600 Hz. Sketch the amplitude spectrum.

(b) List all the frequencies in the product $x_c(t)$ cos 2π500t, where $x_c(t)$ is given in part (a)

QUESTIONS

1. What is signal? Give classification of signal.
2. What is signal spectrum? Explain signal bandwidth.
3. Explain various signals processing operation in:
   a) Analog communication systems
   b) Digital communication systems
4. Explain various signals processing operation:
   a) Filtering
   b) Sampling
5. Explain multiplexing. Also Explain TDM and FDM
6. Explain in detail:
   a) Noise and its type
   b) Interference and its type

CHAPTER 6
COMMUNICATION SYSTEMS

TOPICS COVERED IN THIS CHAPTER

1. **Introduction to Communication Systems**
2. **Transmission lines**
3. **Waveguides**
4. **Antenna fundamentals**
5. **Electronics communication system**
6. **Radio broadcasting**
7. **Television broadcasting**
8. **Mobile communication systems**
9. **Digital communication system**
10. **GPS**
11. **QUESTIONS**

6.1 Introduction to Communication Systems

- A communication system is the means of conveying the information from one place to other, this information can be of different type such as sound, picture, music, computer data etc.
- The communication between human being can be verbal, non-verbal, via body language, facial expression, written words etc.
- Second World War played an important role in development of various communication systems.
- After 1947 invention of transistor, the communication systems like satellite communication system & fiber optic communication system are developed.

6.1.1 Communication Channels:

In any communication system, the communication channel provides transmitter and receiver. Any one of the following communication channels can be used:

- A pair of conducting wires.
- Coaxial cable
- An optical fiber cable.
- An underwater ocean channel. (Acoustical transmission of information)
- Free space for transmission using electromagnetic waves.
- Data storage media such as optical disc, magnetic disc etc.
- Figure 1 shows the frequency bands of the electromagnetic spectrum.

Wired Communication Channels:

- The wire line channels (twisted pair wires) are used primarily for telephone networks that carry voice, data and video information but have bandwidth limitations.
- The co-axial cables are used for higher frequencies and they are guided Electromagnetic Channels.
- The advantages of optical fiber cables are it has very large bandwidth, small size & less interference made it reliable channel for communication.

Wireless Communication Channels:

- The Wireless communication systems use the free space as their communication medium they do not need the wires for sending the information from one place to the other.
- The radio or TV broadcasting; satellite communication are the examples of the wireless communication. These systems transmit the signal using a transmitting antenna in the space.
- The transmitted signal is in the form of electromagnetic waves. A receiving antenna will Pick up this signal and feed it to the receiver.
- Wireless communication can be used for the long distance communication from country to the other or even from one planet to the other.
- The size of transmitting antenna depends on the wavelength of the signal being transmitted.

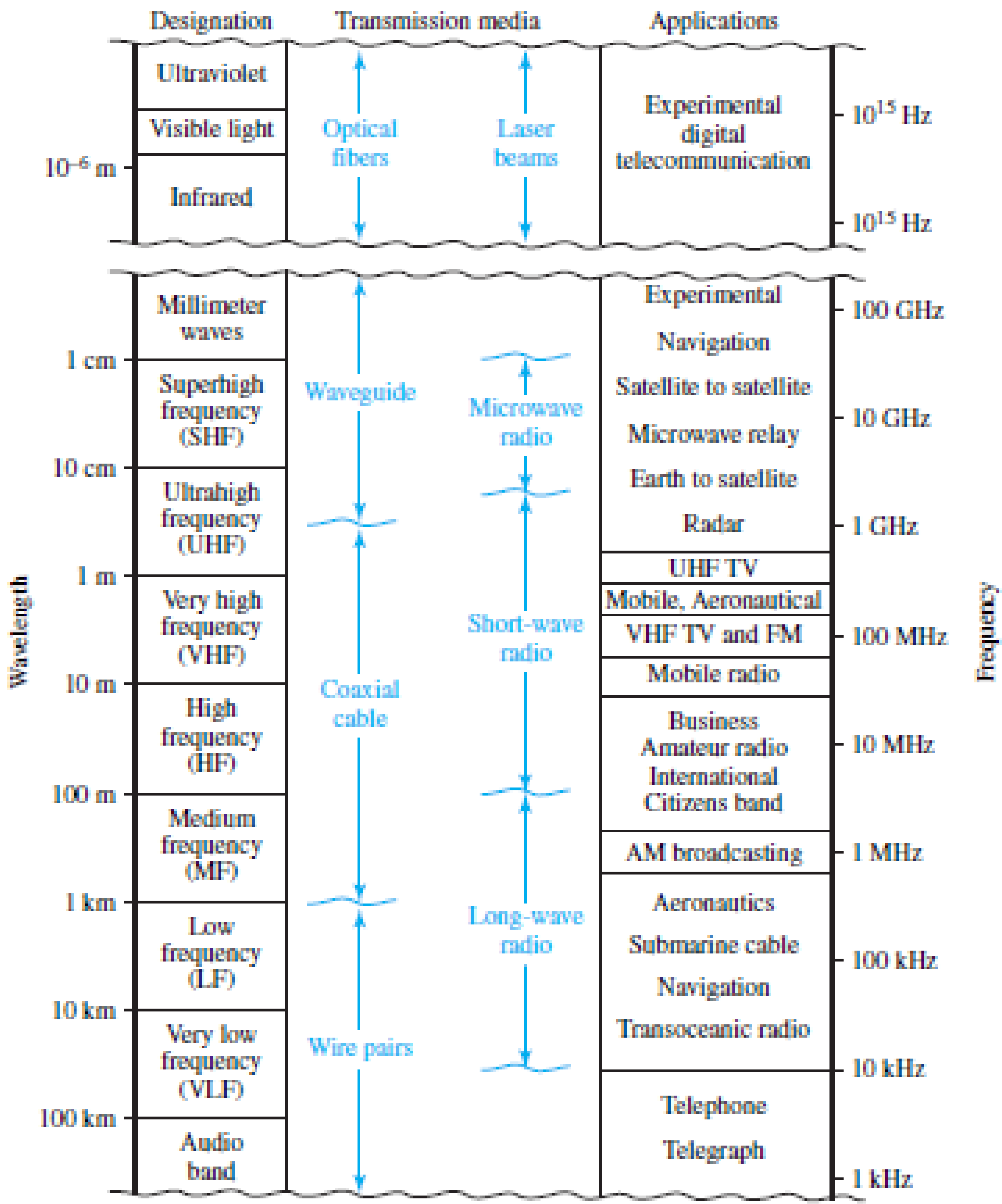


Figure-1 Frequency bands of the electromagnetic spectrum. (*Source:* A. Carlson, *Communication Systems,* 3rd ed., McGraw-Hill, New York, 1986.)

6.1.2 Modes of propagation of EM Waves

- The three possible modes in which an EM wave can propagate through space are:
    - Ground wave propagation.
    - Sky wave propagation
    - Space wave propagation or line of sight (LOS) propagation
- Once the signal leaves the antenna it can take any of the following three paths.
- The three basic paths that a radio signal can take are:
    1) Along the surface of the earth (ground wave propagation).
    2) Up to the layer called "ionosphere" and back to the earth (sky wave propagation).
    3) From transmitter to receiver in a straight line. (Space wave propagation)
- The type of propagation is decided by the path taken by the signal to reach the receiver from the transmitter.
- The path taken by a radio signal depends on many factors including the frequency of the signal, atmospheric conditions and the time of day.
- The figure 2 shows the relation between the type of propagation and frequency.

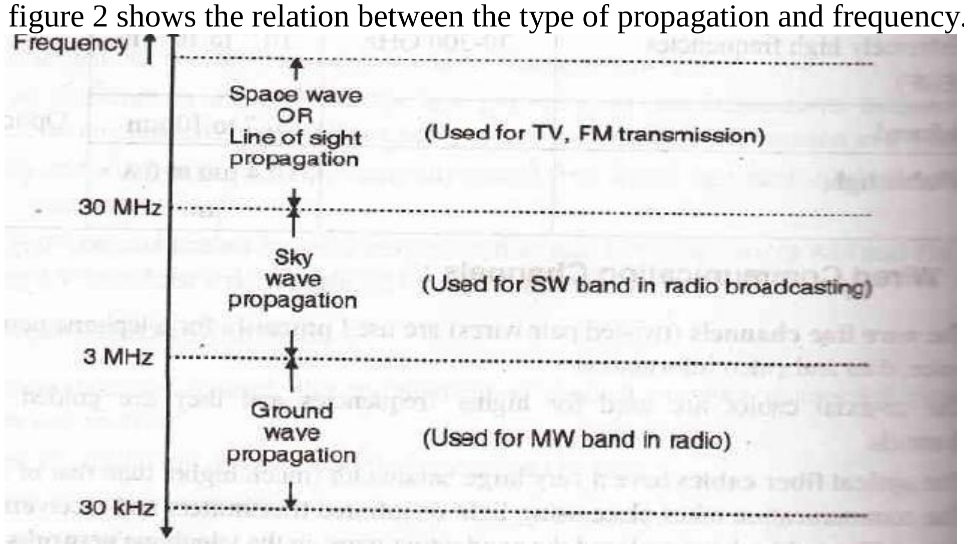


Figure-2 Relation between the type of propagation and frequency

- **Ground Wave propagation:**

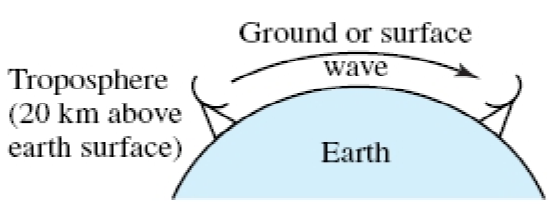


Figure-3 Ground Wave propagation

- The ground or surface wave leaves the antenna and remains closed to the earth. The ground wave will actually follow the curvature of the earth and therefore can travel a distance beyond the horizon. Figure 3 shows the Ground wave propagation.

- The ground wave propagation is the strongest at the low and medium frequency ranges. The ground wave is the path chosen by the signal when the frequency is between 30 kHz and 3 MHz.
- In the AM radio broadcasting operating in MW band.
- The VLF transmission is used for ship communication such as radio navigation and marine mobile communications.

➢ **Sky Wave propagation - The ionosphere:**

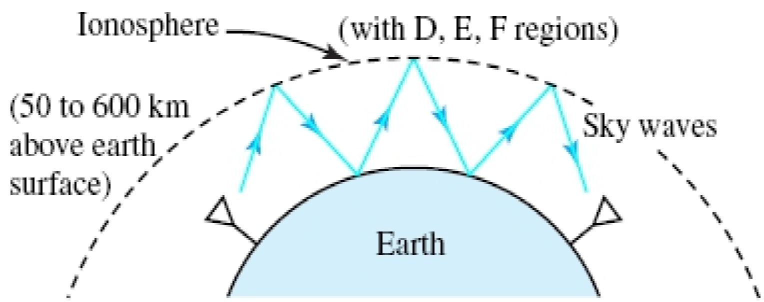


Figure-4 Sky Wave propagation

- In sky wave propagation, the transmitted signal travels into the upper atmosphere where it is bent or reflected back to earth.
- This bending or reflection of the signal takes place due to the presence of a layer called as Ionosphere in the upper atmosphere.
- Due to ionization, this part of the atmosphere becomes electrically charged. The atoms take an extra electron or lose one to become negative or positive ions respectively.
- The free electrons are also present. This layer of ions is known as ionosphere. It is a thick but invisible layer.
- Along with the ultraviolet radiation, the alpha-α, beta-β and gamma-γ radiations from the sun and the cosmic rays are also responsible for the ionization.
- Sky wave propagation is preferred for the short wave (SW) band of frequencies (3 MHz-30 MHz).
- Above 30 MHz the EM waves get absorbed by the ionosphere so, communication is not possible.

➢ **Space Wave propagation (Line of Sight propagation):**

- For the frequencies above 30 MHz the space wave propagation is used.it takes place by the space waves or direct waves. Figure 5 shows the space wave propagation.
- These waves travel in a straight line directly from the transmitting antenna to the receiving antenna.
- Application of space wave propagation is: TV broadcasting, FM radio broadcasting, Microwave links, Satellite Communication.

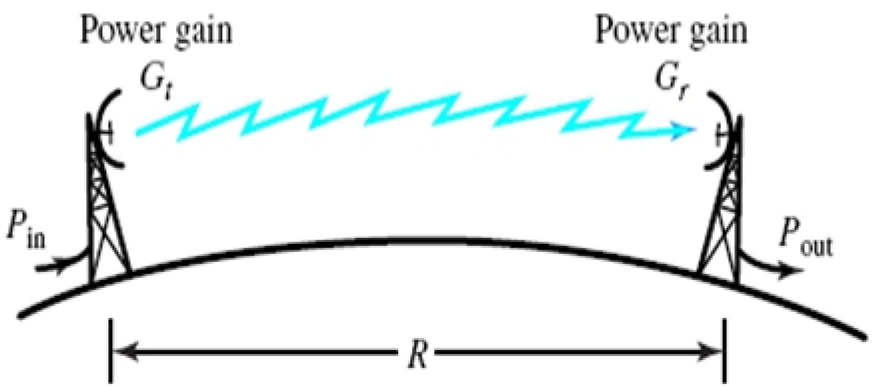


Figure-5 Space Wave propagation

6.1.3 Electromagnetic (EM) Waves

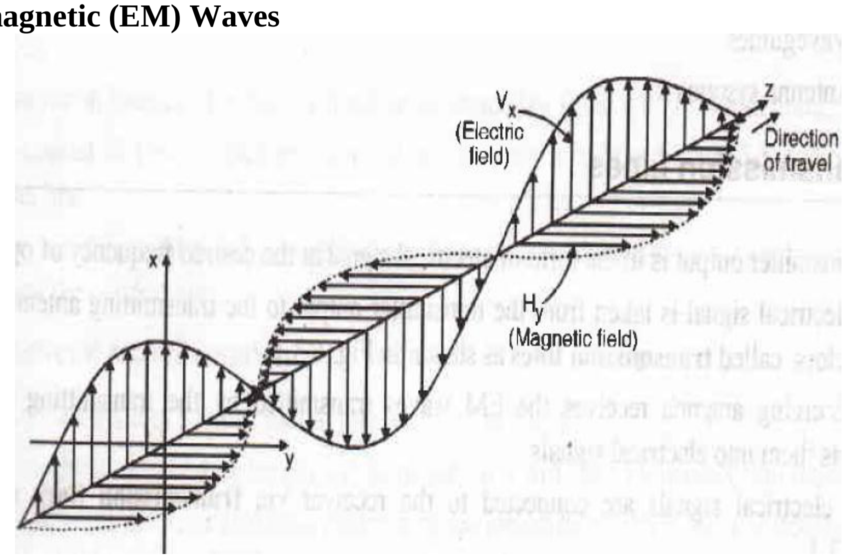


Figure-6 EM Wave

- Figure 6 shows the EM wave. The electromagnetic waves are oscillations, which propagate through free space. They travel through free space at the speed of light.
- These waves are known as electromagnetic waves because the electric and magnetic fields are simultaneously present.
- The directions of these fields are perpendicular to each other and to the direction of propagation of the wave.

Characteristic Impedance:

- The ratio between the electric field intensity and magnetic field intensity is called as characteristic impedance of the medium and it is expressed in ohms.

Polarization:

- The polarization of a plane EM wave is simply the orientation of the electric field vector with respect to the earth surface.
- If the polarization remains constant then it is called as the linear polarization. The linear polarization can be of two types:
    - Horizontal polarization
    - Vertical polarization

6.2 Transmission Lines

- The transmitter output is in the form of electrical signal at the desired frequency of operation.
- This electrical signal is taken from the transmitter output to the transmitting antenna by special conductors, called *"transmission lines"* as shown in figure 7.

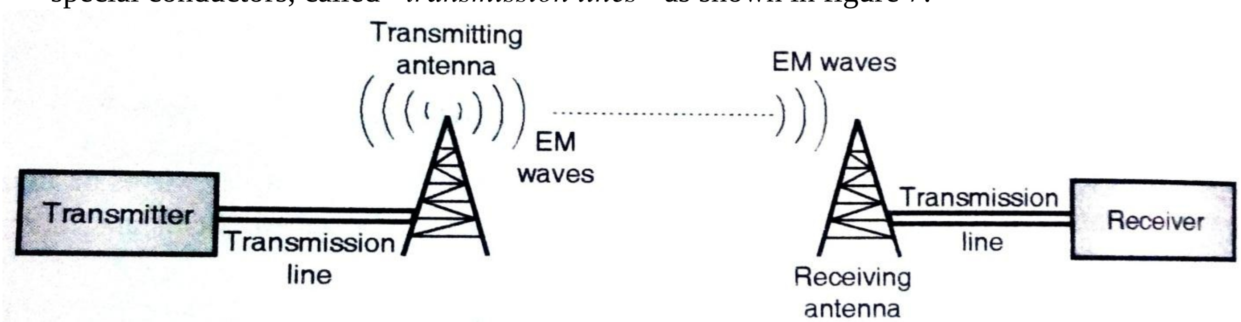


Figure-7 Application of transmission line

- The receiving antenna receives the EM waves transmitted by the transmitting antenna and converts them into electrical signals.
- These electrical signals are connected to the receiver via transmission lines.
- Transmission lines are conducting wires used for connecting points that are some distance apart from each other.
- They are also used for impedance matching between transmitter and antenna and from receiving antenna to receiver.
- All the practical transmission lines are arranged in some uniform pattern, to simplify the calculations, reduce cost and increase convenience.

6.2.1 Types of transmission lines:

- There are two types of commonly used transmission lines as follows :
    - Coaxial (unbalanced) line and
    - Parallel-wire (balanced) line
- The transmission lines can be used for wired communication up to frequencies as high as 18 GHz.
- Transmission lines can be rigid or flexible; air spaced or filled with different dielectric materials.
- They consist of two conductors.
- Figure-8 shows the types of transmission lines.

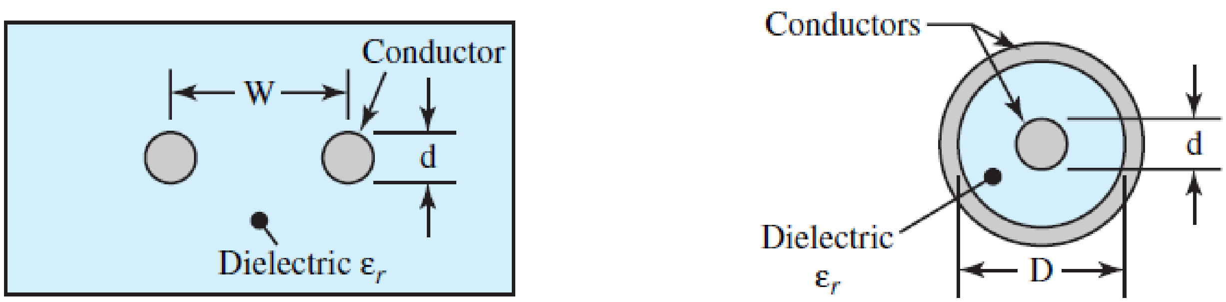


Figure-8 Types of Transmission Lines (parallel wire & coaxial line)

6.2.2 Equivalent circuit representation:

- Figure-9 shows the general equivalent circuit of transmission line.
- Each conductor has certain length and diameter, so it will have inductance and resistance.
- The wires are separated by a medium called dielectric which is not a perfect insulator; some current will flow through it from one conductor to the other. This will give rise to conductance G.
- There is small capacitance present between the two conductors.

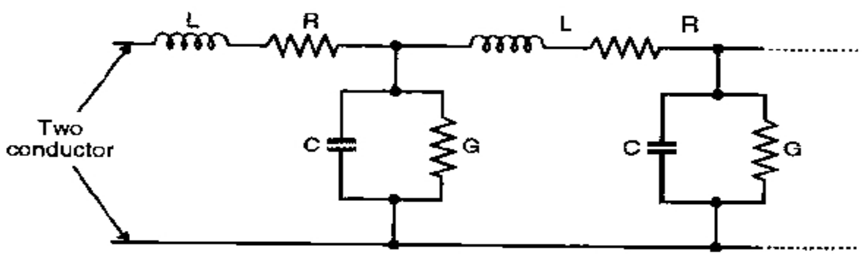


Figure-9 General equivalent circuit of transmission line

6.2.3 Primary line constants:

- The four constants are: distributed series inductance, a distributed shunt distributed series resistance and a distributed shunt conductance.
  - R is series resistance in ohms/unit length
  - L is series inductance in henries/ unit length
  - C is shunt capacitance in farads/unit length
  - G is shunt conductance in suscptance/unit length.

  These are known as primary line constants or primary parameters of line.

6.2.4 Secondary Line Constant:

- The secondary line constants or line parameters are:
  - Characteristic impedance
  - Propagation constant

- ***Characteristic impedance:***
  The characteristic impedance is defined as:

$$Z_o = \left[\frac{R + j\omega L}{G + j\omega C}\right]^{1/2}$$

  Where $\omega = 2\pi f$

- ***Propagation constant:***

  The propagation constant of a transmission line is defined as follows:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

  Where α is called attenuation constant and β is the phase constant.

6.2.5 Advantages:

- Simple construction
- Flexibility
- High mechanical strength
- Not very expensive

6.2.6 Disadvantages:

- It can be used only for point to point communication.
- Power loss increase with increase in frequency.
- It cannot handle high voltage.

6.3 Waveguides

- Any system of conductors and insulators carrying EM waves is called waveguide. It is used at microwave frequencies (300 MHz to 300 GHz)
- At higher frequency, when power levels are large and attenuation in transmission line is significant, connections between system components are often made through waveguide.
- Which are usually hollow, closed, rigid conductor configurations through which wave propagate.
- Waveguides are preferred to transmission lines because at very high frequency they are less lossy than transmission lines.
- Figure-10 shows the simplest shape of the waveguides: rectangular and circular.

6.3.1 Advantages:

- Small size.
- Reduced losses as compared to a transmission line.

- Operation at very high frequencies is possible (up to 325 GHz).
- They are simpler to manufacture.
- Due to absence of the inner conductor, there is no possibility of flashover or sparking between the inner and outer conductors.
- Large power handling capability.

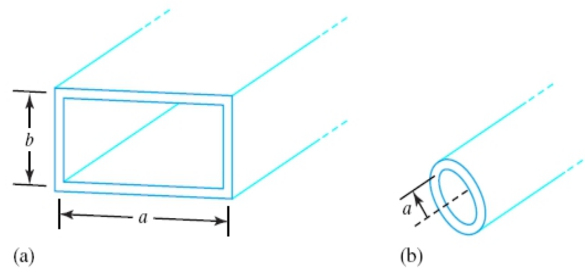


Figure-10 Types of Waveguides (a) Rectangular (b) Circular

6.3.2 Disadvantages:

- Low frequency operation is not possible as they become bulky at low frequencies.
- Absolute efficiency is low.

6.4 Antenna Fundamentals

- Antenna is a metallic object, often a wire or collection of wire which is used to perform following functions :
  - It couples the transmitter output to the free space, or the received input to the receiver.
  - It must be capable of radiating or receiving the electromagnetic waves.
  - It converts the high frequency current into electromagnetic waves and vice versa. Figure-11 shows how an antenna can be used in communication system.

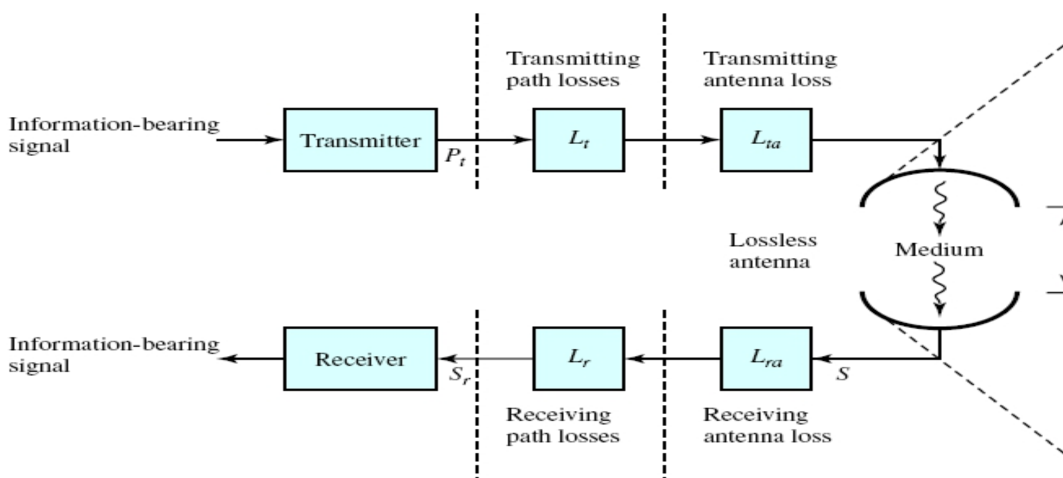


Figure-11 Communication system involving antennas

Where,
P_t =Power generated by the transmitter
$L_{t,}$ L_r= Transmitting-path and receiving-path loss
L_{ta}, L_{ra}= Transmitting antenna loss and receiving antenna loss
R= Distance of separation between the antennas
S= Signal power available at the lossless antenna output
S_r= Signal power available at the receiver input

6.4.1 ISOTROPIC RADIATOR:

- An isotropic radiator is a point source antenna which radiates equally in all the directions.
- All the points at distance "r" from the source lie on the surface of the sphere and have equal power densities.
- The electromagnetic waves spread uniformly in all the directions in space.
- The isotropic radiator is used for study the radiation patterns of other antennas. Figure-12 shows the isotropic radiator.

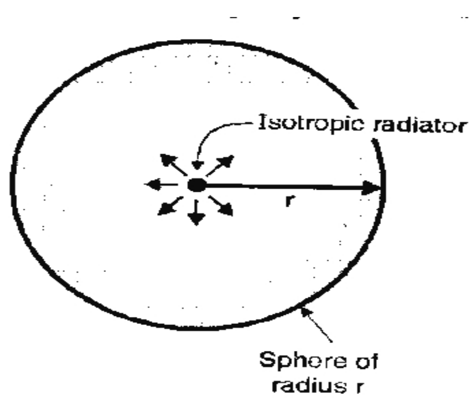


Figure-12 Isotropic radiator

6.4.2 Important Terms and Definitions:

Radiation pattern of Antenna:

- A graph or diagram which tells us about the manner in which an antenna radiates power in different directions is known as the Radiation pattern of antenna. Figure-13 shows radiation pattern of an antenna.
- For a receiving antenna the diagram is known as the directional pattern of the antenna.

Directive Gain:

- The power gain of an antenna is defined as ratio of power fed to an isotropic antenna to the power fed to a directional antenna to develop the same field strength at the same distance, in the direction of maximum radiation.

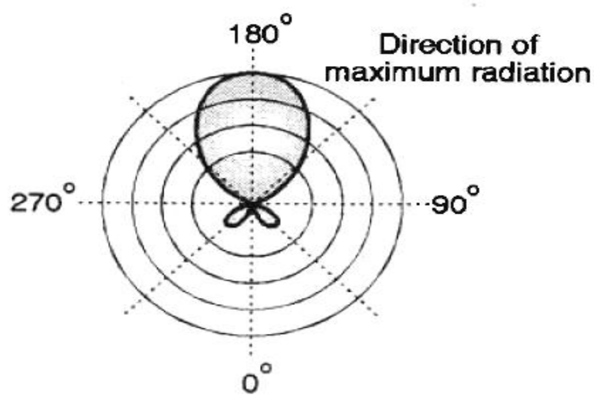


Figure-13 Radiation pattern of an antenna

Antenna Resistance:

- The antenna resistance has two components
    - Radiation resistance
    - Resistance due to the actual losses in the antenna.

Beam width of Antenna:

- Beam width of an antenna is defined as the frequency range over which the operation is satisfactory. Figure-14 shows the beam width of antenna.
- It is the frequency difference between the half power points.
- There are two types of bandwidths. One is related to the radiation pattern and the other one is related to its input impedance.
- The angular separation between two 3 dB down points on the field strength of radiation pattern of antenna.
- Beam width is expressed in degrees.

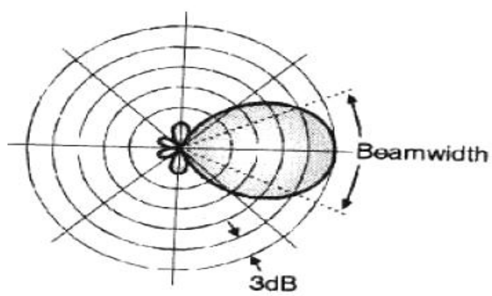


Figure-14 Beam width of antenna

6.4.3 Types of Antennas:

- Different types of antennas are being used depending on the frequency range, gain requirement etc.

Half wave Dipole Antennas:

- The dipole antennas are also called as the resonant antennas.
- A resonant transmission line has a resonant length which is multiple of half wavelength ($\lambda/2$).
- The dipole antennas have lengths of which are all multiples of $\lambda/2$. Hence the dipole antennas are resonant antennas.

Radiation Pattern:

- The radiation is maximum at right angles to the dipole and reduces on both the side of the maximum. The radiation is zero in line with the antenna. Figure-15 shows half wave dipole with its radiation pattern.

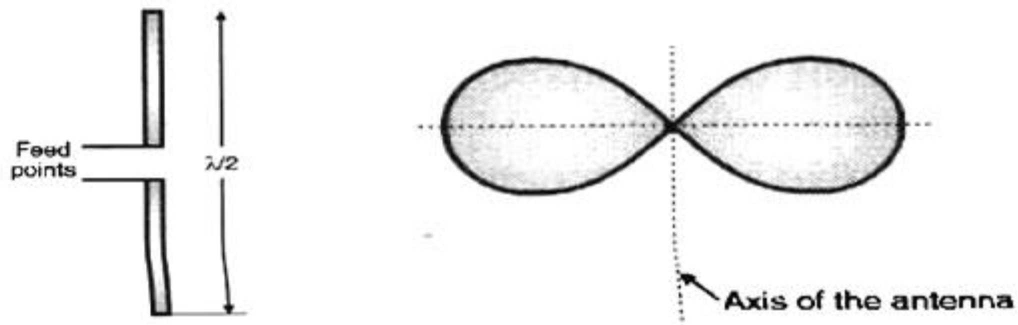


Figure-15 Half wave dipole with its Radiation pattern

- **Folded Dipole Antenna:**

- The dipole antenna used in the Yagi antenna is folded dipole. The folded dipole is a single antenna but it consists of two elements. Figure 16 shows the folded dipole antenna.
- The first element is fed directly while the second one is coupled inductively at its ends.
- The radiation pattern of the folded dipole is same as that of the straight dipole.
- But the input impedance of the folded dipole is four times higher than that of the straight dipole.

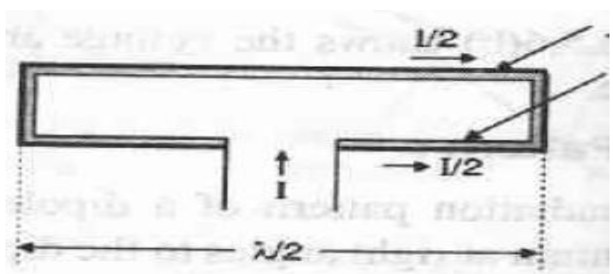


Figure-16 Folded dipole antenna

Advantages of Folded Dipole:

- Higher input resistance.
- Higher bandwidth.
- Ease of construction.

- Cost efficient.
- Impedance matching.

➢ **Yagi-Uda Antenna:**

- The Yagi antenna or Yagi-Uda antenna it is called group of dipole antenna and one or more parasitic element.
- Figure-17 and 18 shows the Yagi antenna & radiation pattern of respectively.
- The Yagi antenna can be used as a transmitting or receiving antenna.
- The YAGI-UDA array is commonly used for television reception. It is usually seen with 3 to 12 elements, although even 40 elements are sometimes employed. Design frequencies from 100 to 1000 MHz are typical.
- A half-wave or folded half-wave dipole is the active element. The array consists of parallel dipoles, all lying in the same plane. The reflector, which reflects waves back toward the active element, enhances radiation in the axis of the array.
- The radiation pattern, with linear polarization, exhibits a principal lobe in the direction. The other elements, called directors, are designed to enhance radiation in the direction. Gain increases with the number of elements and is often in the range of 10 to 20 dB. The Bandwidth is usually small.

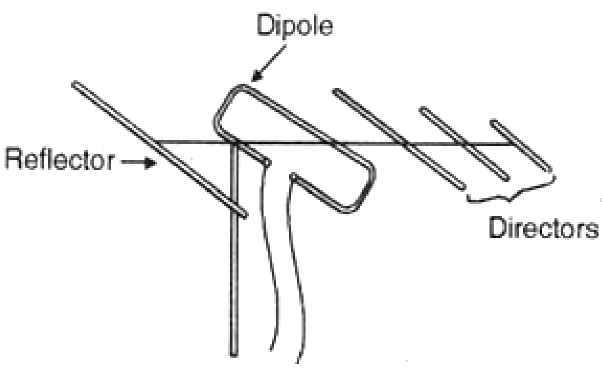


Figure-17 Construction of Yagi antenna

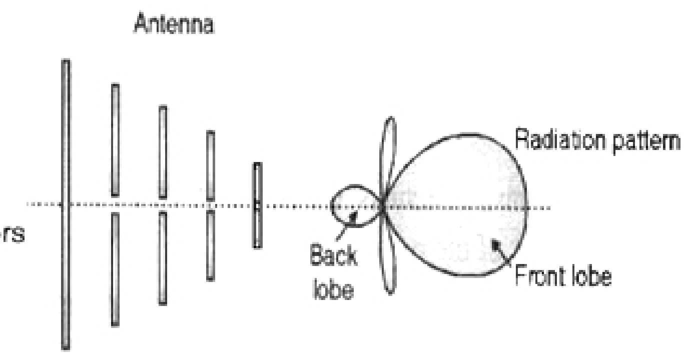


Figure-18 Radiation pattern of Yagi-antenna

Advantages:

- It is a directional antenna.
- It has a moderate gain of about 7dB.
- It is very compact.
- Large bandwidth.
- Can be used at high frequencies.
- Adjustable front-to-back Ratio.

Disadvantages:

- The gain is not very high.
- Needs a large number of elements to be used.

Applications:

- Yagi antenna is used as HF transmitting antenna.
- It is also used at higher frequencies at VHF as TV receiving antenna.

➢ **Microwave Antennas:**

- The frequency band from l to 100 GHz is called as the microwave frequency antennas operating in this frequency band are called as the microwave antennas.
- The microwave antennas are expected to be highly directional.
- Figure-19 shows Dish antenna with a parabolic reflector and direction pattern.
- Types of microwave antennas are:
  - Dish antenna
  - Horn antenna

➢ **Dish Antenna:**

- Dish antennas are the microwave antennas which use parabolic reflectors.
- The special geometric property of a parabolic reflector makes it very useful as a microwave light reflector.

Radiation pattern:

- The radiation pattern includes a narrow main lobe in the desired direction AB and small side lobes in all other directions.

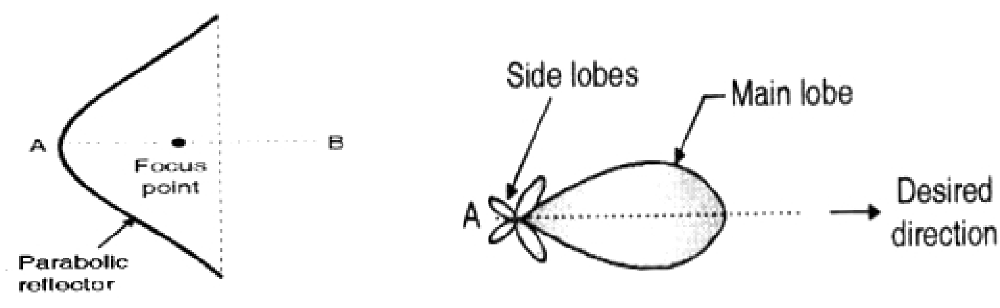


Figure-19 Dish antenna with a parabolic reflector and direction pattern

Advantages:

- Very high gain.
- High directivity.
- Relatively narrow beam width.

Disadvantages:

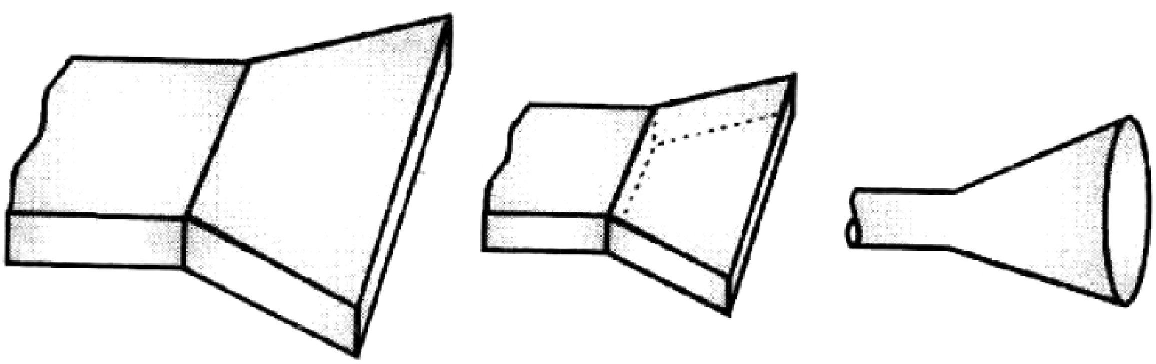
- Presence of side lobes.
- Reception of signals from undesirable sources
- Side lobes created due to diffraction.

Applications:

- Satellite TV reception.
- Point to point microwave links.

➢ **Horn Antennas:**

- When a waveguide is opened, it becomes an electromagnetic horn. Thus waveguides are terminated into horns.
- If the impedance matching is ensured, then all the energy travelling in the forward direction to be radiated.
- The directivity is improved and diffraction is reduced. Figure-20 & 21 shows different types of horn antennas & radiation pattern respectively.
- The three most commonly used horn configurations are:
  1. Sectoral
  2. Pyramidal
  3. Circular



(a) Sectoral (b) pyramidal (c) Circular

Figure-20 Horn antennas

Applications:

- As feed antennas.
- All highly directional microwave antennas
- Satellite antennas.
- Antennas on the spacecrafts.

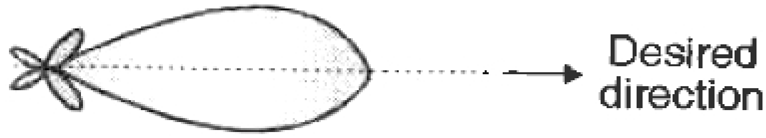


Figure-21 Radiation pattern of horn antennas

6.5 ELECTRONIC COMMUNICATION SYSTEMS

- Classification of electronic communication systems on the basis of nature of information signal.
  1. Analog communication systems
  2. Digital communication systems.

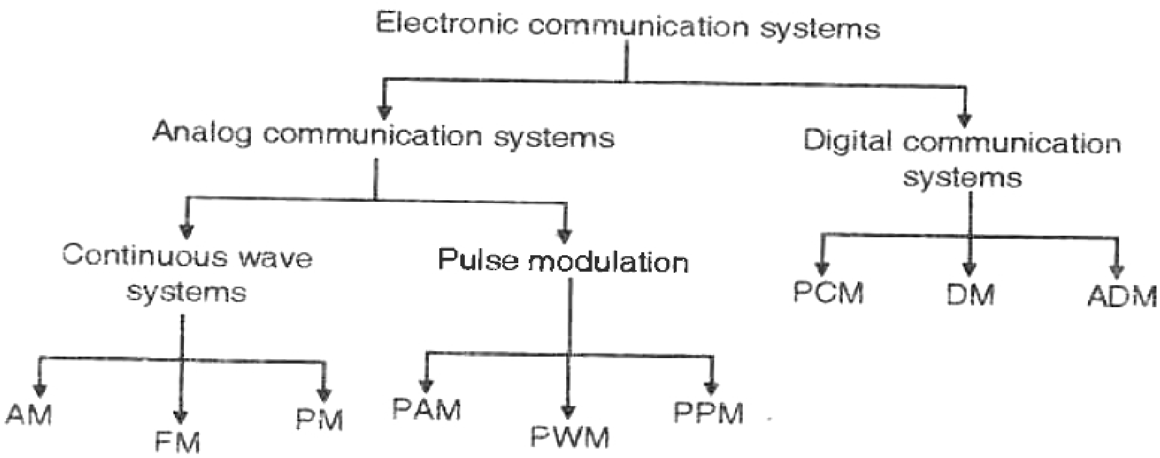


Figure-22 Classification based on analog and digital communication

Examples of analog modulation:

- Amplitude Modulation (AM)
- Frequency Modulation (FM)
- Phase Modulation (PM)
- Pulse Amplitude Modulation (PAM)
- Pulse Width Modulation (PWM)
- Pulse Position Modulation (PPM)

Applications:

- Radio broadcasting (AM and FM.)
- TV broadcasting
- Telephones
- In the subsequent sections we are going to discuss the analog modulation systems one by one.

6.5.1 Amplitude Modulation (AM)

- Amplitude modulation (AM) is the type of analog modulation in which the amplitude of the carrier is varied in proportion with the instantaneous value of information signal.

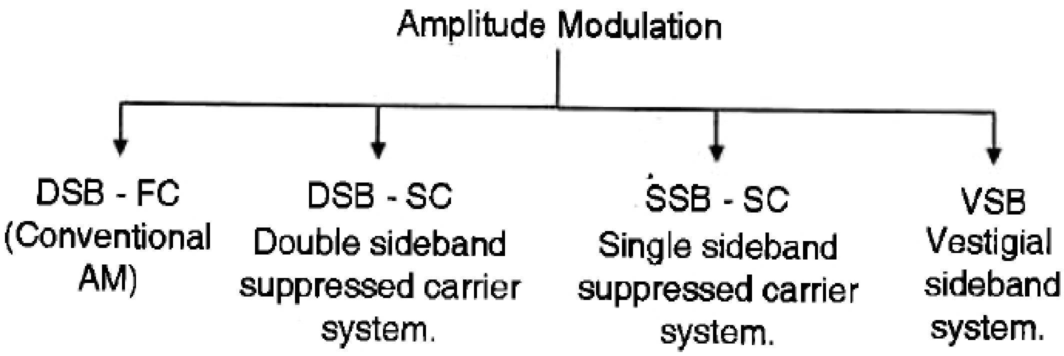


Figure-23 Different types of AM

Conventional AM (DSB-FC):

- The conventional AM is also known as double side band full carrier (DSB-FC) system.
- The frequency of the sinusoidal carrier is much higher than that of the modulating signal.

Generation of AM wave:

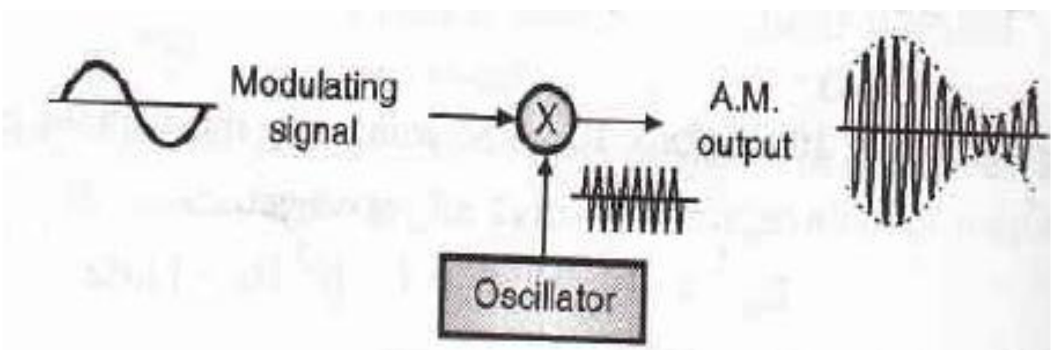


Figure-24 Generation of AM wave

- Figure-24 shows the principle of generation of AM waves. The oscillator produces a sinusoidal carrier of desired frequency.
- This carrier along with the modulating signal is applied to a multiplier circuit. At the output of the multiplier we get the AM wave. Figure-25 shows AM waveforms.

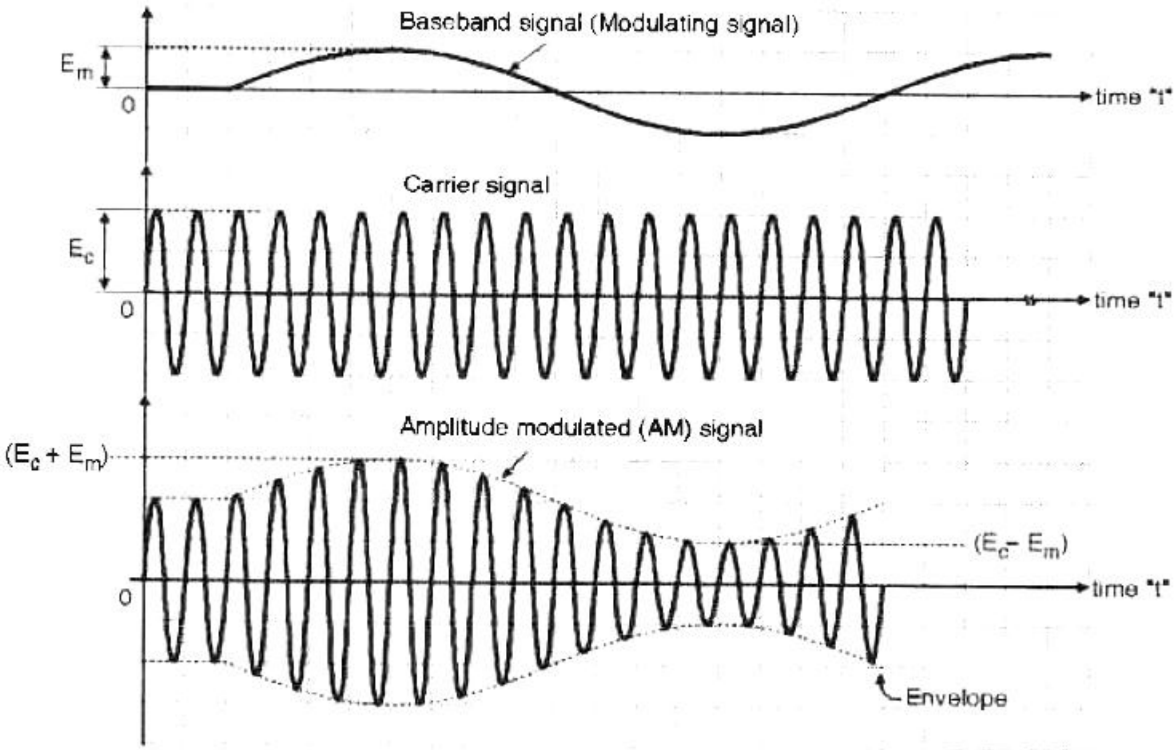


Figure-25 AM waveform

Expression of AM wave

- Let the modulating signal be sinusoidal and be represented as,

$$e_m = E_m \cos(\omega_m t)$$

Where, e_m is the instantaneous amplitude of the modulating signal,

E_m is the peak amplitude,

$\omega_m = 2\pi f_m$ and

f_m= frequency of the modulating signal.

- Let the carrier signal also be sinusoidal at a much higher frequency than that of the modulating signal. The instantaneous carrier signal e_c is given by,

$$e_c = E_c \cos(\omega_c t)$$

Where, E_c = Peak carrier amplitude,

f_c = Carrier frequency and

$\omega_c = 2\pi f_c$.

- The AM wave is expressed by the following expression,

$$e_{AM} = E_c(1 + \frac{E_m}{E_c}\cos(2\pi f_m t))\cos(2\pi f_c t)$$

- Let $m = \frac{E_m}{E_c}$, be the modulation index.

- When $E_m \leq E_c$, the modulation index m has values between o and 1 and no distortion is introduced in the AM wave. But if $E_m > E_c$ then m is greater than 1. This will distort the shape of AM signal. The distortion is called as over modulation

$$e_{AM} = E_c(1 + m\cos(2\pi f_m t))\cos(2\pi f_c t)$$

- This expression represents the Amplitude Modulated (AM) signal i.e. DSB-FC in time.

Frequency spectrum of the AM wave

- So consider the equation for AM wave

$$e_{AM} = (E_c + E_m\cos(\omega_m t))\cos(\omega_c t) = E_c(1 + \frac{E_m}{E_c}\cos(\omega_m t))\cos(\omega_c t)$$

- As per the definition of the modulation index,

$$m = \frac{E_m}{E_c}$$

$$e_{AM} = E_c(1 + m\cos(\omega_m t))\cos(\omega_c t)$$

- Simplify we get, $e_{AM} = E_c\cos(\omega_c t) + E_c m\cos(\omega_m t))\cos(\omega_c t)$
- For the second term in the above expression use the following identity: $2\cos A\cos B = \cos(A + B) + \cos(A - B)$
- Gets simplified as follows:

$$e_{AM} = E_c\cos(\omega_c t) + \frac{mE_c}{2} m\cos(\omega_m + \omega_c)t + \frac{mE_c}{2} m\cos(\omega_m - \omega_c)t$$

- In the above equation first term is carrier, second is upper sideband (USB) and third is lower sideband (LSB)
- The bandwidth of the AM signal is given by the subtraction of the highest and the lowest frequency component in the frequency spectrum.

$$BW = f_{USB} - f_{LSB} = (f_c + f_m) - (f_c - f_m)$$
$$BW = 2f_m$$

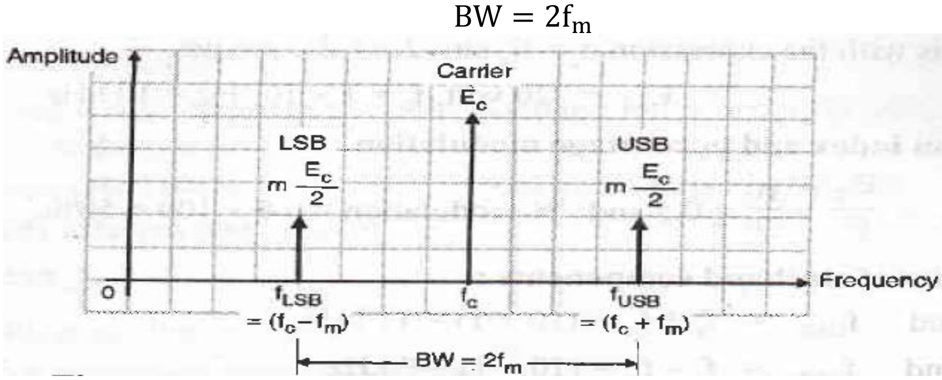


Figure-26 Single sided frequency spectrum of AM wave

Advantages:

- AM transmitters are less complex.
- AM receivers are simple, detection is easy.
- AM receivers are cost efficient.
- AM waves can travel a longer distance.

- Low bandwidth.

Disadvantages:

- Power wastage takes place.
- AM needs larger bandwidth.
- AM wave gets affected due to noise.

Applications:

- Radio broadcasting.
- Picture transmission in a TV system.

AM message demodulation: Standard AM envelope Detector:

- The modulated signals are transmitted by the transmitter via air medium or wire medium.
- At the receiver, the original information signal is separated from the carrier. This process is called as demodulation or detection. Detection is exactly the opposite process of modulation.
- The original modulation signal is recovered back from the AM signal by the process of detection.
- Thus the process of detection or demodulation is the process of recovering the message signal from the received modulated signal

Operation:

- The standard AM wave is applied at the input of the detector.
- In detector, at every positive half cycle of the input the detector diode is forward biased.
- It will charge the filter capacitor C connected across the load resistance R to almost the peak value of the input voltage.

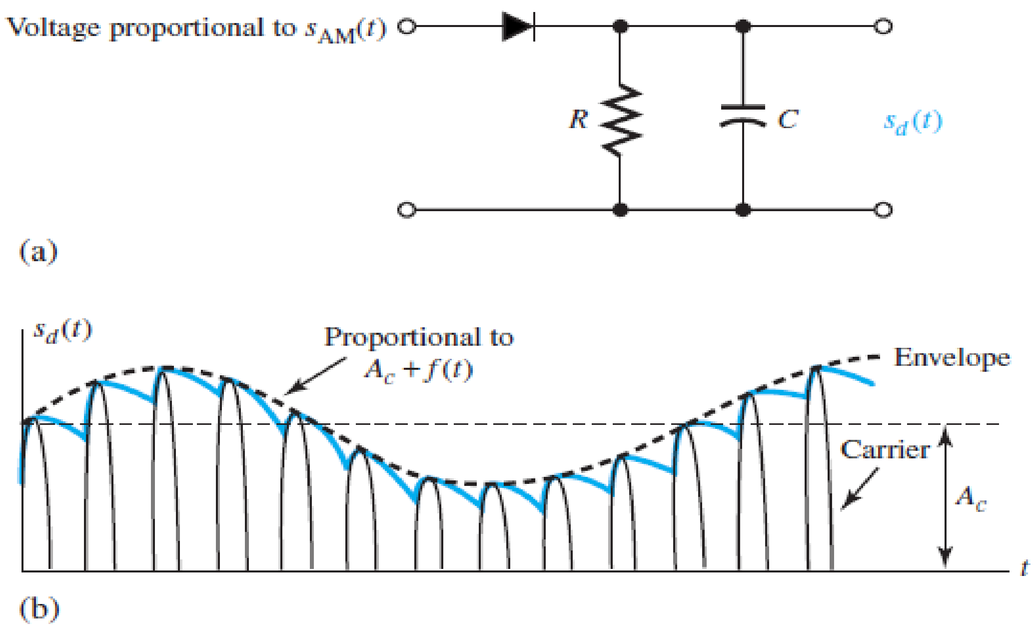


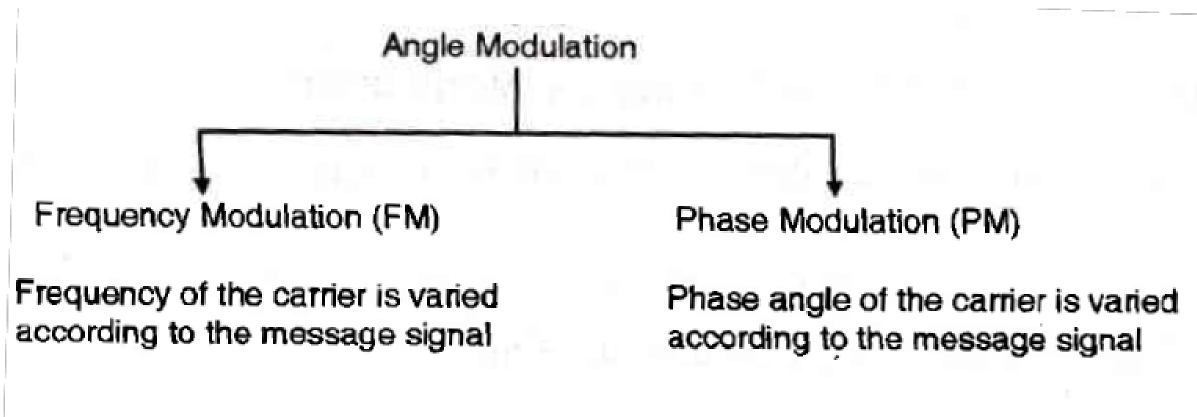
Figure-27 (a) Standard AM envelope detector (b) its response

- As soon as the capacitor charges to the peak value, the diode stops conducting. The capacitor will discharge through R between the positive peaks.
- The discharging process continues until the next positive half cycle. When the input signal becomes greater than the capacitor voltage, the diode conducts again and the process repeats itself.
- The input-output waveforms for the envelope detector shows the charging discharging of the filter capacitor and the approximate output voltage.
- The envelope of the AM wave is being recovered successfully.
- This time constant should not be too long which will not allow the capacitor discharge at the maximum rate of change of the envelope.

$$\frac{1}{f_c} \ll RC \ll \frac{1}{f_m}$$

6.5.2 Angle Modulation:

- In angle modulation either frequency or phase of the carrier is varied according to the message signal, but the carrier amplitude is constant.



6.5.3 Frequency Modulation (FM)

- In sinusoidal frequency modulation (FM), the modulating signal $x(t) = E_m \cos(2\pi f_m t)$ is a pure sinusoidal signal. The carrier signal c(t) is also a sinewave at much higher frequency.
- FM is a system of modulation in which the instantaneous frequency of the carrier is varied in proportion with the amplitude of the modulating signal. The amplitude of the carrier signal remain constant. Thus the information is conveyed via frequency changes.
- The amount by which the carrier frequency varies from its unmodulated value is called as deviation. The deviation (δ) is made proportional to the instantaneous value of modulating voltage.
- The rate at which this frequency variation or oscillations takes place is equal to the modulating frequency(f_m). Figure 28 shows FM waveforms.

- The amplitude of the FM wave always remains constant. This is the biggest advantage of FM.

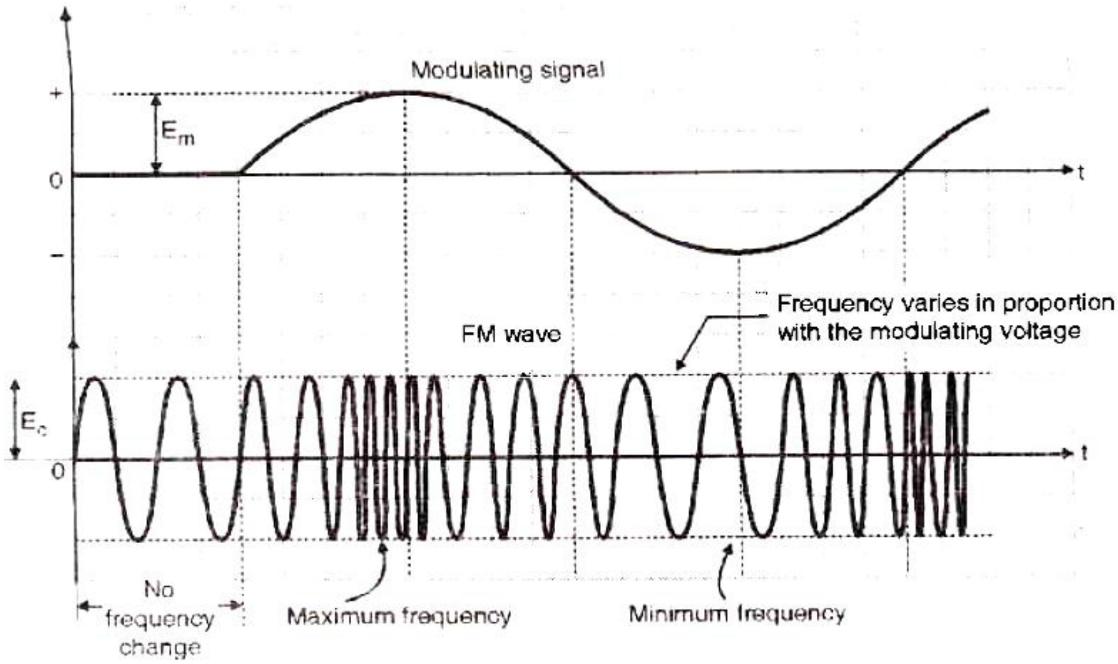


Figure-28 FM waveform

- For the FM wave the modulating signal $x(t)$ be a sinusoidal signal of amplitude E_m and frequency f_m.

$$x(t) = E_m \cos(2\pi f_m t)$$

- The unmodulated carrier is represented by the expression,

$$e_c = A\sin(\omega_c t + \phi)$$

- In FM, the frequency (f) of the FM wave varies in accordance with the modulating voltage. The instantaneous frequency of the FM wave is denoted by $f_i(t)$ and is given by,

$$f_i(t) = f_c + k_f\, x(t) = f_c + k_f\ E_m\ \cos(2\pi f_m t) = f_c + \delta \cos(2\pi f_m t)$$

- Where $\delta = k_f\ E_m$ and it is called as frequency deviation. Where k_f is a constant with units Hz/Volts.
- Frequency deviation δ represents the maximum departure of the instantaneous frequency $f_i(t)$ of the FM wave from the carrier frequency f_c.
- Since $\delta = k_f\ E_m$ the frequency deviation is proportional to the amplitude of modulating voltage E_m and it is independent of the modulating frequency f_m.
- The Maximum frequency of FM wave is,

$$f_{max} = f_c + \delta$$

The minimum frequency of a FM wave is

$$f_{min} = f_c - \delta$$

Equation for the FM wave is

$$e_{FM} = E_c \sin\left(\omega_c t + \frac{\delta}{f_m} \sin \omega_c t\right)$$

- But $\frac{\delta}{f_m} = m_f$ i.e. the modulation index of FM wave. Hence the equation for FM wave is given as,

$$e_{FM} = E_c \sin(\omega_c t + m_f \sin \omega_c t)$$

- The **modulation index of an FM wave** is defined as,

$$m_f = \frac{\text{frequency deviation } (\delta)}{\text{modulating frequency } (f_{m)}}$$

- In AM the maximum value of the modulation index m is 1. But for FM the modulation index can be greater than 1.

$$\text{Deviation ratio} = \frac{\text{maximum deviation}}{\text{maximum modulating frequency}}$$

- Percentage modulation of FM wave

$$\%\ \text{modulation} = \frac{\text{actual frequency deviation}}{\text{maximum allowed deviation}}$$

- **Bandwidth** of FM is greater than AM.

Generation of FM wave:

- Figure 29 shows the block schematic of a simple FM modulator. It is a simple voltage controlled oscillator (VCO).
- The modulating signal instantaneous value. Thus we get an FM wave at the output of the VCO.

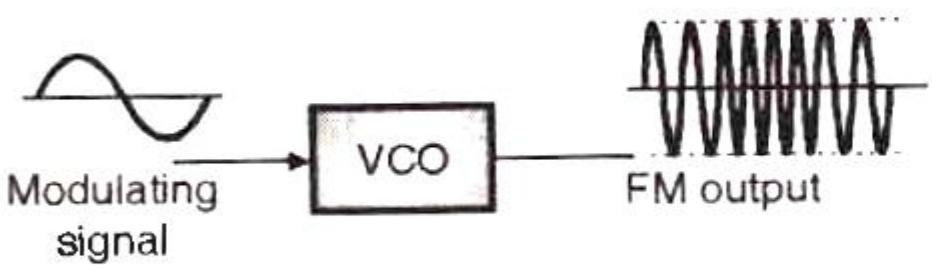


Figure-29 generation of FM signal

Advantages of FM:

- Improved noise immunity
- Low power is required to be transmitted to obtain the same quality of received signal at the receiver.
- Covers a larger area with the same amount of transmitted power.

- Transmitted power remains constant.
- All the transmitted power is useful.

Disadvantages of FM:

- Very large bandwidth is required.
- Since the space wave propagation is used, the radius of transmission is limited by the line of sight.
- FM transmission and reception equipment's are complex.

Application of FM:

- Radio broadcasting
- Satellite communication
- Point to point communication
- Sound broadcasting in TV
- Police wireless

Figure-30 FM detector

Demodulation of FM:

- A detector or demodulator is a circuit which receives an FM wave at its input and produces the message signal or modulating signal at its output.
- Demodulator or detector is exactly opposite to the modulation process.
- The AM detector is basically an envelope detector. But FM demodulator is basically a frequency to amplitude converter.
- It is expected to convert the frequency variation in FM wave at its input into amplitude variations at its output to recover the original modulating signal. Figure 30 shows FM detector.

Requirements of FM detector:

The FM demodulator must satisfy the following requirements

- It must convert frequency variations into amplitude variations
- This conversion must be linear and efficient.
- The demodulator circuit should be insensitive to amplitude changes. It should respond only to the frequency changes.
- It should not be too critical in its adjustment and operation.

A general FM demodulator:

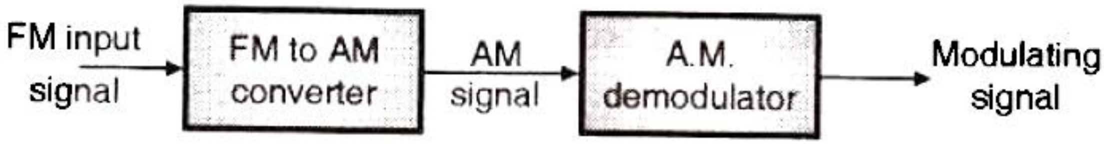


Figure-31 General FM demodulator

- Figure-31 shows the block diagram of a general FM demodulator. The FM signal which is to be demodulated is applied at the input of the FM to AM converter. This block converts the frequency variations in FM signal into proportional amplitude variations.
- The FM demodulator is a simple envelop detector discussed earlier which demodulates AM and recovers the original modulating signal.

Pre-emphasis:

- In, FM the noise has a greater effect on the higher modulating frequencies. This effect can be reduced by increasing the value of modulation index (m_f) for higher modulating frequencies(f_m).
- This can be done by increasing the deviation δ and δ can be increased by increasing the amplitude of modulating signal at higher modulating frequencies.
- Thus if we boost the amplitude of higher frequency modulating signals artificially then it will be possible to improve the noise immunity at higher modulating frequencies.
- The artificial boosting of higher modulating frequencies is called as **pre-emphasis**.
- Boosting of higher frequency modulating signal is achieved by using t pre-emphasis circuit at the transmitter.

De-emphasis:

- The artificial boosting given to the higher modulating frequencies in the process of pre-emphasis is nullified or compensated at the receiver by a process called de-emphasis.
- The artificially boosted high frequency signals are brought to their original amplitude using the de-emphasis circuit. This is done at the FM receiver.

6.5.4 Phase Modulation (PM)

- Phase modulation is very similar to the frequency modulation. The only difference is that the phase of the carrier is varied instead of varying the frequency. The amplitude of the carrier remains constant.
- As shown in figure 32 as the modulating signal goes positive, the amount of phase lag increases with the amplitude of the modulating signal. The effect is that the carrier signal is stretched or its frequency is reduced.

- When the modulating signal goes negative, the phase shift becomes leading. This causes the carrier wave to be effectively compressed. The effect is as if the carrier frequency is increased.
- Thus phase modulation produces frequency modulation.
- Note that the PM wave of figure 5 is the same as the FM wave produced by dx(t)/dt i.e. the derivative of x(t) with respect to time.
- So in figure-32 we have plotted the derivation of x(t) which is original x(t) shifted by 90°.

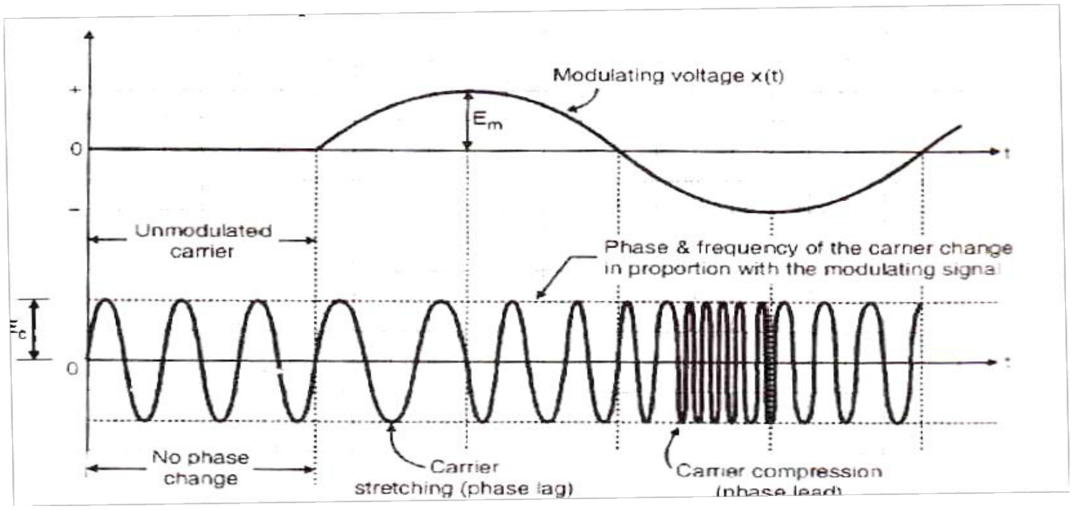


Figure-32 PM modulation waveform

- From the discussion it is clear that the difference between FM and PM waves can be made only by comparing with the original modulating wave.
- The PM wave is obtained by varying the phase angle ϕ of a carrier in proportion with the amplitude of the modulating voltage.

  If the carrier voltage is expressed as,

  $$e_c = A\sin(\omega_c t + \phi)$$

  Then the PM wave can be expressed as,

  $$e_{PM} = A(\sin(\omega_c t + \phi_m \sin(\omega_m t))$$

- Here ϕ_m= maximum phase change corresponding to the maximum amplitude of the modulating signal. For the sake of uniformity let us modify the equation as,

  $$e_{PM} = A(\sin\left(\omega_c t + m_p \sin(\omega_m t)\right)$$

  Where $m_p = \phi_m$= modulating index of PM.

- The FM and PM waves look identical when their modulating index are identical. However if we change the modulating frequency f_m then m_f will change but there is no change in the value of m_p.

Generation of PM:-

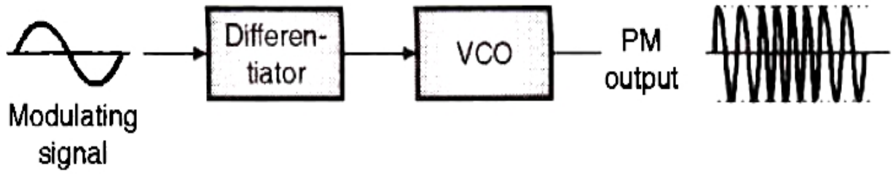


Figure-33 General PM demodulator

- Figure-33 shows the generation of PM wave. The modulating signal is applied to a differentiator.
- Then the differentiated modulating signal is applied to the VCO to produce the PM wave.
- **Bandwidth** of PM is less than that for FM. The bandwidth of a PM signal can be calculated from the maximum modulating frequency and the maximum amplitude of the modulating signal.

6.6 RADIO BROADCASTING

- Radio (AM and FM) and television broadcasting are the most common familiar forms of communication via analog transmission systems.
- The receiver used in radio and TV broadcasting are known as the super heterodyne receivers.

6.6.1 Super heterodyne AM radio receiver:

- In super heterodyne AM every selected RF signal converted to a fixed lower frequency called as the intermediate frequency (IF).
- This frequency contains the same modulation as the original carrier. The IF signal is then amplified and detected to get back the modulating signal.
- As the IF is lower than the lowest RF signal frequency, the possibility of oscillations and instability is minimized.
- Also the required value of Q for constant Bandwidth does not depend on the frequency of desired signal, because the IF is constant and same for all the incoming RF signals.
- Thus the super heterodyne receiver solves all the problems associated with the RF receiver.
- The radio and TV receivers operate on the principle of super heterodying. The block diagram of a super heterodyne AM radio receiver is shown in figure-34.

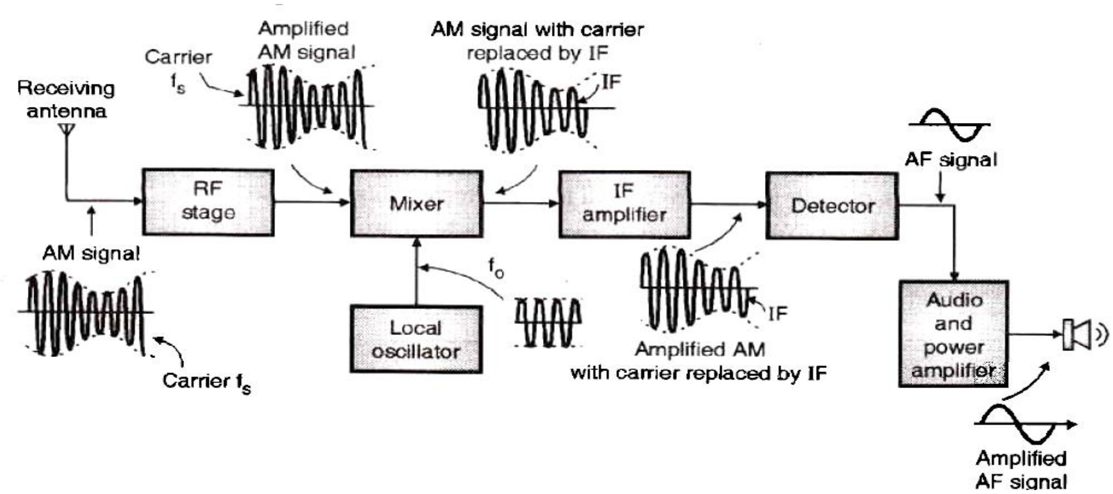


Figure-34 The Superheterodyne AM radio receiver

Operation:

- The DSB-FC or AM signal transmitted by the transmitter travels through the air and reaches the receiving antenna. This signal is in the form of electromagnetic waves. It induces a very small voltage into the receiving antenna.
- **RF:** The RF stage is an amplifier which is used to select the wanted signal and reject other out of many, present at the antenna. It also reduces the effect of noise. At the output of the RF amplifier we get the desired signal at frequency f_s.
- **Mixer:** The mixer receives signals from the RF amplifier at frequency f_s and from the local oscillator at frequency f_o such that $f_o > f_s$.
- **Intermediate frequency (IF):** The mixer will mix these signals to produce signals having frequencies f_o, f_S, $(f_o + f_S)$ and $(f_o - f_S)$. Out of these the difference of frequency component i.e. $(f_o - f_S)$ is selected and all others are rejected. This frequency is called as the intermediate frequency (IF). IF = $(f_o - f_S)$
- In order to maintain constant difference between the local oscillator frequency and the incoming frequency, ganged tuning is used. This is simultaneous tuning of RF amplifier, mixer and local oscillator and it is achieved by using ganged tuning capacitors.
- The intermediate frequency signal is then amplified by one or more IF amplifier stages. IF amplifier provide most of the gain and the bandwidth requirements of receiver. Therefore the sensitivity and selectivity of this receiver do not change much with changes in the incoming frequency.
- The amplified IF signal is detected by the detector to recover the original modulating signal. This is then amplified and applied to the loudspeaker.
- AGC means automatic gain control. This circuit controls the gains of the RF and IF amplifiers to maintain a constant output voltage level when the signal level at the receiver

input is fluctuating. This is done by feeding a controlling dc voltage to the RF and IF amplifiers. The amplitude of this dc voltage is proportional to the detector output.

6.6.2 Super heterodyne FM radio receiver:

- Figure 35 shows the block diagram of a FM receiver. The first thing that strikes us is it's similarly with the AM receiver.
- The FM receiver also operates on the principle of super heterodyning, as the AM receiver.
- The **FM receiver is different from the AM receiver** in the following way:
  - ➢ The operating frequency in FM are much higher than in AM.
  - ➢ The FM receivers need the circuits like limiter and de-emphasis.
  - ➢ The FM demodulators are different from AM detectors.
  - ➢ The method to obtain the AGC is different in FM receiver.

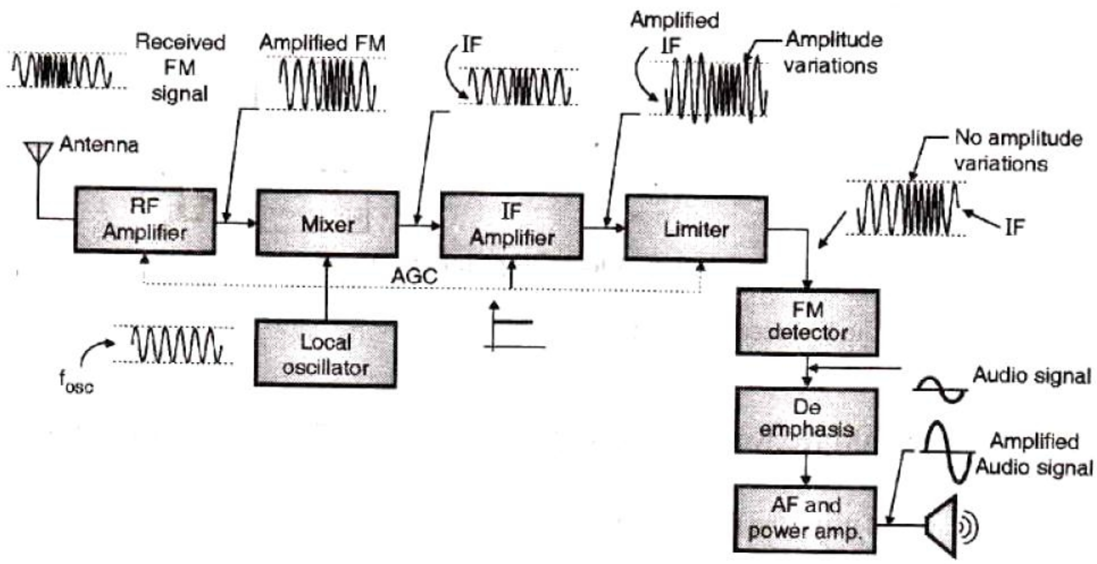


Figure-35 The Super heterodyne FM radio receiver

- The advantages of using the RF amplifier are:
  - o It improves the signal to noise ratio.
  - o It will match the receiver input impedance to the antenna impedance.
- Similar to AM receivers, the mixer stage in FM receiver will down convert the received signal to intermediate frequency (IF).
- The IF produced at the output of the mixer is at 10.7 MHz in FM receivers as compared to 455 to 470 kHz in AM receivers.
- IF amplifier used in FM receivers are similar to those used in the AM receiver but the IF and the bandwidth required are much higher than those in the AM receivers. Here the IF is 10.7 MHz and the bandwidth is 200 kHz.

- Due to the large bandwidth the gain per stage will be low. Therefore two or more IF amplifiers are required to be used.
- The FM wave which is transmitted by the transmitter has constant amplitude. But while travelling, noise and other unwanted signals get added to it and change its amplitude. These unwanted amplitude changes in the received FM signal must be removed before the signal goes for demodulation.
- Otherwise distortion appears in the demodulated signal as the demodulators react to amplitude changes as well as frequency changes.
- The amplitude limiter will remove all the unwanted amplitude variations from the received signal and it is always placed before the FM detector.

6.7 Television broadcasting

6.7.1 Introduction:

- The meaning of the word "Television" is to see at a distance. The first demonstration of television was made in 1925 by J. L. Baird in London and by C. F. Jenkins in USA.
- In a TV system both picture and sound are to be transmitted simultaneously and then they are to be received at a long distance.
- To transmit the sound + picture information, a transmitter along with a transmitting antenna is needed and to receive this signal we need a receiving antenna and a TV receiver.
- A television system is required to reproduce the shape of each object the brightness and tonal contents, movements, sound, color and perspective contents of picture.
- The camera tube at the transmitter acts like a human eye. It converts the image into an equivalent electrical signal- whereas the picture tube at the receiver converts the electrical signal into an image. About 150,000 effective elements are displayed in each scene.
- There are 25 pictures or frames shown per second. This is related to the 50 Hz ac line frequency. This is the minimum picture rate required to see a flicker free picture.
- A special type of scanning process called as the interlaced scanning is used by TV system in order to reduce the effect of flicker.

6.7.2 TV camera:

- A TV camera tube is a transducer which converts, all the picture information into an equivalent electrical signal.
- The amplitude of the electric signal produced at the output of camera tube is proportional to the brightness of the point being scanned by the electron Beam.
- In order to synchronize the TV receiver with the TV transmitter, synchronizing (sync) information is transmitted along with the picture information.
- At the receiver, the sync signals derived from the transmitted sync signal are used for controlling the vertical and horizontal sync circuits.
- The transmission and reception of black and white pictures take place in this way.

6.7.3 TV Transmitter:

- Color TV needs more information. It has to transmit brightness information like black and white TV and also the color information.
- The color information is transmitted via chrominance or chrome signal.
- Actually the Red, Green and Blue colors are indicated, but all the other colors can be made from these three basic colors.
- By adding these basic colors together in proper proportion, it is possible to generate any color.
- FDM is used for interleaving the chrominance signal with the luminance signal. Figure 36 shows color TV transmitter.

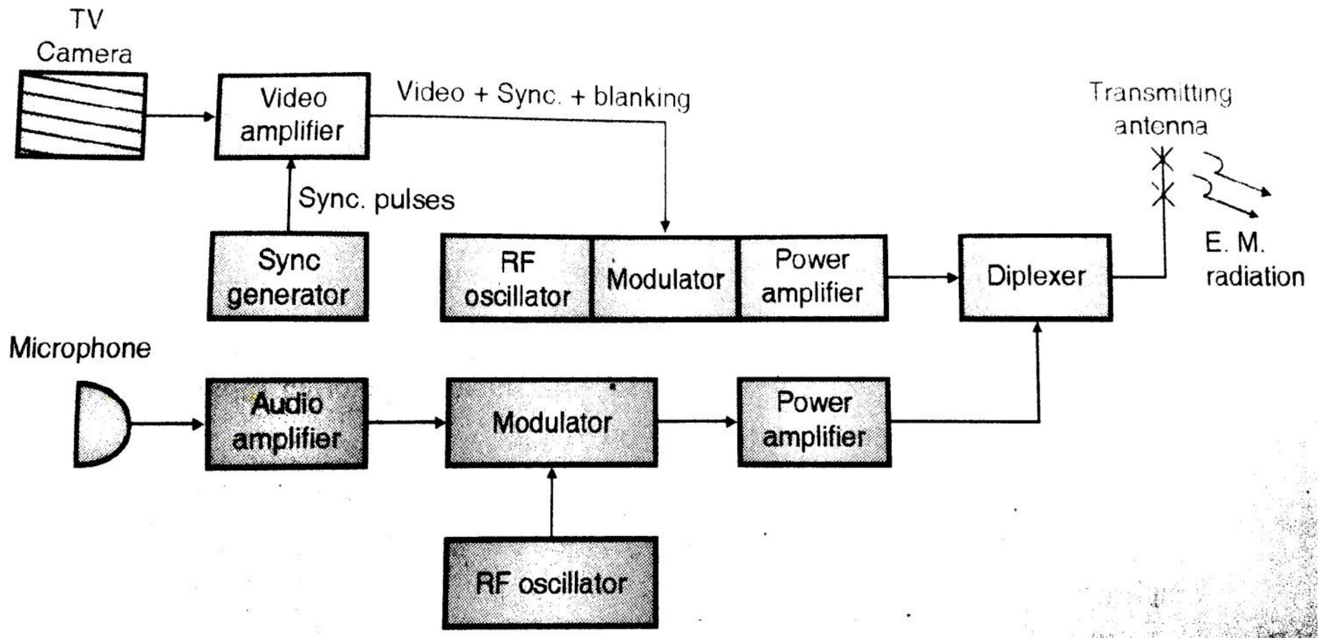


Figure-36 TV Transmitter

TV camera:

- The first block in the TV transmitter is a TV camera which converts the optical information such as intensity; color etc. into an electrical signal.
- The electrical signal obtained at the output of TV camera is called as "video signal.

Video amplifier:

- The video signal obtained at the output of the TV camera is weak. Therefore a video amplifier is used to amplify it to an adequate level.
- This amplifier is a wideband amplifier with a bandwidth of about 5 kHz. This is necessary because video signals can have frequencies anywhere between 0 to 5 MHz

Modulator:

- The amplified video signal acts as a "modulating signal." The other input to the modulator is the "carrier" signal generated by an RF oscillator.

Power amplifier:

- The power amplifier is used to raise the power content of the amplitude modulated video signal to the required level, so that the transmitted signal can reach a long distance.

Diplexer:

- This is a combining network which connects the outputs of video section as well as the sound section to a common transmitting antenna. It isolates these two sections from each other.

Microphone:

- A microphone is used to convert the sound signal into an equivalent electrical signal.

Sound amplifier:

- An audio amplifier is used to amplify$ the weak audio signal coming from the microphone. This amplified audio signal is then applied as a "modulating signal" to the sound modulator.

Sound modulator:

- The audio signal from the audio amplifier will "frequency modulate" the carrier signal produced by the RF oscillator. Thus sound is frequency modulated (FM).

Transmitting antenna:

- The transmitting antenna converts the AM video signal + FM sound signal into electromagnetic waves. These waves are transmitted in all the directions. The transmitting antennas are installed on hill tops or on tall building in order to increase the range of transmission.

6.7.4 TV reciever:

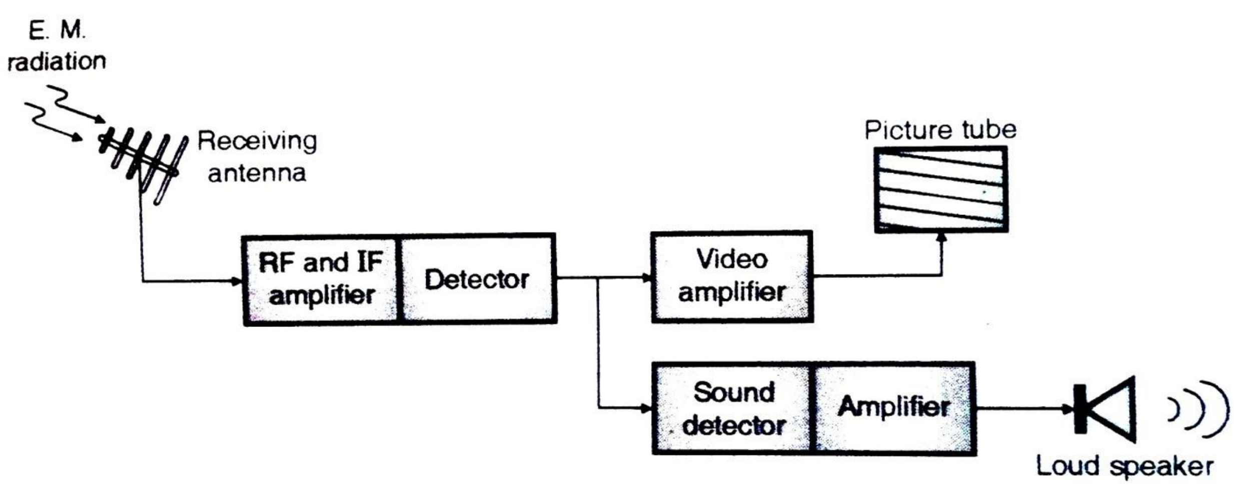


Figure-37 TV receiver

Figure-37 shows color TV receiver which has following blocks.

Receiving antenna:

- A special receiving antenna called "Yagi-Uda" antenna is used to receive the TV transmission.

RF and IF amplifier:

- The weak signal coming from the antenna is amplified by the RF amplifier to and translated to a lower frequency called intermediate frequency (IF)

Detector:

- The detector will separate the video, audio and sync. Signals from each other.

Video amplifier:

- The demodulated video signal is amplified by a video amplifier to an adequate voltage level. The video amplifier is a wideband amplifier having bandwidth of approximately 5 MHz

Picture tube:

- The amplified video signal is applied to the cathode of a picture tube. It will convert the electrical signal into an optical signal, to reproduce the originally transmitted picture.

Sound detector:

- The separated sound signal is applied to the sound detector. It is an FM demodulator which demodulates the frequency modulated audio signal to produce the original audio signal.

Audio amplifier:

- The demodulated sound signal is amplified by the audio amplifier and the amplified signal is used to drive a loud-speaker. The loud-speaker then converts this signal into sound waves.

6.8 Mobile Communication Systems (Cellular System)

6.8.1 Cellular Concept

- Cellular phone is wireless communication just like cordless phone.
- In cell phone distance is not restricted to within home but one can travel in the city or even - outside the city without interruption in communication.
- In the cellular system city is divided into small areas called ‘cells’. Each cell is around l0 square kilometer.
- Each cell is (The cellular network is as shown in Fig. linked to central location called the Mobile Telephone Switching Office (MTSO).
- MTSO coordinates all mobile calls between an area comprised of several cell sites and the central office. Figure 38 shows the cellular network.

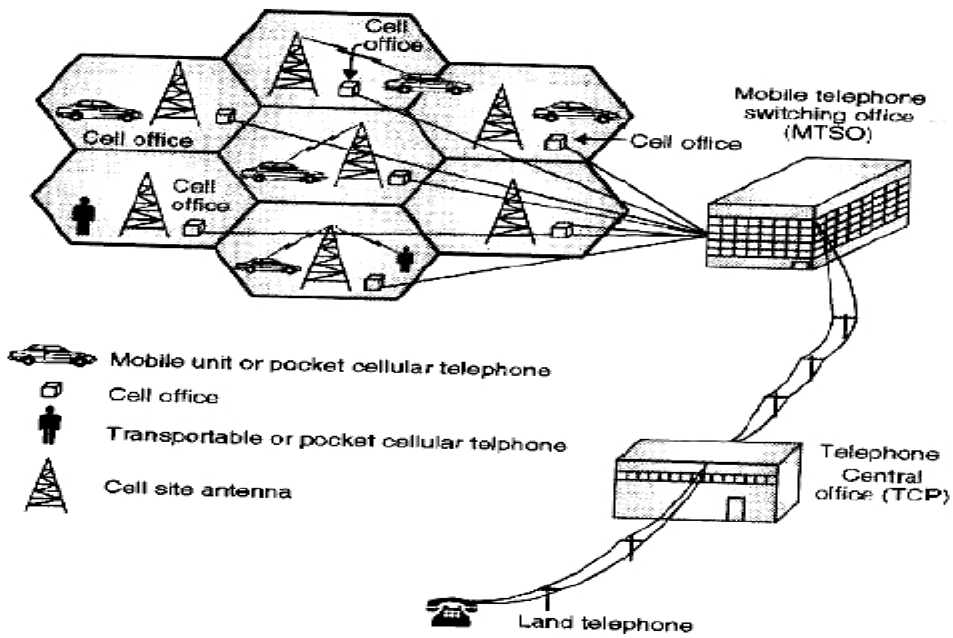


Figure- 38 The Cellular Network

1. *Cell:*
   - The basic geographic unit of a cellular communication system is called as a cell.
   - Its shape is hexagonal.
   - Cells are the base stations transmitting over small geographic areas.
   - The size of a cell is not fixed. practically the shape of the cell may not be a perfect hexagon figure 39 shows cell structure.

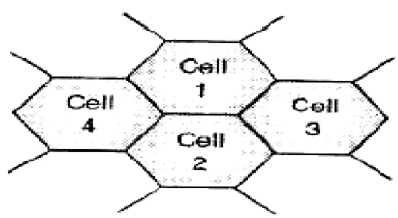


Figure-39 Cell structure

2. *Cluster:*
   - A group of cells is called as a cluster.
   - The cluster size (n) is not fixed. It depends on the requirements of a particular area.
   - The l-G (first generation) cellular systems were analog systems. They use FM signal for communication.
   - The 2-G (Second Generation) cellular systems are all digital systems. The two most widely used 2-G systems are :

     l. GSM

     2. IS-95.

6.8.2 Basic Structure of Mobile phone System:

- In the mobile communication either the transmitter or the receiver or both are going to be movable. Figure 40 shows the basic structure of mobile telephone network.

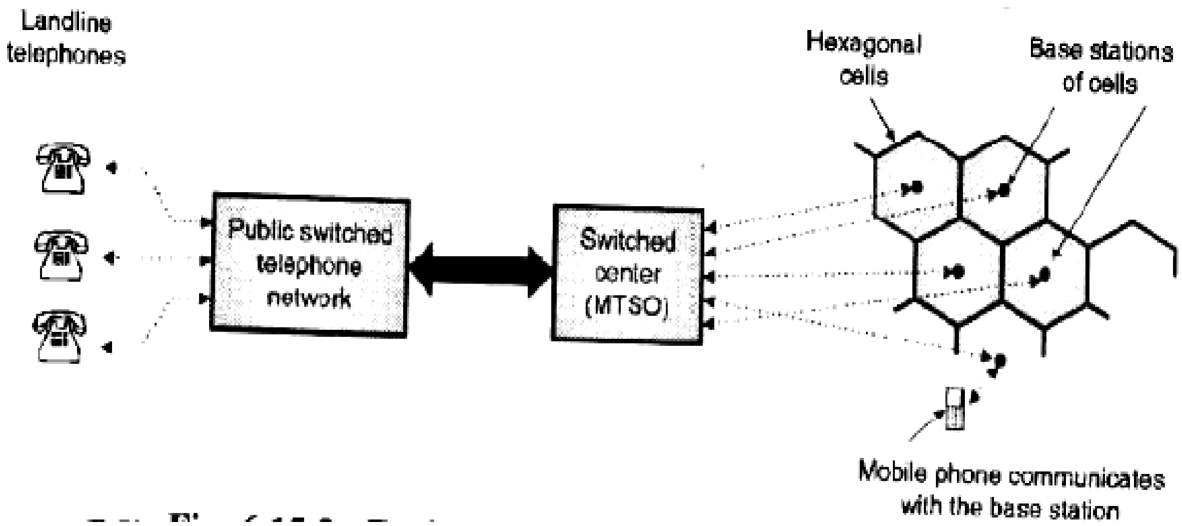


Figure-40 Basic structure of mobile telephone network

Description:

- The task of the base stations is to act as an interface between the mobile phone and the cellular radio system.
- The switching center acts as the interface between the public switched Telephone Network (PSTN).
- If a mobile subscriber travels from one cell area to the other then it automatically gets connected
- To base station of that cell. Thus the service provided to a mobile subscriber is continuous without any break.

Functions of MTSO:

- The MTSO controls all the cells and provides the interface between each cell and the main telephone office.
- As the vehicle moves from one cell to the next the system automatically switches from one cell to the next.

Advantages of using MTSO

- It operates at a much higher frequency. So more spectrum space is available and so more number of channels can be accommodated.
- Due to MTSO, the cellular system can use the concept of frequency reuse. This will allow the cells to use the same frequency.
- The cells of different size can be accommodated.

6.8.3 Evolution of Mobile Phones

- **The first generation (1G)** of cellular telephony was designed for voice communication using analog signals.
- One of the important first generation (1G) mobile systems used in North America is AMPS (Advanced Mobile Phone System). It is one of the leading analog cellular systems in North America.
- It makes use of FDMA (Frequency Division Multiple Access) to separate channels in a link.
- **The second generation (2G)** of cellular telephony was developed in order to improve the quality of communication.
- The second generation was designed for digital voice. Three major systems in the second generation (2G) are: 1 D-AMPS 2 GSM 3.IS-95
- **The 3G (third generation)** wireless systems show a tremendous improvement in the field of wireless access. Multimegabit internet access, voice over internet is some of the examples of 3G systems.
- The users are able to receive live music, conduct interactive web sessions and have a simultaneous voice and data access using a mobile handset.
- The 3G version of 2G CDMA systems is called as CDMA 2000.
- The 3G evolution for GSM, 15-136 and PDC systems has lead to wideband CDMA (W-CDMA) which is also known as Universal Mobile Telecommunication Service (UMTS).

Applications of 3G Cellular Services

- Personal applications
- Content applications
- Communications applications
- Productivity applications
- Business application

6.9 Digital Communication System

6.9.1 Introduction

- **Definition:** The modulation system or technique in which the transmitted signal is in the form of digital. Pulses of constant amplitude, constant frequency and phase are called as digital modulation System.
- **Examples:** Pulse Code Modulation (PCM) and Delta Modulation (DM) are the examples of digital modulation.
- In the PCM and DM, a train of digital pulses is transmitted by the transmitter.

Applications of digital communications:

- Long distance communication between earth and space ships.
- Satellite communication.

- Military communications which needs coding.
- Telephone systems.
- Data and computer communications.

6.9.2 Pulse Code Modulation:

- The PCM output is in the coded digital form. It is in the form of digital pulses of constant amplitude, width and position.
- The information is transmitted in the form of code words. A PCM system consists of a PCM encoder (transmitter) and a PCM decoder (receiver).
- The essential operations in the PCM transmitter are sampling, quantizing and encoding figure 41 shows PCM transmitter.

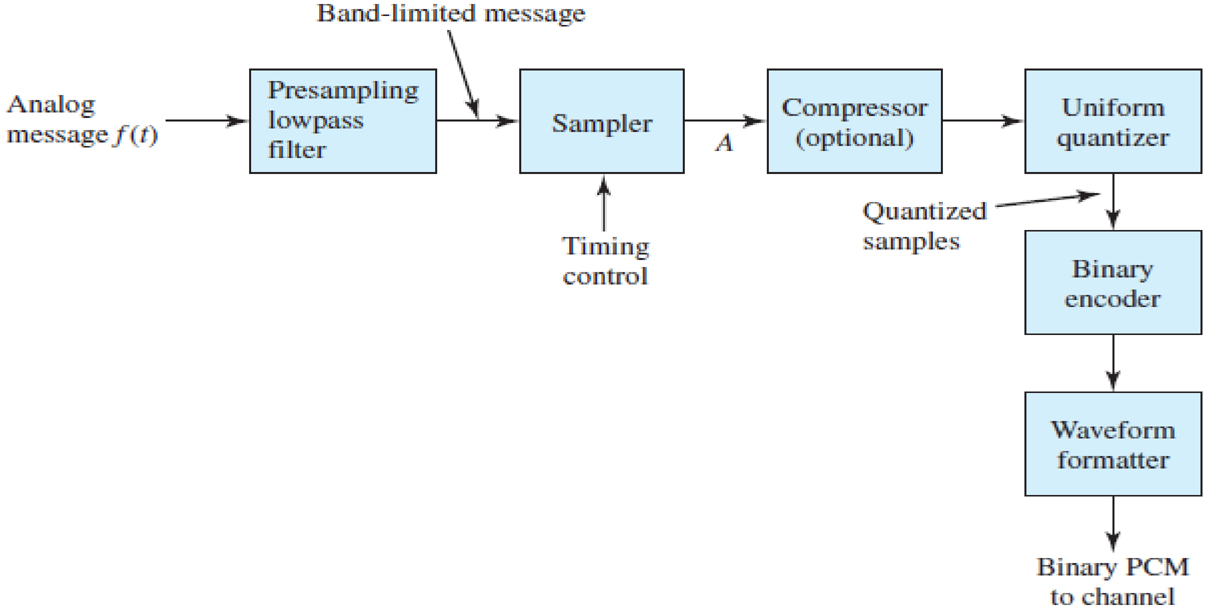


Figure-41 PCM transmitter

- PCM is the simplest and oldest waveform coding scheme for processing an analog signal by sampling, quantizing, and binary encoding.
- Sampling of an analog signal makes it discrete in time.
- Quantization consists of rounding exact sample values to the nearest of a set of discrete amplitudes called quantum levels.
- The quantizer is said to be uniform when the step size between any two adjacent quantum levels is a constant.
- After the quantization of message samples, the digital system will then code each quantized sample into a sequence of binary digits (bits) 0 and 1.
- After quantization and coding the samples of the message, a suitable waveform has to be chosen to represent the bits. This waveform can then be transmitted directly over the channel figure-42 shows PCM receiver.

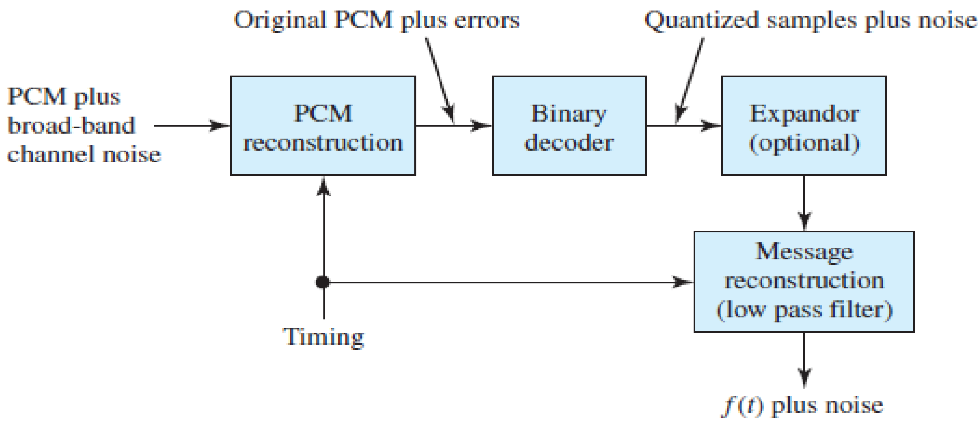


Figure-42 PCM receiver

- The operations of the receiver are basically the inverse of those in the transmitter.
- The first and most critical receiver operation is to reconstruct the originally transmitted PCM signal as nearly as possible to minimize the effect of noise. Figure 43 shows PCM waveform.

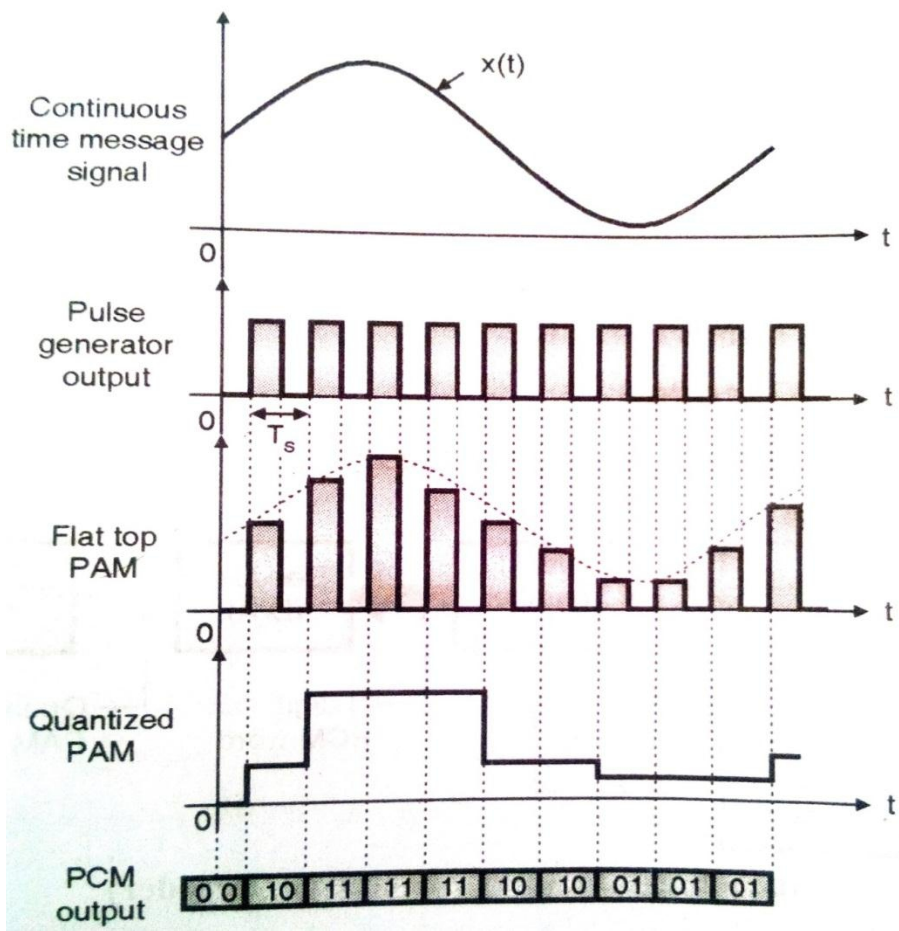


Figure-43 Waveforms at different points in PCM transmitter

Advantages:

- Very high noise immunity.
- Due to digital nature of the signal, repeaters can be placed between the transmitter and the receivers.
- The repeaters actually regenerate the received PCM signal. This is not possible in analog systems.
- It is possible to store the PCM signal due to its digital nature.
- It is possible to use various coding techniques so that only the desired person can decode the received signal. This makes the communication secure.

Disadvantages:

- The encoding, decoding and quantizing circuitry of PCM is complex.
- PCM requires a large bandwidth as compared to the other systems.

Applications:

- In telephony systems.
- In the space communication space craft transmits signals to earth.

Shape of the PCM signal:

- Figure-44 shows input to and output of a PCM system. It is important to understand that the output is in the form of binary codes. Each transmitted binary code represents a particular amplitude of the input signal. Hence the "information" is contained in the "code" which is being transmitted.
- The range of input signal magnitudes is divided into 8-equal levels. Each level is denoted by a three bit digital word between 000 and 111.

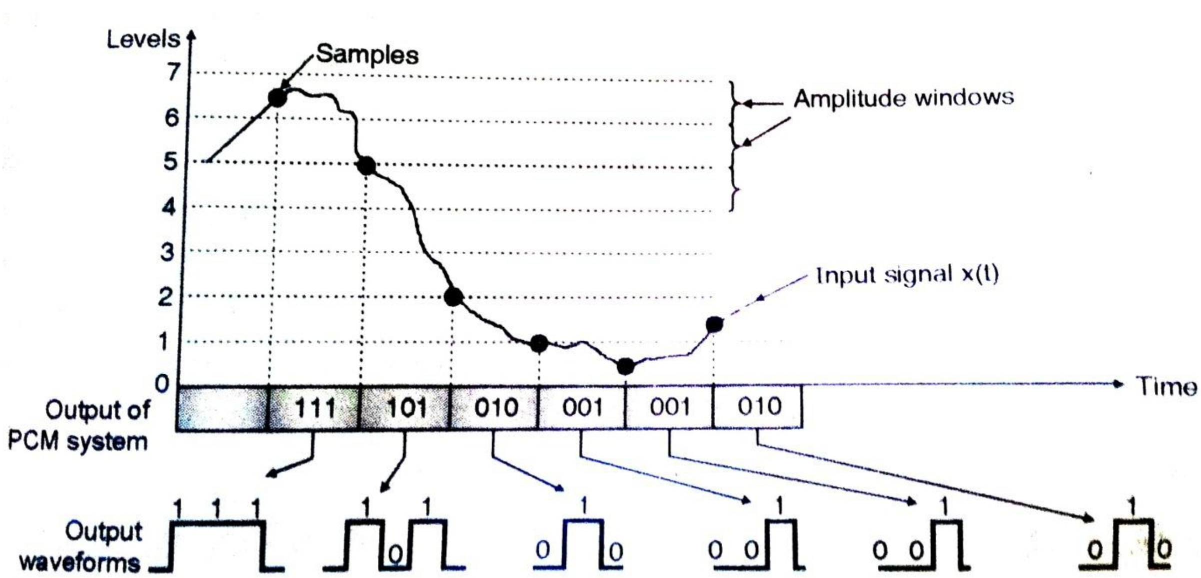


Figure-44 Input and output waveforms of a PCM system

- Input signal x(t) is sampled. If the sample is in the 5th window of amplitude then a digital word 101 is transmitted. If the sample is in the 2nd window then the transmitted word is 010 and so on.
- In this example we have converted the amplitudes into 3 bit codes, but in practice the number of bits per word can be as high as 8, 9 and 10.

6.9.3 QUANTIZATION PROCESS

- Quantization is a process of approximation or rounding off.
- Quantizer converts the sampled signal into an approximate quantized signal which consists of only a finite number of predefined voltage levels.
- The input signal x (t) is assumed to have a peak to, peak swing of V_H, to V_L volts. This entire voltage range has been divided into 'Q' equal intervals each of size 'S'
- "S" is called as the step size and its value is given as, S= V_H - V_L / Q
- The quantized signal xo (t) is thus an approximation of x (t). The difference between them is called as quantization error or quantization noise. Figure-45 Process of Quantization.

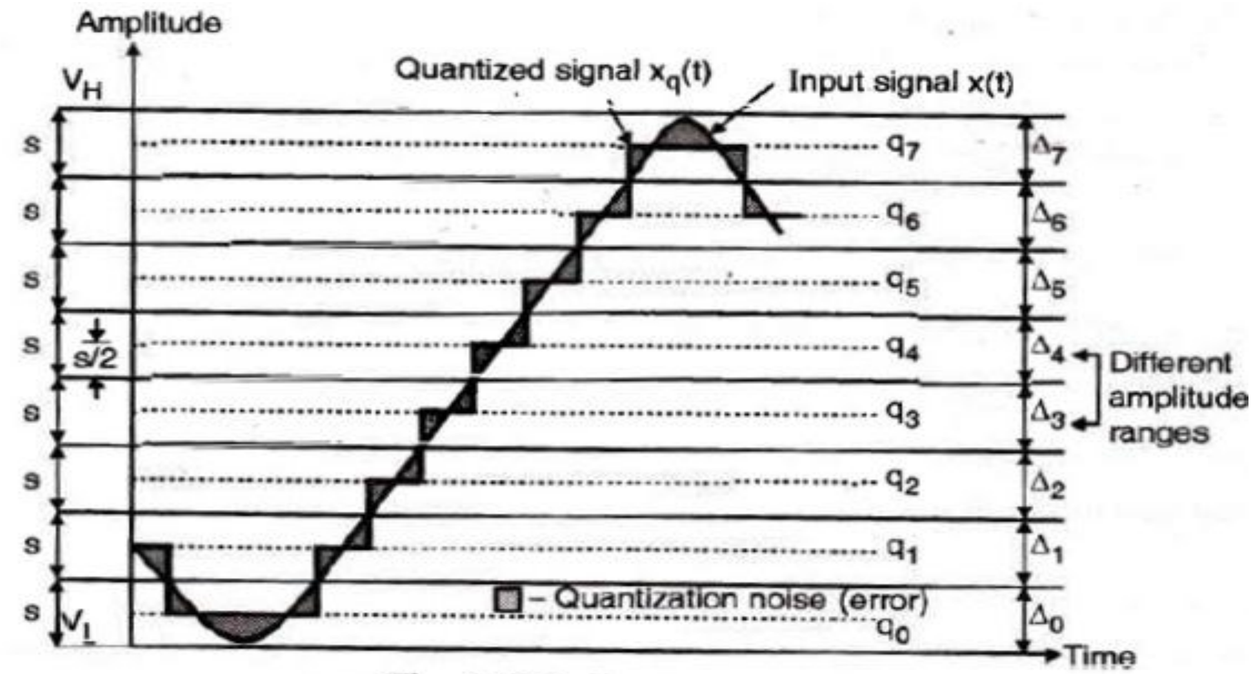


Figure-45 Process of Quantization

6.9.4 Signal Multiplexing

- When the data from many sources in time are interlaced the interlacing of data is called as time multiplexing, in which a single link can handle all sources.
- When a large number of PCM signals are to be transmitted over a communication channel, multiplexing of these PCM signals is required.

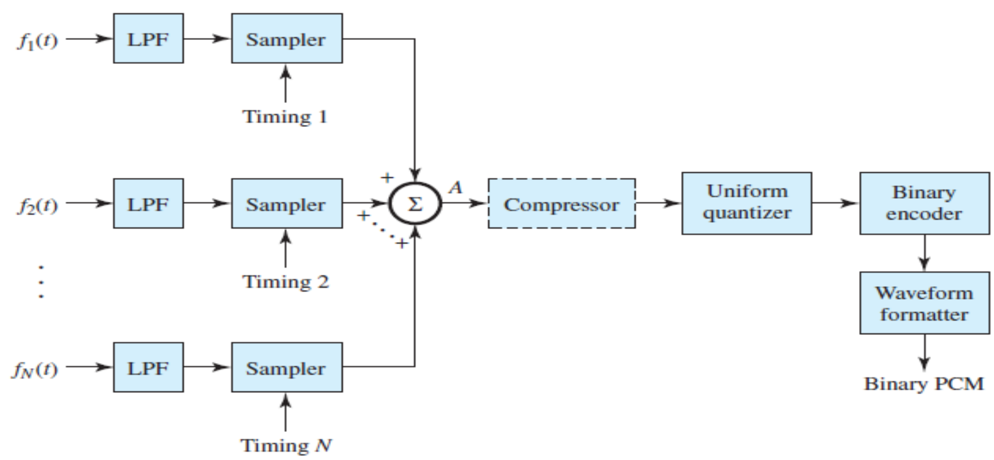


Figure-46 Time multiplexing for similar analog signal

6.9.5 Digital Signal (DS) Service

- The telephone companies implement TDM (time division multiplexing) through the hierarchy of digital signals. This is called as Digital Signal (DS) Service.
- A DS0 signal is the basic input signal which is a single digital channel (64 kbps PCM channel).
- 24 DS0 signals are multiplexed using TDM to produce a DSI signal. The bit rate of DSI is24 x 64 kbps = 1.544 Mbps plus 8 kbps of overhead.
- 4 DS1 signals are multiplexed at the second level of multiplexing to obtain the DS2 signal.
- 7 DS2 signals are multiplexed to produce a DS3 signal. Finally 6 DS3 lines are multiplexed to obtain a DS4 signal. Its bit rate is 274.176 Mbps.

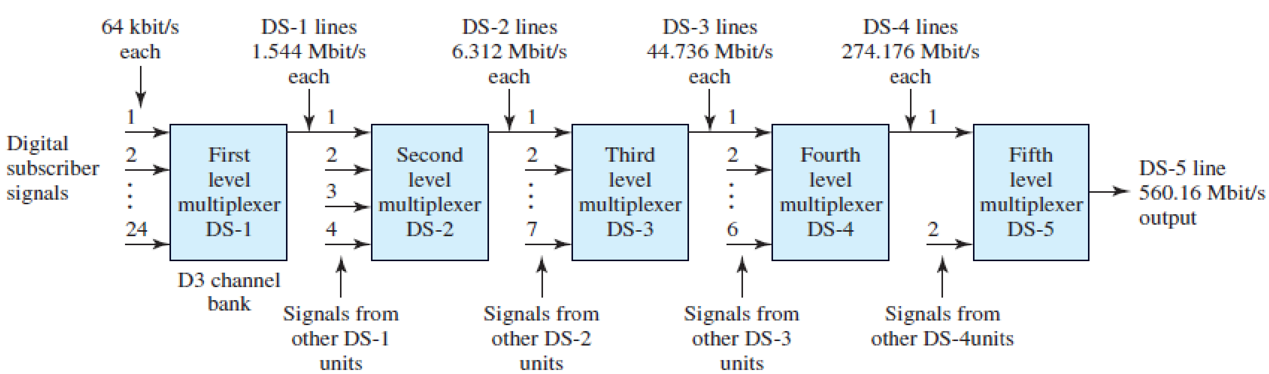


Figure-47 Digital TDM hierarchy for North American telephone communication system

6.9.6 Digital to Analog Conversion

- The digital signals at the input are converted in analog signals. These analog signals are transmitted over long distances.

- The most familiar use of this transformation is for transmitting digital data through the public telephone network.

Figure-48 Digital data to analog signal

Types of Digital Carrier Modulation:

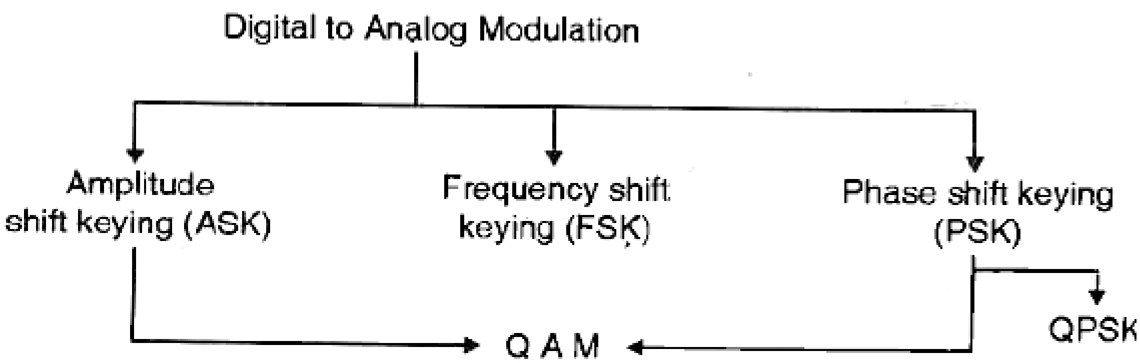


Figure-49 Types of digital to analog modulation

6.9.7 Amplitude Shift Keying (ASK) or Digital Amplitude Modulation

- Amplitude shift keying (ASK) is the simplest type of digital CW modulation.
- It is also called as digital amplitude modulation.
- The carrier is a sine wave of frequency f_c. We can represent the carrier signal mathematically as follows :

$$e_c = \sin(2\pi f_c t)$$

- The digital signal from the computer is a unipolar NRZ (non-return to zero) signal which acts as the modulating signal.
- The ASK modulator is nothing but a multiplier followed by a band pass filter.
- Due to the multiplication, the ASK output will be present only when a binary "1" is to be transmitted.
- The ASK output corresponding to a binary "0" is zero.

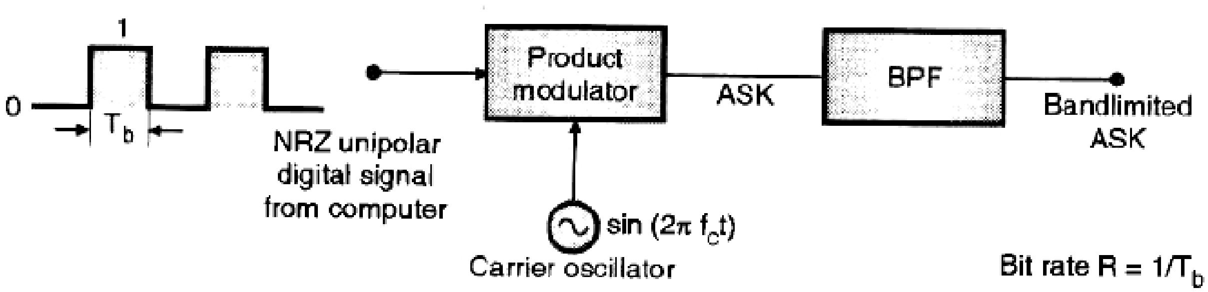


Figure-50 ASK generator

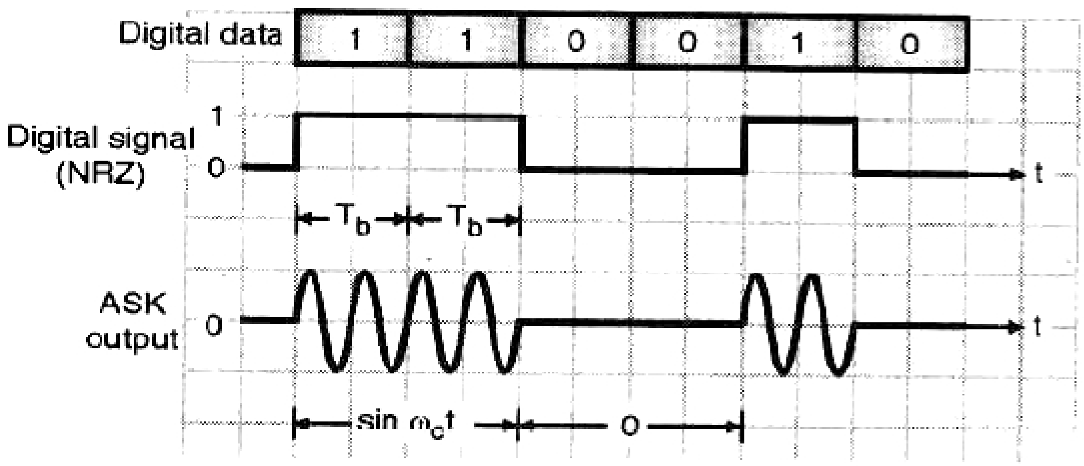


Figure-51 ASK Waveforms

6.9.8 Frequency Shift Keying (FSK)

- In frequency shift keying (FSK), the frequency of a sinusoidal carrier is shifted between two discrete values.
- One of these frequencies (f1) represents a binary “1 and the other value (f0) represents a binary “0”
- There is no change in the amplitude of the carrier.

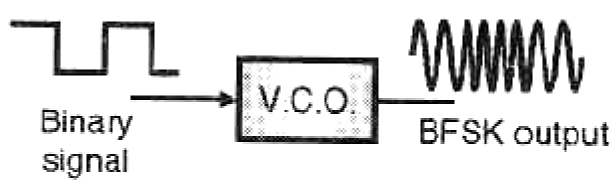


Figure-52 FSK generation

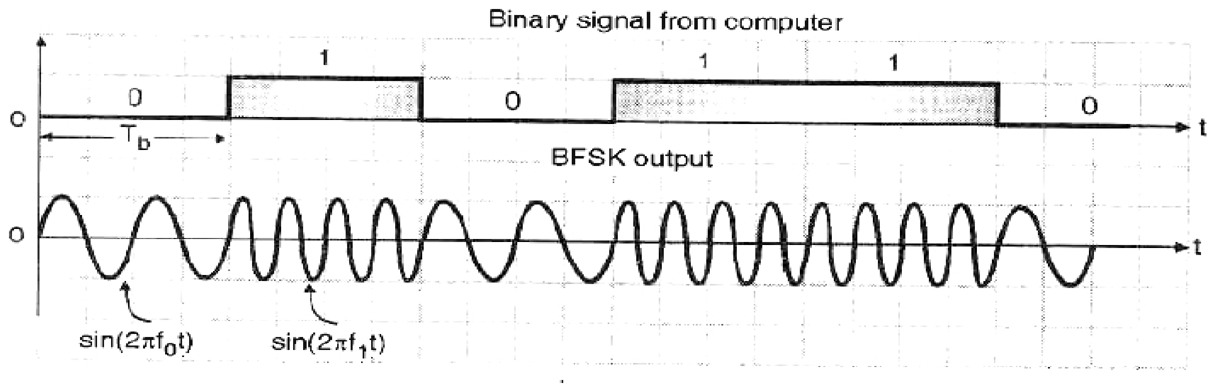


Figure-53 FSK waveform

Disadvantages:

- High bandwidth requirement
- FSK is extensively used in low speed modems having bit rates below 1200 bits/sec.
- The FSK is not preferred for the high speed modems because with increase in speed, the bit rate Increases.

Advantages:

- FSK is relatively easy to implement.
- It has better noise immunity than ASK. Therefore the probability of error free reception of data is high.

6.9.9 Phase Shift Keying (PSK)

- This is another form of digital CW modulation.
- Phase shift keying (PSK) is the most efficient of the three modulation methods.

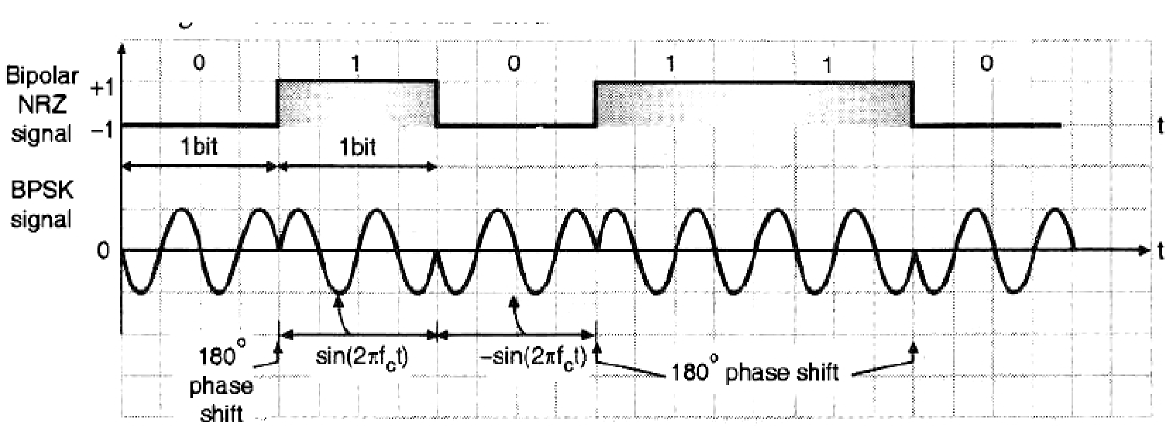


Figure-54 Binary phase shift keying (BPSK)

BPSK Generation:

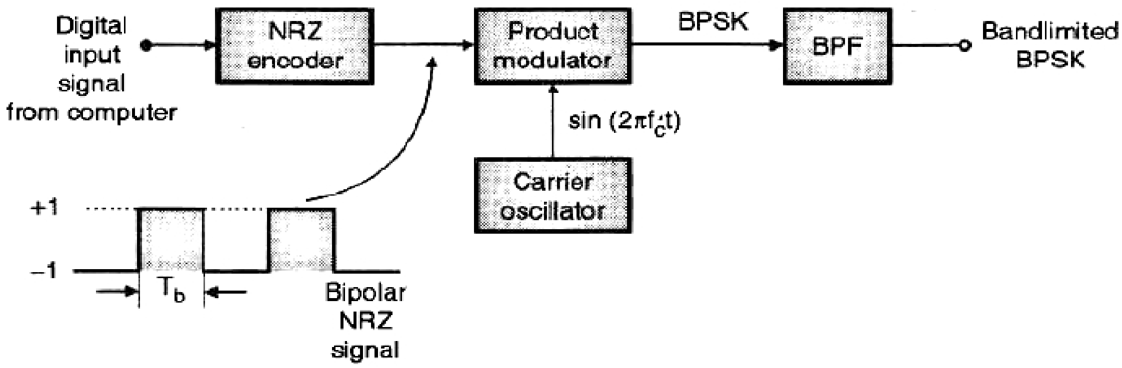


Figure-55 BPSK generation

Advantages:

- BPSK has a bandwidth which is lower than that of a BFSK signal.
- BPSK has the best performance of all the systems in presence of noise. It gives the minimum Possibility of error.
- BPSK has very good noise immunity.

Disadvantage:

- Generation and detection of BPSK is not easy.

Applications:

- Phase shift keying is the most efficient of the three modulation methods and it is used for high bit rates even higher than 1800 bit/sec.
- Due to low bandwidth requirement the BPSK modems are preferred over the FSK modems, at higher operating speeds.

6.9.10 Comparison of Binary Modulation Systems:

Sr. No.	Parameter	Binary ASK	Binary FSK	Binary PSK
1.	Variable characteristic.	Amplitude	Frequency	Phase
2.	Bandwidth (Hz)	2R	$\lvert f_1 - f_0 \rvert + (1 + r) f_b$	$(1 + r) f_b$
3.	Noise immunity.	low	high	high
4.	Error probability	high	low	low
5.	Performance in presence of noise.	Poor	Better than ASK	Better than FSK
6.	Complexity	Simple	Moderately complex	Very complex
7.	Bit rate	Suitable upto100 bits/sec.	Suitable upto about 1200 bits/sec.	Suitable for high bit rates.
8.	Detection method.	Envelope	Envelope	Coherent

Figure-56 Comparison of Binary Modulation Systems

6.10 GPS (Global Positioning System):

- GPS allows a person to determine the time and his exact location (in terms of latitude, longitude and altitude) anywhere on the earth.
- The GPS uses MEO (Medium Earth orbit) satellite system in which multiple satellites are orbiting at an altitude of about 18000 km above the earth.
- GPS operates on the principle of triangulation.
- The practical GPS system uses 24 satellites in six orbits.
- The orbits and the location of the satellites in each orbit are designed in such a way that at any instant of time, four satellites are visible from any point on the earth. This will satisfy the necessity of four spheres required for exact positioning.
- A GPS receiver can also show your position on the map.

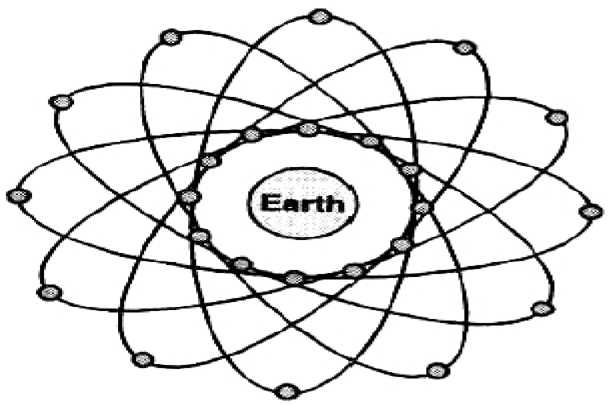


Figure-57 Practical GPS

Applications of GPS:

- GPS is used by military forces.
- Navigation. The driver of a car can find the location of his vehicle.
- GPS gives the location and database uses this information to find a path to the destination.
- Surveying
- Geological applications.

QUESTIONS

MCQ

1) In the given pulse modulations, which one is not the type of pulse modulation?
   (a) PWM, (b) PSK, (c) PPM, (d) PAM
2) A radio station transmitting AM wave with 1 MHz frequency band having a wavelength of ________.
   (a) 3 meter, (b) 300 meter, (c) 0.3 meter, (d) 30 meter
3) Commercial FM radio broadcasting utilizes a frequency band ______.
   (a) 90 MHz to 110 MHz, (b) 70 MHz to 120MHz, (c) 110 MHz to 180MHz, (d) 88MHz to 108MHz
4) In Which process Sampling is used?
   (a) Frequency Division, (b) Signal amplification, (c) Signal attenuation, (d) Digital Modulation
5) Minimum bandwidth of an AM wave is ________.
   (a) fm (b) 2fm (c) 0.5fm (d) 4fm
6) PCM is a _________Pulse Modulation Technique.
   (a) Analog (b) Digital (c) Hybrid (d) None of Above
7) The first cellular systems were ___________.
   (a) Analog (b) Digital (c) Hybrid (d) None of Above
8) Electrical energy at a frequency of 1454 Hz is in what frequency range?
   (a) Radio (b) Audio (c) High (d) Super-high
9) In a frequency modulation receiver, the __________ is in between the antenna and the mixer.
   (a) audio frequency amplifier (b) high frequency oscillator
   (c) intermediate frequency amplifier (d) radio frequency amplifier

Descriptive Questions

1) Explain in detail Pulse modulation with necessary diagrams.
2) Explain in brief Product Modulation and Demodulation with necessary diagrams.
3) Write short not on Cellular communication system.
4) Define Waveguide, Transmission lines and Antenna.
5) Define The following Terms. (a) Interference (b) Noise Margin
6) What do you understand about multiplexing? Explain any one of the Multiplexing technique.
7) Draw & Explain the functional description of digital communication system in brief.
8) Draw block diagram of Pulse code Modulation.
9) What do you understand about isotropic radiator?
10) Classify the standard based on 2G & 3G.
11) What do you understand about frequency reuse concept & Why it is used in cellular system?
12) Compare DSB-FC, DSB-SC, SSB, VSB.

13) Explain in brief cellular concept in mobile radio system.
14) State the need of modulation and what are the other advantages of modulation in communication system?
15) Define the following terms: (a) Reflection (b) Directivity (c) Isotropic Radiator
16) What is transmission medium? What are the different types of transmission medium?
17) Explain transmission line and waveguide.
18) Explain parameters of antenna and also explain Yagi-Uda antenna.
19) What is modulation? Explain AM modulation and demodulation
20) Difference between AM, FM, & PM and analog & digital comm. system
21) Explain super-heterodyne receiver with block diagram.
22) What is digital communication? Also explain PCM.
23) Explain GPS.
24) Explain cellular system.

CHAPTER 7
BASIC CONTROL SYSTEMS

TOPICS COVERED IN THIS CHAPTER

1. **Introduction**
2. **Classification of Control System**
3. **Classification of Feedback Control System**
4. **Block Diagram Reduction**
5. **Transfer Function**
6. **Introduction to Time Response**
7. **Classification of Various Control Action (P, I, D, PI, PD & PID)**
8. **Digital Control System**
9. **A Case Study : Digital Process Control**
10. **Questions**

7.1 INTRODUCTION:

- In modern times control system play a very important role in our daily life.
- From a simple bread toaster to a complex modern power plant, there is a series of control principles that affect our life.
- Advanced examples of launching a satellite, regulating the generation in a power plant, tracking an enemy plane on radar etc.
- The principle of control theory is applicable to engineering as well as non-engineering fields.

➢ **Important Definitions:**

- **System:-**
  "A system is an arrangement of or a combination of different physical components connected or related in such a manner so as to form an entire unit to attain a certain objective."
  Thus a system is a collection of objects etc. in such a manner so as to achieve an aim or output. Thus a system has an input, an output and a way to achieve this input-output combination.
- **Output:-**
  The actual response obtained from a system is called output.
- **Input:-**
  The stimulus or excitation applied to a control system from an external source in order to produce the output is called input.
- **Control:-**
  It means to regulate, direct or command a system so that the desired objective is attained.

  Combining the above definitions
- **Control System:-**
  It is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself to achieve a certain objective. (or command some other system to achieve a certain objective.)

- **Error Detector**
  It is the comparator which compares the output of the feedback element with the reference input signal and the difference of these two signals appears as the output of the error detector.

- **Feedback Element**
  Feedback element converts the controlled variable to a form which is comparable to the reference input signal. For e.g., if the reference input is voltage and the controlled variable is speed, then the feedback element (called transducer) converts speed into voltage which is compared with the reference input signal which is also in the form of voltage.

- **Feedback Control**

  The feedback control is an operation where the output is fed back to the input. The feedback output may be in-phase or out-of-phase with respect to the input and accordingly it is termed positive or negative feedback respectively. All control systems are usually negative feedback systems. The oscillators are the examples of positive feedback systems. In a negative feedback system, the difference between the reference input and the output produces an error which is reduced gradually to achieve the desired output in accordance with the reference input signal.

➢ **Requirements of a Good Control System:-**

1) Accuracy:

Accuracy is very high as any error arising should be corrected. Accuracy can be improved by using feedback element. Because of feedback element, system becomes closed loop system.
In closed loop control system, steady state error tends to zero. To increase the accuracy error detector should be present in control system.

2) Sensitivity:

A control system senses changes in output due to environmental or parametric changes, internal disturbance or any other parameters and corrects the same.
Any control system should be insensitive to such parameters but very sensitive to the input signal.

3) Noise:

Noise is an undesired input signal. A good control system should be in sensitive to such input signal. A good control system should be able to reduce the effects of wise or undesired input.

4) Stability:

Stability means bounded input and bounded output. In the absence of the time increase. A good control system response is stable for all variations.

5) Bandwidth:

Operating frequency range decides the bandwidth of any system. For frequency response of good control system, bandwidth should be large.
The required output means maximum possible output without overshoots and it should be stable for required input frequency.

6) Speed:

A good control system should have high speed. That is output of system should be as fast as possible.

7) Oscillation:

For a good control system, oscillations of output should be constant or sustained oscillation which follows the barkhausein's criteria.

7.2 CLASSIFICATION OF CONTROL SYSTEMS:

- "Control action is that quantity responsible for activating the system to produce the output"
  (1) Open loop control system
  (2) Close loop control system

1. Open loop control system:

- "A system in which the control action is totally independent of the output of the system is called as open loop system"

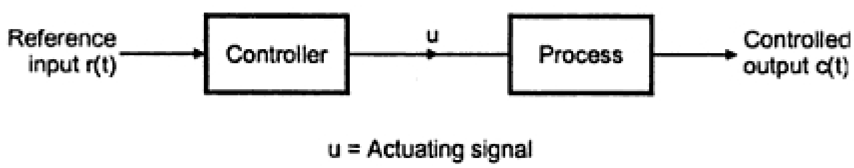


Figure 7.1: Open loop control system

- Refer figure 7.1, Reference input r(t) is applied to the controller which generates the actuating signal u(t). u(t) actuates the process to give controlled output c(t). The Control action has nothing to do with status of output c(t). Hence system is open loop.

- **Traffic Light Controller**

  A traffic flow control system used on roads is time dependent. The traffic on the road becomes mobile or stationary depending on the duration and sequence of lamp glow. The sequence and duration are controlled by relays which are predetermined and not dependent the rush on the road.

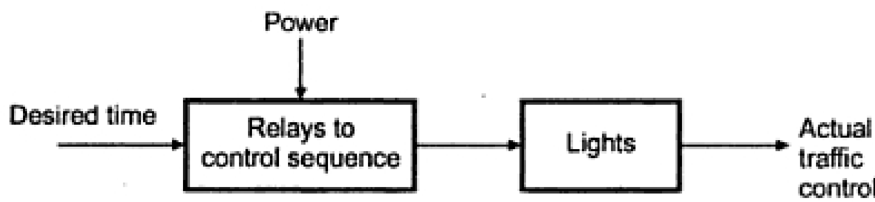
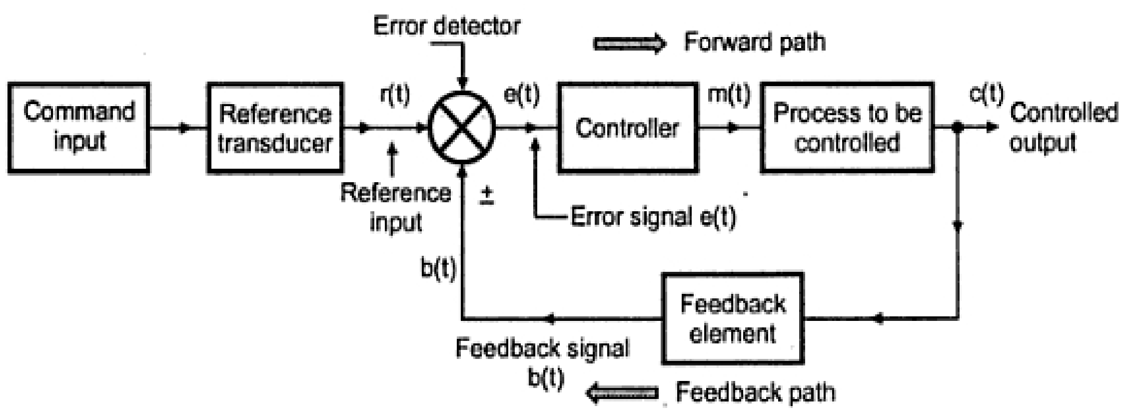


Figure 7.2: Traffic Light Controller

- **Other Example:** Automatic hand driver, Automatic washing machine, bread toaster, automatic coffee server, automatic milk server, electric lift, theatre lamp dimmer, cold drinks-milk bottling etc.

2. Close loop control system:

- "A system in which the control action is somehow dependent on the output is called as close loop system" or
- "Feedback is that property of the system which permits the output to be compared with the reference input so that appropriate control action is formed"



Figue 7.3 Close Loop Control System

➢ **Home heating system**

- In this system, the heating system is operated by a valve. The actual temperature is sensed by a thermal sensor and compared with the desired temperature. The difference between the two, actuates the valve mechanism to change the temperature as per the requirement.

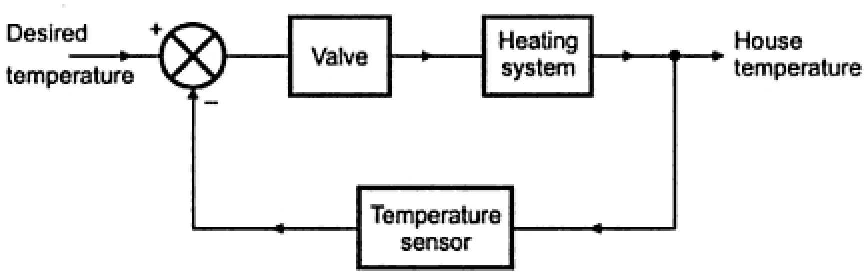


Figure 7.4: Domestic Heating System

- **Other Example:** automatic electric iron, DC motor speed controlled by tacho feedback ,railway reservation status display, missile launched and auto tracked by radar, Servo voltage stabilizer, water level controller etc…

➢ **Advantages:**

- Accuracy of such system is always very high because controller modifies and manipulated the actuating signal such that error in the system will be zero.
- Such system senses environmental changes, as well as internal disturbances and accordingly modifies the error
- In such system, there is reduced effect of nonlinearities and distortions.
- Bandwidth of such system i.e. operating frequency zone for such system is very high.

➢ **Disadvantages:**

- Such systems are complicated and time consuming from design point of view and hence costlier.
- Due to feedback, system tried to correct the error from time to time. Tendency to overcorrect the error may cause oscillations without bound in the system. Hence system has to be designed taking into consideration problems of instability due to feedback. The stability problems are severe and must be taken care of while designing the system.

1.2.1 Difference between closed loop and Open loop control system

Open Loop Control system	Close Loop Control system
No feedback element. Hence feedback elements absent	Feedback exists. Hence feedback elements exists
No error detector	Error detector is present
It is inaccurate	It is accurate
Highly sensitive to parameter change	Less sensitive to parameter changes
Small bandwidth	Large bandwidth
Stable	May become unstable

Economical	Costly
Example : Coffee maker	Example : Temperature control of oven

7.3 CLASSIFICATION FEEDBACK CONTROL SYSTEMS:

➢ **According to the method of analysis and design**

1. Linear Control system

- Linear feedback control system are idealized models that are conceived by the analyst for the sake of simplicity of analysis and design.
- For the design and analysis of linear systems there exist a wealth of analytical and graphical techniques.

2. Non Linear control system

- Nonlinear system are very difficult to analysis mathematically, and there are no general methods that can be used for a broad class of nonlinear system.

3. Time-invariant systems

- When the parameters of a control system are stationary with respect to time during the operation of the system, the system is known as a time-invariant system.

4. Time varying system

- A system in which certain quantities governing the system's behavior change with time, so that the system will respond differently to the same input at different times.

➢ **According to the types of signal found in the system**

1. Continuous time system

- A continuous time system is one in which the signals at various parts of the system are all functions of the continuous time variable t.

2. Discrete-time System

- A discrete time system is one in which the signal at various parts of the system are all functions of the discrete time variable t.

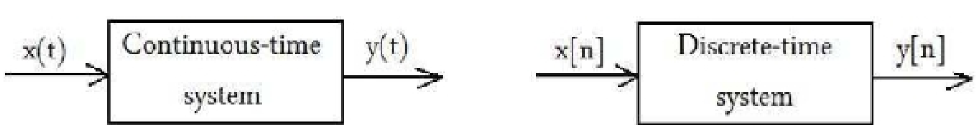


Figure 7.5: Continuous and Discrete time System

➢ **According to the type of system components**

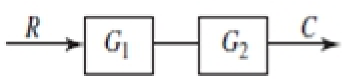
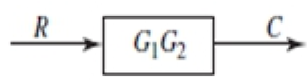
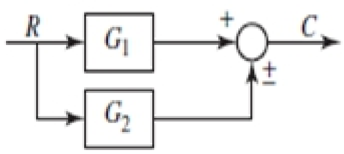
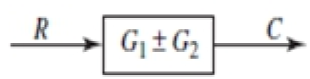
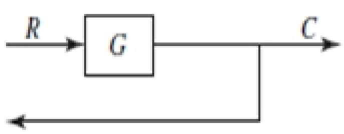
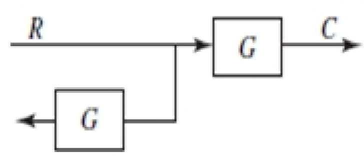
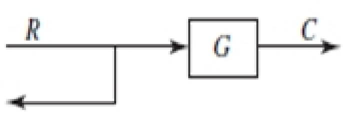
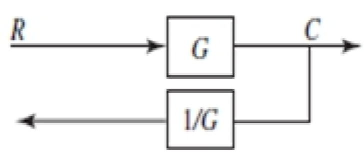
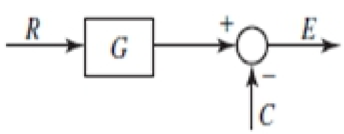
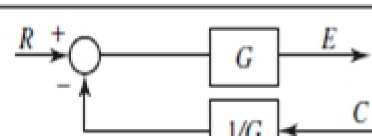
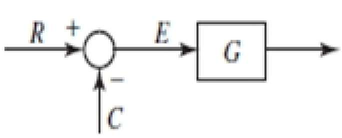
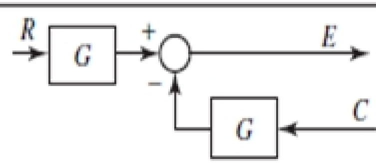
- Electromechanical control system
- Hydraulic control system
- Pneumatic control system
- Biological control system

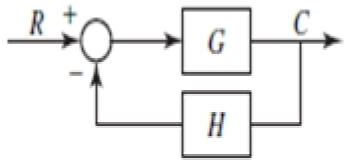
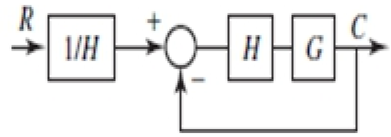
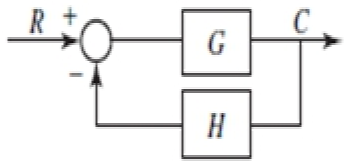
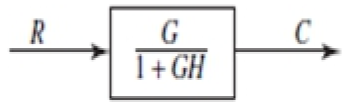
➢ **According to the main purpose of the system**

- Position control system : Here the output position, such as the shaft position on motor exactly follow the variations of the input position
- Velocity control system
- Regulators : Speed control of a motor
- Servomechanisms : Automobile Power steering

7.4 BLOCK DIAGRAM REDUCTION:

Table 7.1: Block Diagram Reduction

Original Block Diagram	Manipulation	Modified Block Diagram
	Cascaded elements	
	Addition or subtraction (eliminating auxiliary forward path)	
	Shifting of pick-off point ahead of block	
	Shifting of pick-off point behind block	
	Shifting of summing point ahead of block	
	Shifting of summing point behind block	

	Removing H from feedback path	
	Eliminating feedback path	

7.5 TRANSFER FUNCTION:

- ***Transfer function** is defined as the ratio of the Laplace transform of output to the Laplace transform of input with all initial conditions as zero*. The concept of transfer function is applicable to single-input-single-output, linear time-invariant systems. The dynamics of linear time-invariant system are represented by a linear differential equation such as:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_1 \frac{du(t)}{dt} + b_0 u(t)$$

➢ **Derivation of Transfer Function of Closed Loop Control System**

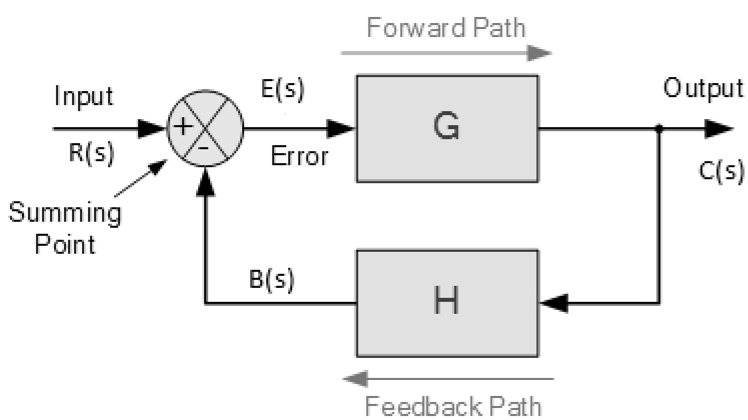


Figure 7.6: Simple form of Closed Loop Control System

- In the above fig., the control system is **negative feedback closed loop control system.** The feedback is added negatively to the reference input. Various signals are,
  R(s) = Reference Input Signal
  E(s) = Error Signal
  B(s) = Feedback Signal
  C(s) = Output Signal
  G(s) = Transfer Function of Forward path
  H(s) = Transfer Function of Feedback path

Now,

$$C(s) = G(s) \cdot E(s)$$

Also,

$$E(s) = R(s) - B(s) \text{ and } B(s) = C(s) \cdot H(s)$$

$$\therefore E(s) = R(s) - C(s) \cdot H(s)$$

$$\therefore C(s) = G(s) \cdot [R(s) - C(s) \cdot H(s)]$$

$$\therefore C(s) = G(s) \cdot R(s) - G(s) \cdot C(s) \cdot H(s)$$

$$\therefore C(s) + G(s) \cdot C(s) \cdot H(s) = G(s) \cdot R(s)$$

$$\therefore C(s) [1 + G(s) \cdot H(s)] = G(s) \cdot R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s) \cdot H(s)}$$

- If there is a **positive feedback closed loop control system**, then the transfer function will be,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s) \cdot H(s)}$$

7.6 INTRODUCTION TO TIME RESPONSE:

- In time domain analysis, time is independent variable. When a system is given an excitation (input), there is a response (output). This response varies with time, is called the time response.
- Generally the response of any system has two type.
  1. Transient Response
  2. Steady state Response

1. Transient Response

- That part of the time response which goes to zero as time becomes very large is called as transient response. It is denoted by $c_t(t)$.

2. Steady state Response

- It is basically the final value achieved by the system output.
- "The part of response that remains after the transient have died out is called as steady state response"

3. **Steady state error :**

- "The difference between desired output and actual output of system is called as steady state error (e_{ss})"

7.6.1 Transient Response specifications

- In many practical cases, the desired performance characteristics are specified in terms of *time-domain quantities*.
- Frequently, the performance characteristics of a control system are specified in terms of the transient response to a *unit-step input* since it is easy to generate and is sufficiently drastic.

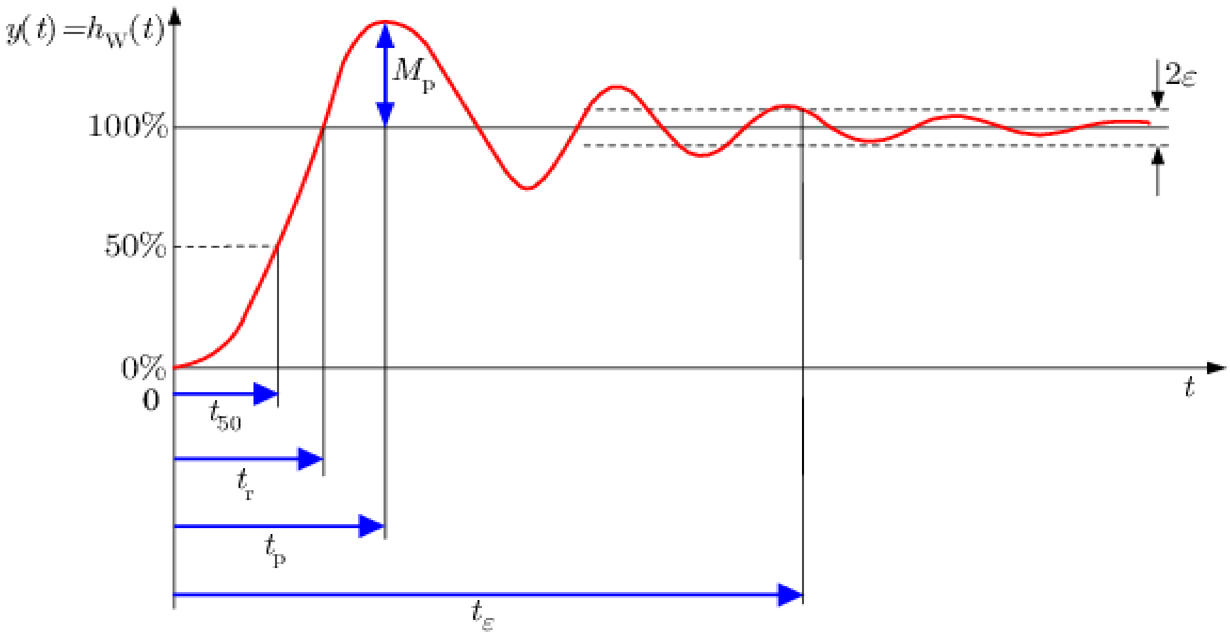


Figure 7.7: Transient Response

1) **Delay Time (T_d)**: It is the time required for the response to reach 50% of the final value in the first attempt.
2) **Rise Time (T_r)**: The time required for the response to rise from 10% to 90% of the final value for over damped systems. (To rise from 0% to100% of the final value for under damped systems.)
3) **Peak time (T_p):** It is the required for the response to reach the first peak.
4) **Peak Overshoot (M_p)**: The maximum overshoot is the maximum peak value of the response curve measured from unity. It is therefore the largest error between input and output during the transient period.
5) **Settling Time (T_s)**: It is the time required for the response curve to reach and stay within a specified percentage of the final value.

7.7 CLASSIFICATION OF VARIOUS CONTROL ACTION:

- Various types of control systems are discussed in the earlier sections. In this section the control systems are discussed from the view point of the controlling action. Various types of systems result.
- These may be
    1. ON-OFF type control
    2. Proportional control (P)
    3. Derivative control (D)
    4. Integral control (I)

5. Proportional Derivative control (PD)
6. Proportional Integral control (PI)
7. Proportional Integral Derivative control (PID)

1. ON-OFF Control:

In this there are two operating states ON and OFF. The correction depends only on the direction of the error and not on its magnitude. As for example in ON-OFF type temperature control system of a furnace, the power is switched on, whenever the temperature is found to be lower than the desired. Another example is water level controller.

2. Proportional control:

In proportional control system, the control action is proportional to the error. The error is detected and amplified and control is applied. Greater is the control. The block diagram is shown.

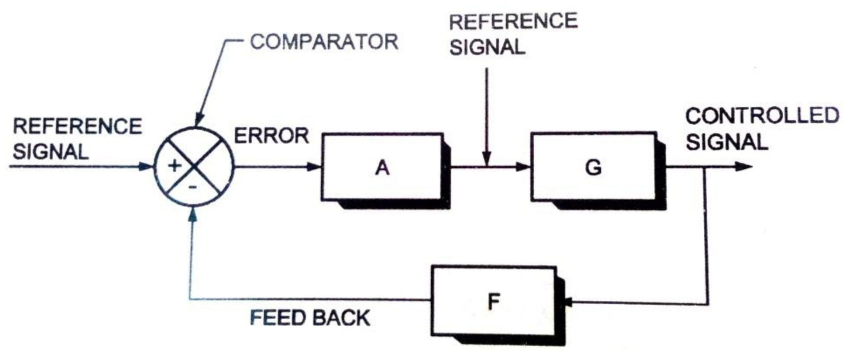


Figure 7.8: Proportional Control

3. Derivative control:

In this type of controller the output of the controller is proportional to the time derivative of the error i.e. the rate of change of error. In this the response becomes rapid for rapidly changing error. This is not suitable for slowly changing error as it causes drift.

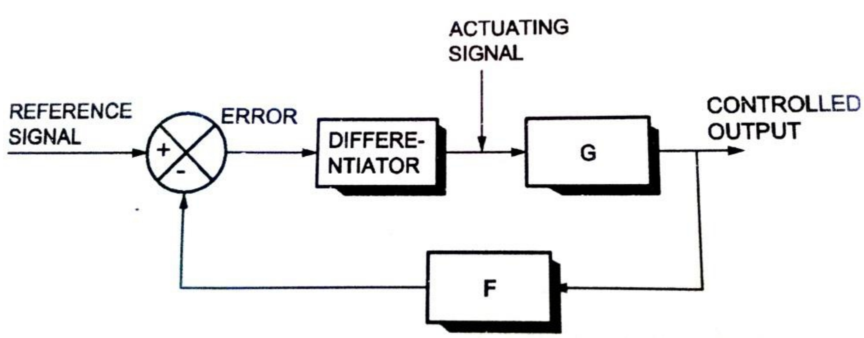


Figure 7.9: Derivative Control

4. Integral control:

In this type of control the controller output is proportional to the time integral of error or the rate of change of output is proportional to the error. This eliminates drift but there will be oscillations or hunting in the system.

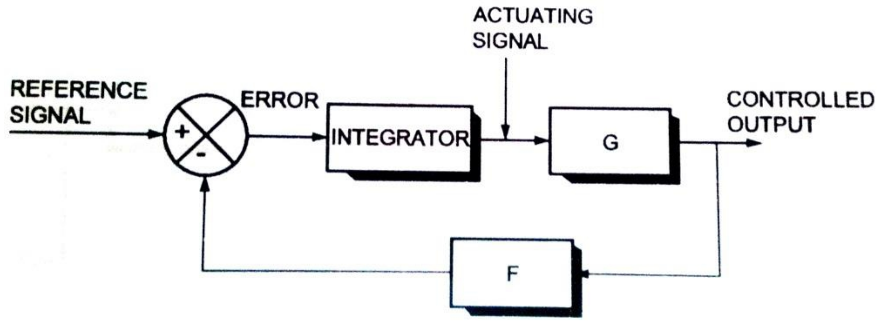


Figure 7.10: Integral Control

It is seen that each of above controls has some limitations. So normally a combined action is used.

5. Proportional Derivative Control (PD):

In this type of control part of the control is proportional to the error and the part of the control is proportional to the time derivative of the error. The block diagram is shown in figure.

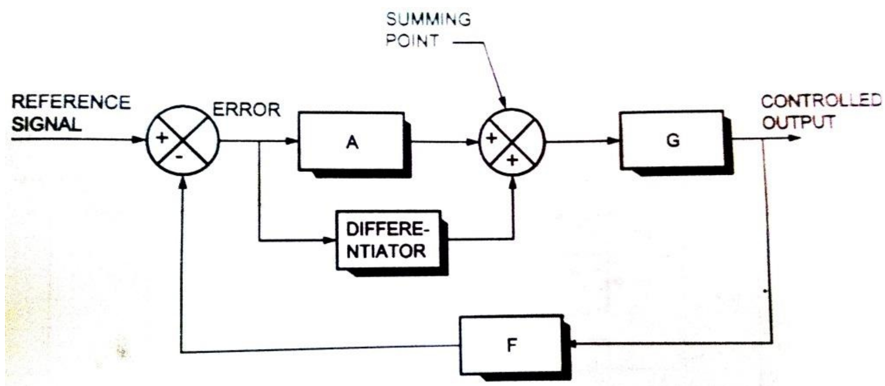


Figure 7.11: Proportional Derivative Control

In this case the oscillations are damped but the steady state error remains.

6. Proportional Integral Control (PI):

In this the part of the control signal is proportional to the error and part of the signal is proportional to the time integral of the error signal.

Due to the proportional component the simplicity is improved. Due to integral action, the steady state error is reduced. But the stability is reduced as the system will oscillate.

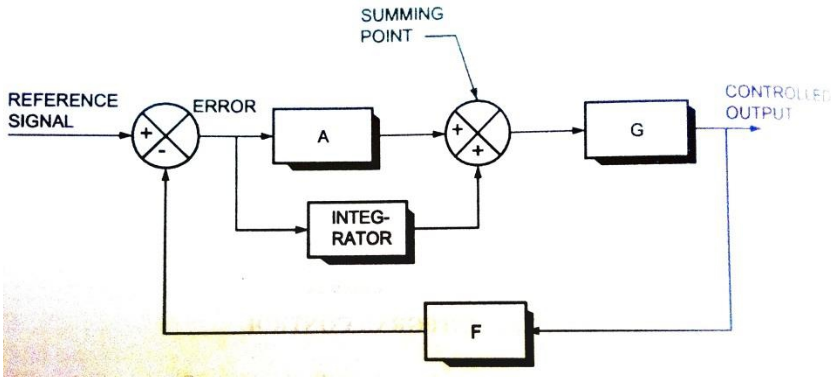


Figure 7.12: Proportional Integral Control

7. Proportional Integral and Derivative Control (PID):

In this case the control action comprises of all the three types i.e. proportional, integral and derivative. The block diagram is shown in figure.

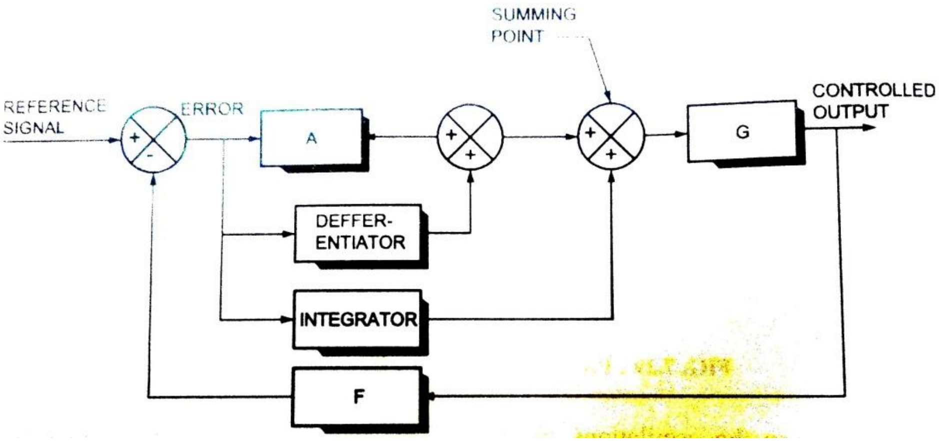


Figure 7.13: Proportional Integral and Derivative Control

Figure 7.14 and 7.15 show the characteristics of various controller actions.

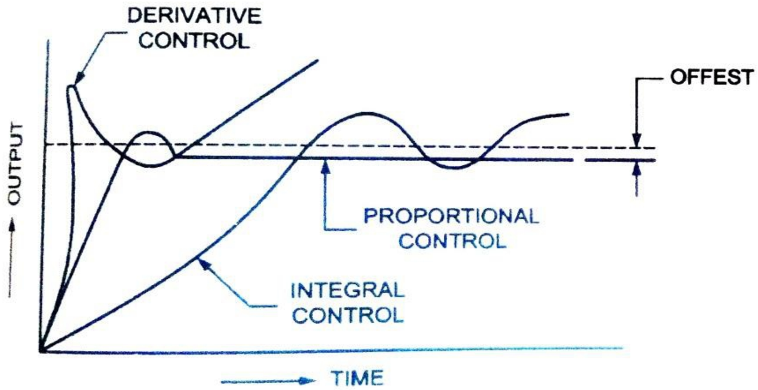


Figure 7.14 Characteristics of Controllers

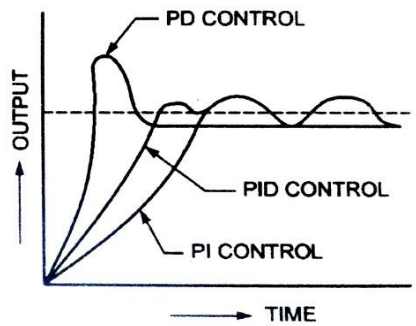


Figure 7.15 Characteristics of Combined Controllers

7.8 DIGITAL CONTROL SYSTEM:

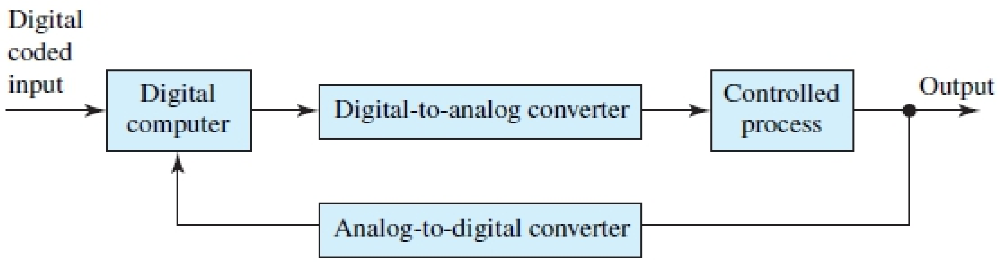


Figure 7.16: Typical digital control system

- Figure 7.16 illustrates a typical digital control system, in which the signal at one or more points of the system is expressed in a numerical code for digital-computer or digital-transducer processing in the system.
- Because of the digitally coded (such as binary-coded) signals in some parts of the system, it becomes necessary to employ digital-to-analog (D/A) and analog-to-digital (A/D) converters.
- In spite of the basic differences between the structures and components of a sampled-data and a digital control system, from an analytical standpoint both types of systems are treated by the same analytical tools.

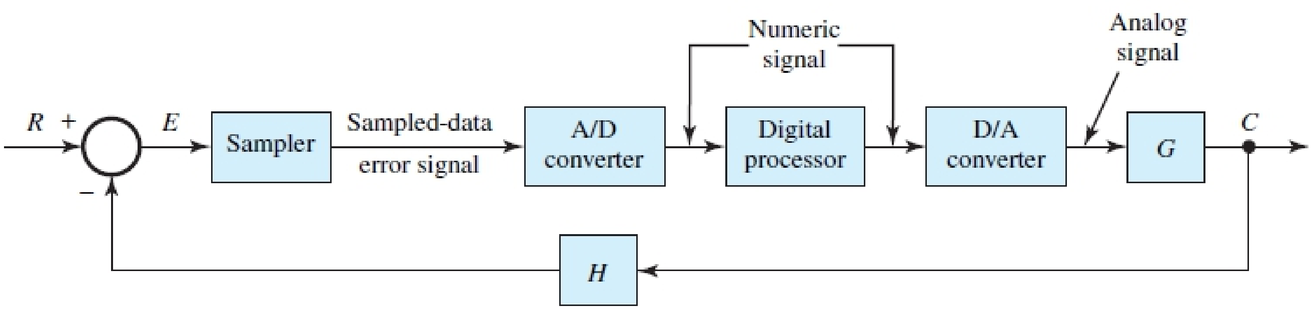


Figure 7.17: Block diagram of a type of digital control system

- The block diagram of Figure 7.17 is a functional representation of a type of digital control system, in which G and H serve the same function as in any feedback system.
- Note that the error signal is sampled and a digital processor is used.
- The controller in this system is the digital processor whose output, reconverted to an analog signal, becomes the excitation for the block G. As usual, G is the subsystem that provides the output to be controlled.
- A central computer which controls several functions could be used as a digital processor; or a microprocessor (special purpose computer) designed for the particular control function may also be used as a digital processor.

- Large, high-speed computers with their speed, memory, and computational ability, as well as programmability, are utilized for central control in large automated manufacturing facilities.

Sampled data and digital control offer several advantages over analog systems:

- More compact and lightweight
- Improved sensitivity
- Better reliability, speed, and accuracy
- More flexibility and versatility (in programming)
- Lower cost
- More rugged in construction
- No drift
- Less effect due to noise and disturbance.

Example of Digital Control System:

Missile Launching and Guidance System:

- The missile launching and guidance system shown in figure which is an example of military application of feedback control.

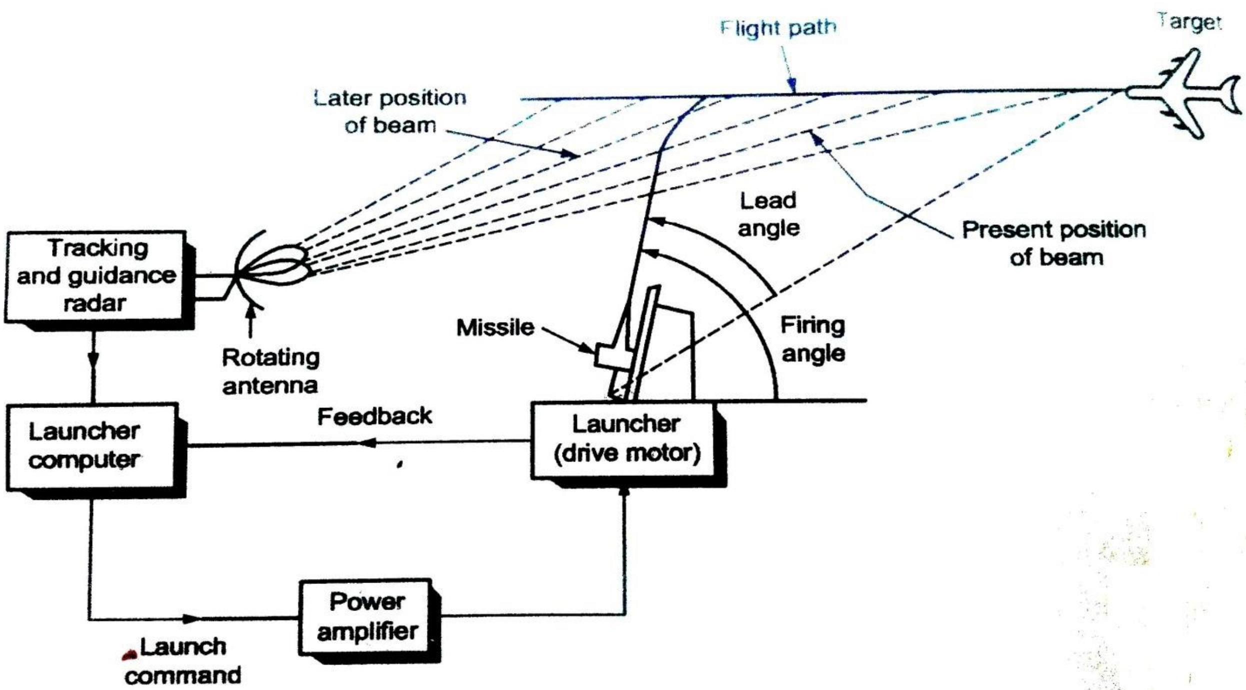


Figure 7.18

- The radar detects the presence of the target aircraft through its rotating antenna and passes the detection signal to the computer indicating the velocity and position of the target.

- The computer calculates the firing angle which is the launch command signal.
- This command signal is passed to the launcher i.e. the drive motor, through the power amplifier. The launcher angular position is feedback to the launch computer and the missile is fired.

Aircraft Control System:

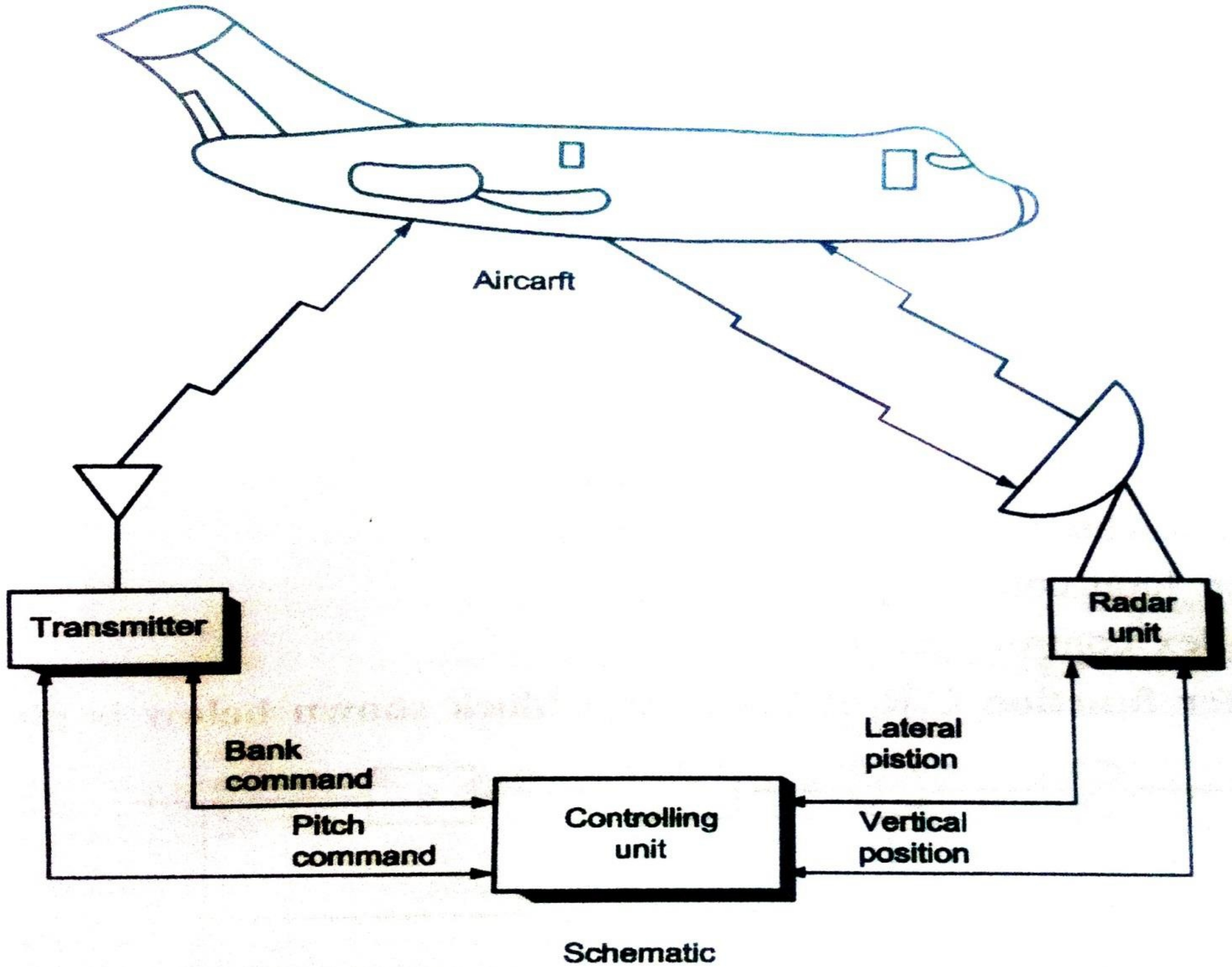


Figure 7.19

- The system consists of three basic parts: The aircraft, the radar unit and the controlling unit.
- The radar unit measures the approximate vertical and lateral positions of the aircraft, which are then transmitted to the controlling unit. From these measurements, the controlling unit calculates appropriate pitch and bank commands.
- These commands are then transmitted to the aircraft autopilots which in turn cause the aircraft to respond.

7.9 A Case Study : Digital Process Control

- Figure 7.20 shows a block diagram for microcomputer-based control of a physical process, such as a chemical plant. A slight variation of the system can be used for automotive instrumentation in which sensors furnish various signals for speed, fuel reserve, battery voltage, oil pressure, engine temperature, and so on.

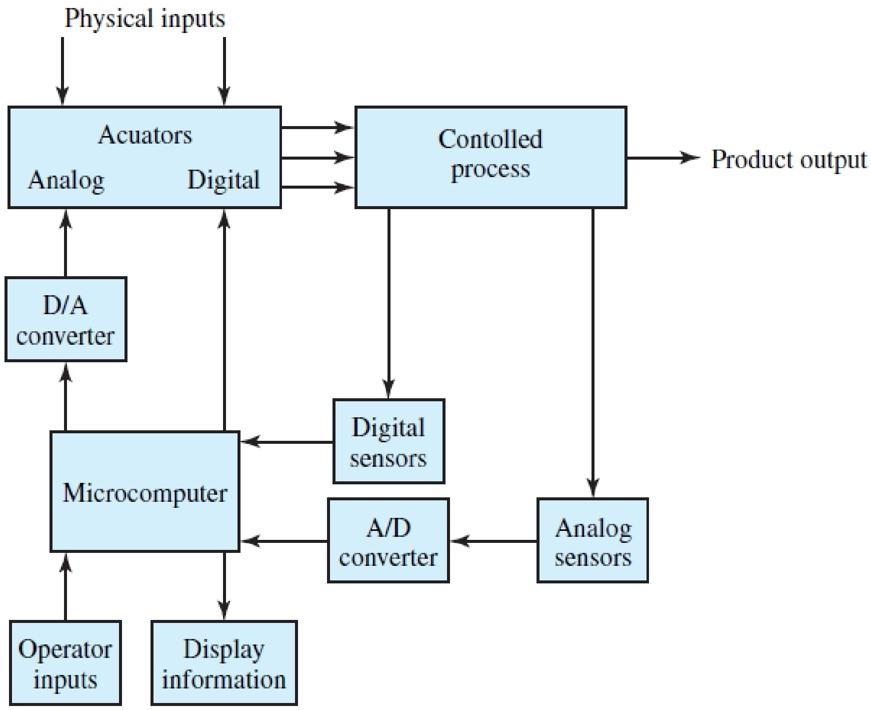


Figure 7.20 block diagram for microcomputer-based control of a physical process

- The data are presented to the driver in one or more displays on the dashboard. In a physical process on the other hand, based on the display information, an operator can assess and direct the operation of the control process through a keyboard or other input devices to the microcomputer.
- Various physical inputs, such as power and materials, are regulated by actuators, which are in turn controlled by the microcomputer.
- Electric signals related to the controlled-process parameters, such as pressure and temperature, are produced by various sensors, which in turn feed the information to the microcomputer. Actuators and sensors may be either analog or digital.
- Digital-to-analog (D/A) converters are used to convert the digital signals to analog form so as to suit the analog actuators, where as analog-to-digital (A/D) converters are employed to convert the analog sensor signals to digital form so as to suit the microcomputer.

- One can think of so many systems in daily practice controlled or monitored by microcomputers. Some examples include monitoring patients in intensive cardiac-care units of hospitals, nuclear-reactor controls, traffic signals, aircraft and automobile instrumentation, chemical plants, and various manufacturing processes.

QUESTIONS

1. What is control system? Explain open loop control system and close loop control system.
2. Compare between open loop control system and close loop control system.
3. Classify feedback control system in details.
4. What is transfer function? Give elementary block diagram of closed loop system and find its transfer function.
5. Defined a transient response and steady state response?
6. Defined following term: Delay time, Rise time, Peak time, Peak Overshoot, Setting time.
7. Explain P, I, D, PI, PD and PID controller.
8. Explain Digital control system with example.